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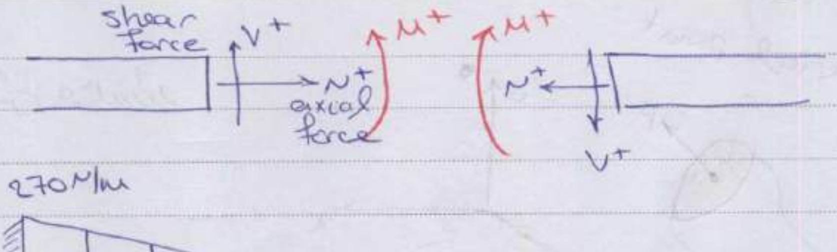
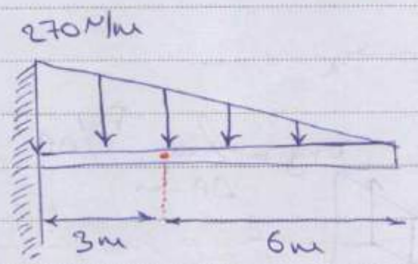
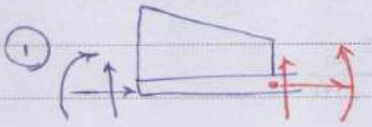
* مقاومت مصالح *

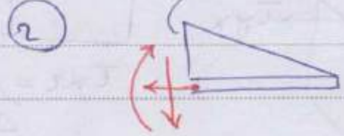




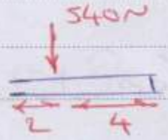
مثال :

$$\frac{\omega}{6} = \frac{270}{9} \Longrightarrow W = 180N$$





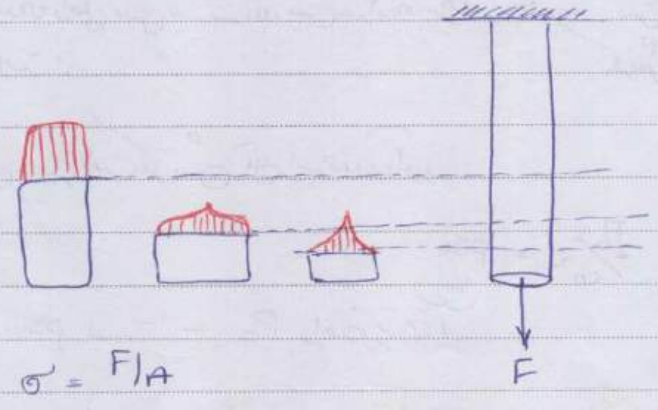
② سمت راست



$N = 0$

$V = 540N$

$M = -1080$ Nm



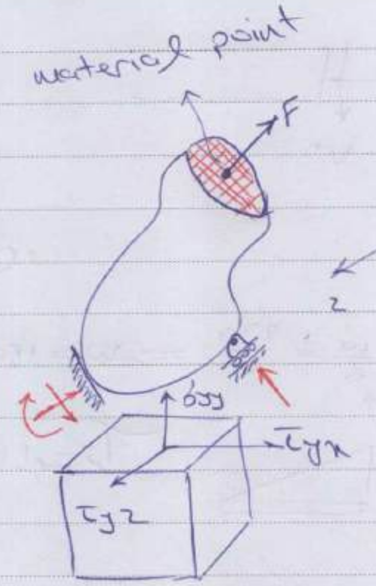
$\sigma = F/A$

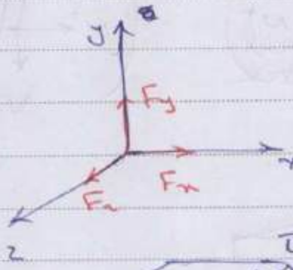
normal stress

تنش عمودی σ

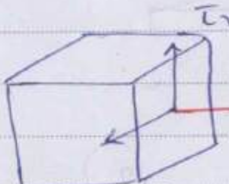
تنش برشی τ

تنش ۹ مؤلفه دارد





$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{F_y}{\Delta A}$$



$$\sigma_{xx} = \lim_{\Delta A \to 0} \frac{F_x}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{F_z}{\Delta A}$$

نقطه مادی آب از آهن بزرگتر است چون که چگالی آب کمتر از آهن است.

material point کوچکترین نقطه جسم است که خواص جسم را به همراه دارد

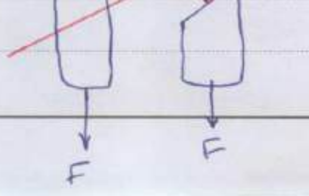
stress تنش ⟹ خود نیروهای داخلی تنش نیستند بلکه تنش مؤلفه‌های نیروی داخلی است.

strain کرنش

کرنش چیزی است که تنش آن را به وجود آورد.

$N/m^2 = Pa$ $\quad lb/in^2 = psi$

psi نسبت به Pa واحد بزرگتری است



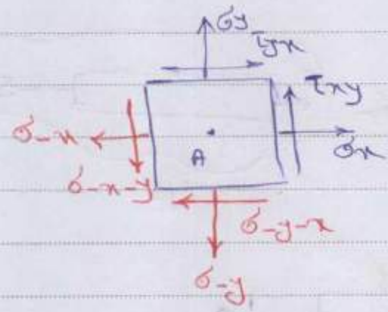
$\sigma = \frac{F_n}{A_0}$ تنش محوری

$\tau = \frac{F_s}{A_0}$ تنش برشی

تنش $= \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$

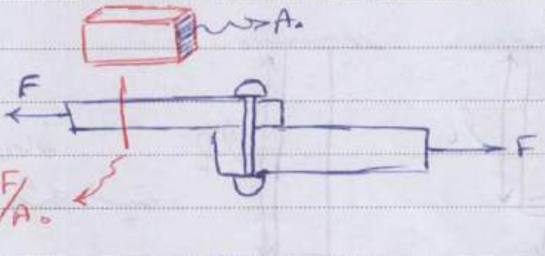
مولفه های تنش در حالت کلی (3 بعدی)

در حالت دو بعدی τ ها حذف می شوند



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z|_A = 0 \; * \end{cases} \Rightarrow \underset{*}{\tau_{xy}} = \tau_{yx}$$

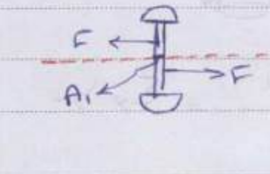
تنش $= \begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix}$



bearing stress
F/td

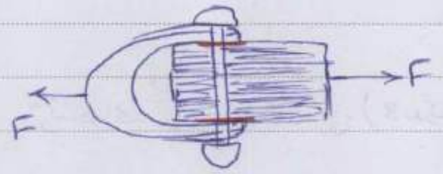
axial stress $\sigma = F/A_0$

single shear
shearing stress

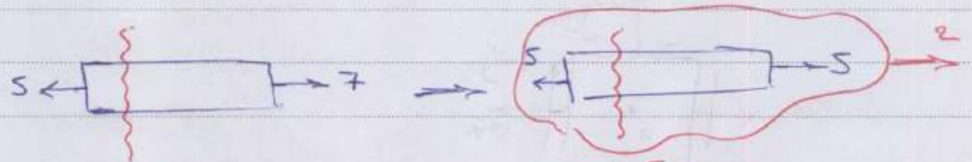


$\tau = F/A_1$

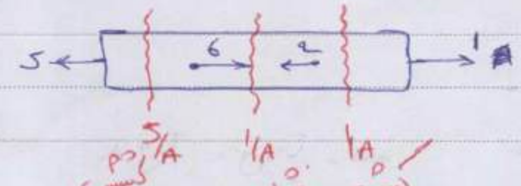
double shear



$$\tau = \frac{F/2}{A} = F/2A$$

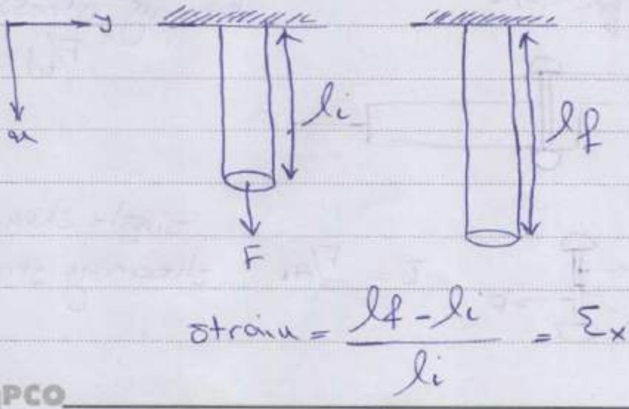


$$\sigma = S/A$$



$$\text{تنش مجاز (allowable stress)} = \frac{\text{تنش تسلیم (کاری)}}{\text{safty factor}}$$

کرنش:



حجم الزاما ثابت نمی ماند

چون ماده ایجاد شده دانسیته تغییر کند

$$\text{strain} = \frac{l_f - l_i}{l_i} = \varepsilon_x > 0$$

طول میله زیاد می شود

$$\frac{d_f - d_i}{d_i} = \varepsilon_y < 0$$

قطر میله کم می شود

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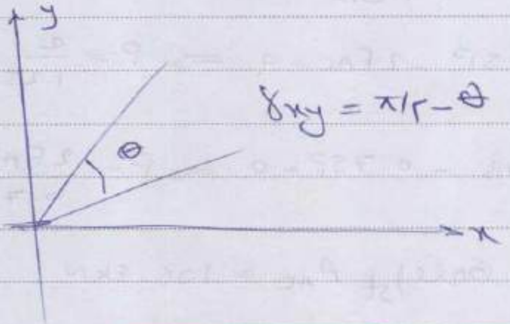
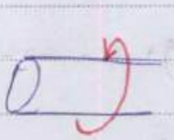
سه‌شنبه 12:30-13:30 کلاس حل تمرین

$$\text{کرنش} = \begin{pmatrix} \varepsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_z \end{pmatrix} \quad \text{3 بعدی}$$

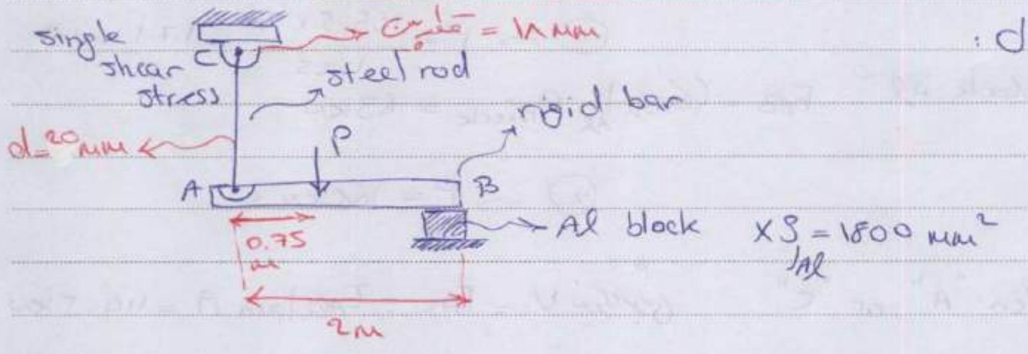
$$\text{کرنش} = \begin{pmatrix} \varepsilon_x & \gamma_{xy} \\ \gamma_{yx} & \varepsilon_y \end{pmatrix} \quad \text{2 بعدی}$$



$$\gamma_{xy} = \pi/2 - \theta$$



مثال:



$X S_{Al} = 1800 \text{ mm}^2$

$\sigma_F)_{st} = 680$ MPa

Failure

$\sigma_F)_{Al} = 70$ MPa

$\tau_F)_{pin} = 900$ MPa

مطلوبست $P_{MAX} = ?$

$S.F = 2$

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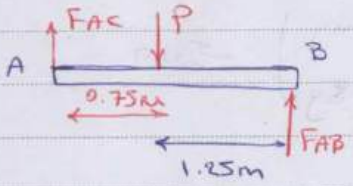
$\sigma_{All})_{st} = \frac{680}{2} = 340\ MPa \qquad \sigma_{All})_{Al} = 35\ MPa$

$\tau_{all})_{pin} = 450\ MPa$



$\sum M_B \curvearrowleft + = 0 \Rightarrow 1.25P - 2F_{AC} = 0 \Rightarrow P = \frac{2}{1.25} F_{AC}$ ①

$\sum M_A \curvearrowleft + = 0 \Rightarrow 2F_{AB} - 0.75P = 0 \Rightarrow P = \frac{2F_{AB}}{0.75}$ ②

"Rod AC" $F_{AC} = \sigma_{All})_{st} A_{AC} = 106.8kN$

① $\Rightarrow P = \frac{106.8 \times 2}{1.25} = 171\ kN$ *

"Block Al" $F_{AB} = (\sigma_{All})_{Al} A_{Block} = 63kN$

② $\Rightarrow P = 168\ kN$ *

Pin "A" or "C" نیروی برشی $V = F_{AC} = \tau_{All})_{pin} A = 114.5kN$

① $\Rightarrow P = \frac{11.4(2)}{1.25} = 183kN$ *

* $\Rightarrow P_{max} = 168\ kN$

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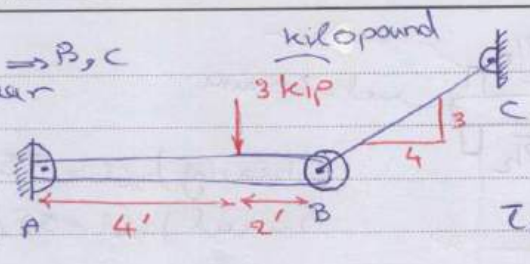
مثال:

Single shear ⇒ B, C



A ⇒ double shear

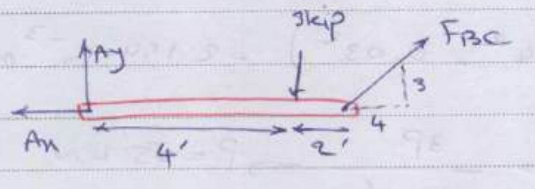
' ft

" inch

$\tau_{All})_{pin} = 12.5$ ksi $\frac{\text{k pound}}{\text{sq. inch}}$

$\sigma_{All})_{CB} = 16.2$ ksi

کوچکترین قطر پین و قطر میله BC را تعیین کنید تا $(\frac{1}{16}")$



* دقت، تنش برشی در نقطه A برشی دار
می شود ناشی کل نیروی A است نه فقط
مؤلفه x نیروی A

$\sum M_B \curvearrowleft^{+} = 0 \Rightarrow 6A_y = 3\times2 \Rightarrow A_y = 1$ kip

$\sum M_A \curvearrowleft^{+} 0 \Rightarrow 3\times4 = 6B_y \Rightarrow B_y = 2$ kip

$F_{BC} = 5/3\, B_y = 3.33$ kip

$A_x = 4/5\, F_{BC} = 2.67$ kip

$V_A = \sqrt{1^2 + (2.67)^2} = 2.85$ kip

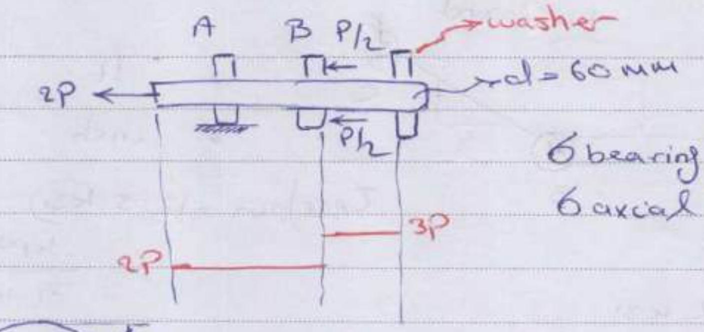
$A_A = \frac{2.85/2}{\tau_{All}} \Rightarrow d_A = 0.381"$

$A_C = \frac{3.33}{12.5} \Rightarrow d_B = 0.625"$

$A_{BC} = \frac{F_{BC}}{\sigma_{All}|_{BC}} = \frac{3.33}{16.2} \Rightarrow d_{BC} = 0.512"$

choose 0.5625"

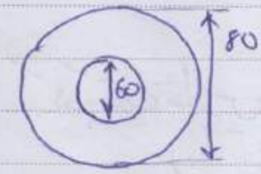
PAPCO



حل:

σ bearing)all = 75 MPa

σ axial)all = 55 MPa



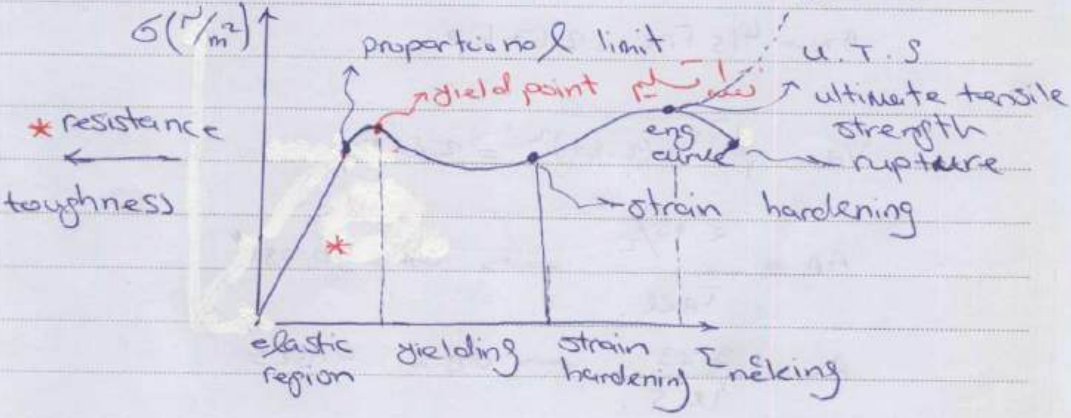
$$55 \times 10^6 = \frac{3P}{\pi(0.03)^2} \longrightarrow P = 51.8 \text{ kN}$$

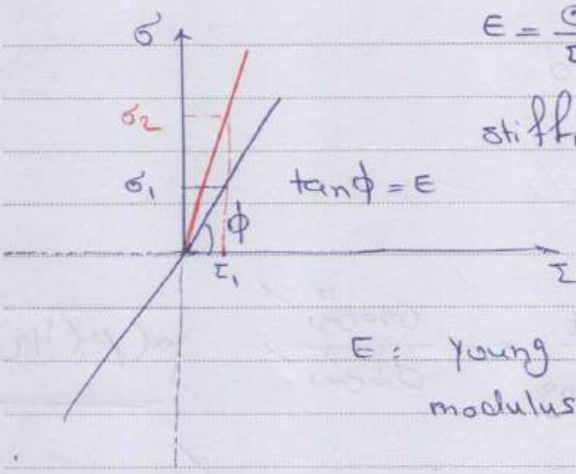
$$A_b = \pi(0.04^2 - 0.03^2) = 2.199 \times 10^{-3} \text{ m}^2$$

$$2.199 \times 10^{-3} = \frac{3P}{75 \times 10^6} \longrightarrow P = 55 \text{ kN}$$

$$P_{max} = 51.8 \text{ kN}$$

ductile materials





$E = \frac{\sigma}{\varepsilon} \longrightarrow \boxed{\sigma = E\varepsilon}$ Hook's Law

stiffness

$\tan\phi = E$

E: young modulus
modulus of elasticity

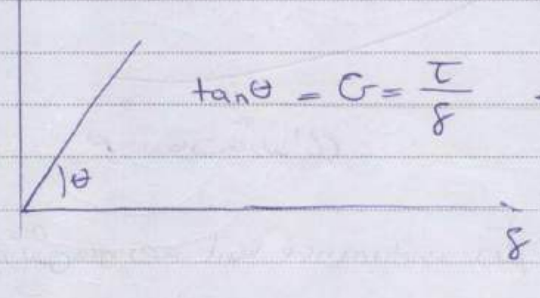
برای اغلب مصالح $E_c = E_t$

$E \equiv \frac{\sigma}{\varepsilon}$ N/m^2

strain energy ⟶ انرژی مصرف شده برای تغییر حجم ΔV

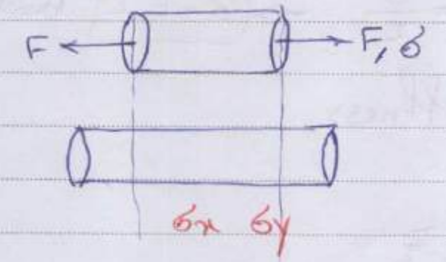
$\Delta U = \frac{1}{2}\,\sigma\,\varepsilon\,\Delta V = \frac{1}{2}\,\frac{F}{L^2}\,\frac{L}{L}\,L^3 = \underbrace{F.L}_{\text{کار = انرژی}}$

انرژی کرنشی واحد حجم (سطح زیر نمودار σ-ε) $= \frac{\Delta U}{\Delta V} = \frac{1}{2}\,\sigma\,\varepsilon = \frac{1}{2}\,\sigma\,\frac{\sigma}{E} = \frac{1}{2}\,\frac{\sigma^2}{E}$



$\tan\theta = G = \frac{\tau}{\gamma} \longrightarrow \boxed{\tau = G\gamma}$

shear elastic modulus



ضریب پواسون
poisson ratio

$$\mu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{\text{کرنش عرضی}}{\text{کرنش طولی}}$$

$$\boxed{0 < \mu < 1/2}$$

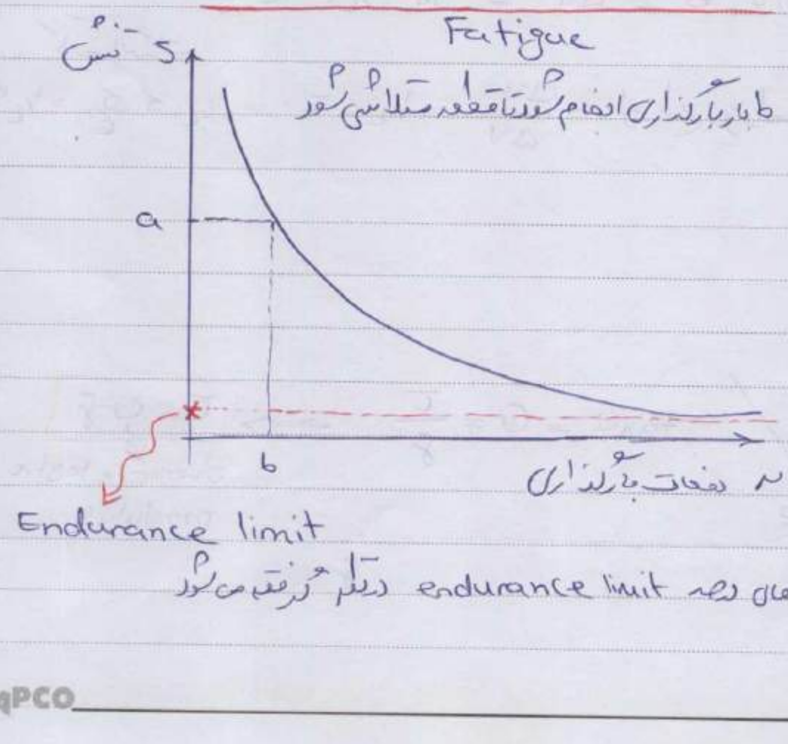
برای مواد غیرقابل تراکم (حجم ثابت) $\Longrightarrow \mu = 1/2$

$$G = \frac{E}{2(1+\mu)}$$

Fatigue

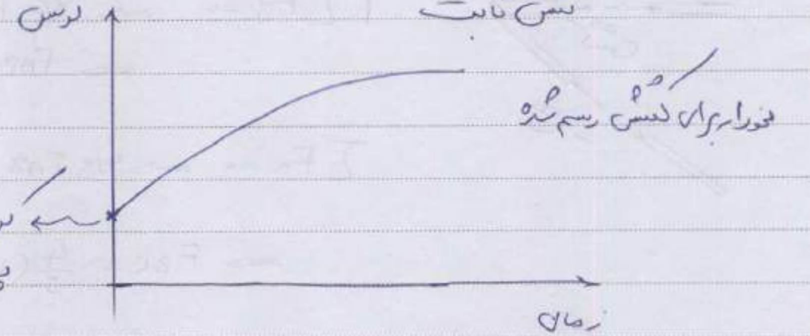
با این تنش a باید b بار بارگذاری انجام شود تا قطعه متلاشی شود

* S-N curve*



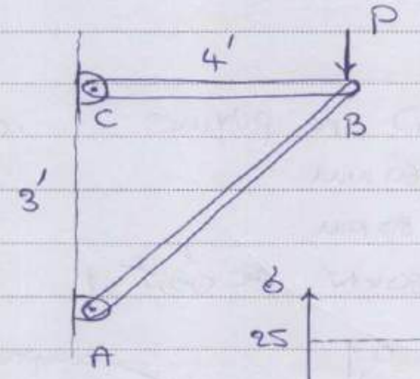
برای یک قطعه تنش‌های زیر endurance limit دچار ترک نمی‌شود

(Creep خزش)



$\varepsilon = \frac{1}{E}\sigma$

مثال:

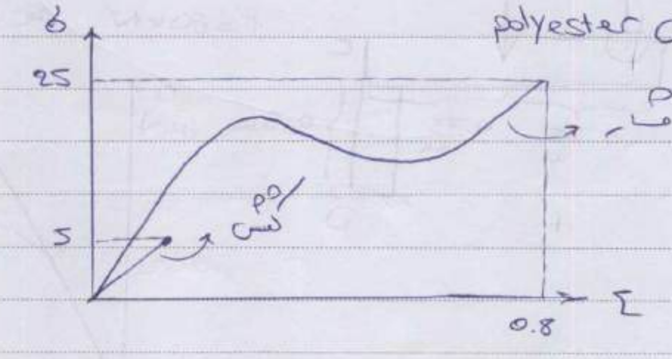


AB مقطع $= 1.5 \text{inch}^2$

BC مقطع $= 4 \text{inch}^2$

از جنس polyester

$P_{max} = ?$ قبل از failure



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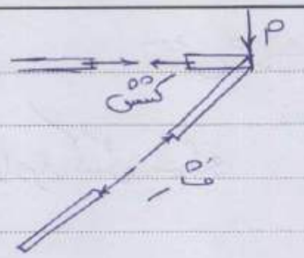


$\uparrow \sum F_y = 0 \Rightarrow 3/5\, F_{AB} - P = 0$

$\Rightarrow F_{AB} = 1.667P$

$\sum F_x = 0 \Rightarrow 4/5\, F_{AB} = F_{BC}$

$\Rightarrow F_{BC} = \frac{4}{5}(1.667P) = 1.333P$

$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} \Rightarrow 5 = \frac{F_{BC}}{4} \rightarrow F_{BC} = 20\text{ kip}$

$\Rightarrow P = \frac{20}{1.333} = 15\text{ kip}$

$25 = \frac{F_{AB}}{\underset{1.5}{\boxed{A_{AB}}}} \Rightarrow F_{AB} = 37.5\text{ kip} \rightarrow P = \frac{37.5}{1.667} = 22.5\text{ kip}$

$\Rightarrow \boxed{P_{max} = 15\text{ kip}}$

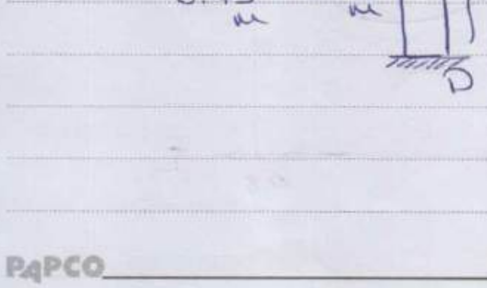
مثال،

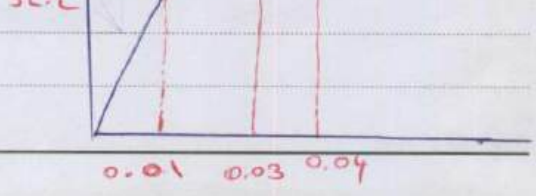
AB & CD are polymers

$D_{AB} = 60\text{ mm}$

$D_{CD} = 80\text{ mm}$

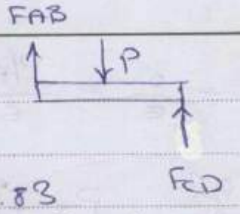
$P = 80\text{ kN}$ AC گردش =?





PAPCO

$F_{AB} = F_{CD} = P/2 = 40\,kN$



$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40\times10^3}{\frac{\pi}{4}(0.04)^2} = 31.83\ MPa < 32.2\ MPa = \sigma_{yield}$

در proportional limit هستیم پس می‌توان از رابطه $\sigma = E\varepsilon$ استفاده کرد

$E = \frac{32.2(10^6)}{0.01} = 3.22\ GPa$

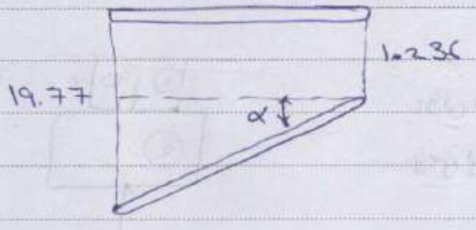
$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83\times10^6}{3.22\times10^9} = 0.0098$

$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0098 \times 2000 = 19.77\,mm$

$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40\times10^3}{\frac{\pi}{4}(0.08)^2} = 7.958\ MPa$ in the proportional limit.

$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958\times10^6}{3.22\times10^9} = 0.00247$

$\delta_{CD} = \varepsilon_{CD} L_{CD} = 0.00247 \times (500) = 1.236\,mm$



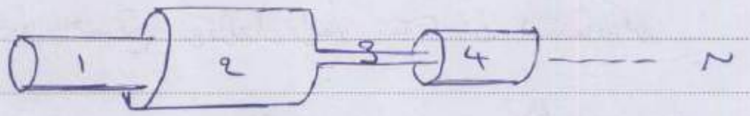
$\tan\alpha = \frac{19.77 - 1.236}{1500}$

$\Rightarrow \alpha = 0.708°$

$$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E} = \frac{P/A}{E} \Rightarrow \frac{\Delta l}{l} = \frac{P/A}{E}$$

$$\Rightarrow \Delta l = \frac{Pl}{AE}$$

$$\Delta l = \sum_{i=1}^{N} \frac{P_i l_i}{A_i E_i}$$

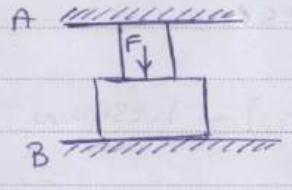


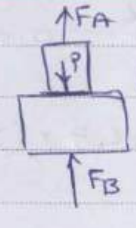
$$\Delta l = \int_0^{l} \frac{F(x)\,dx}{A(x)\,E(x)}$$

E معمولاً به صورت تابعی از x نیست بلکه مثلاً قطعه از 3 جنس تشکیل شده:

$$\sum_{i=1}^{3} \frac{1}{E_i} \int_0^{l_i} \frac{F(x)\,dx}{A(x)}$$

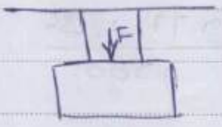
مثال:



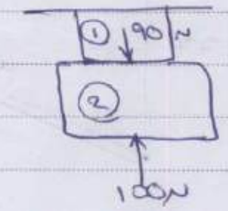


از تعادل استاتیکی داریم $F_A + F_B = P$

[illegible] $\Delta l_1 + \Delta l_2 = 0$

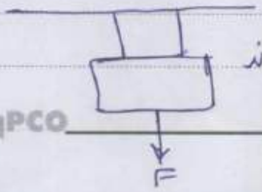


بالایی کشیده می‌شود
پایینی تغییری نمی‌کند

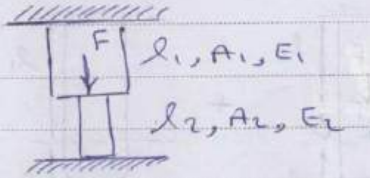


$\sigma_1 = \frac{10}{A_1}$ فشار

$\sigma_2 = \frac{100}{A_2}$ فشار



هر دو کشیده می‌شوند



$$\sum F_y = 0 \Rightarrow R_A - F + R_B = 0 \quad (1)$$

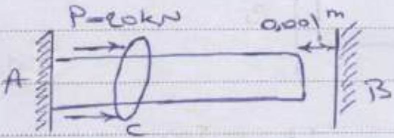
$$\delta = \frac{R_A l_1}{A_1 E_1} - \frac{R_B l_2}{A_2 E_2} = 0 \quad (2)$$

$$\overset{\checkmark}{(1),(2)} \Rightarrow R_A\,,\ \overset{\checkmark}{R_B}$$

مثال :

$E = 200\,GPa$

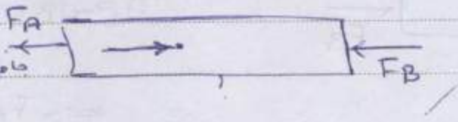
$d = 0.005\,m$



راه حل: تغییر طول AC را حساب کرده و بسنجیم بزرگتر از 0.001 است یا کمتر

تنش در AC و BC

با فرض پر شدن gap



$$F_A + F_B = 20 \times 10^3\,N \quad (1)$$

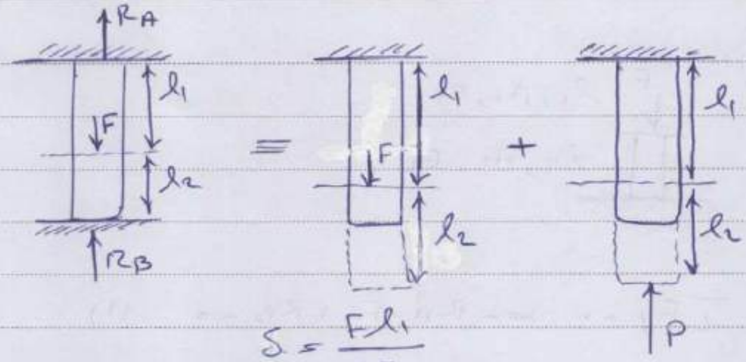
$$\Delta l = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{BC}}{AE} = 0.001$$

$$\Rightarrow 0.4F_A - 0.8F_B = 3927 \quad (2)$$

$$(1),(2) \Rightarrow F_A = 16.6\,kN$$

$$F_B = 3.39\,kN$$

فرض پر شدن gap درست بوده چون FB مثبت شده

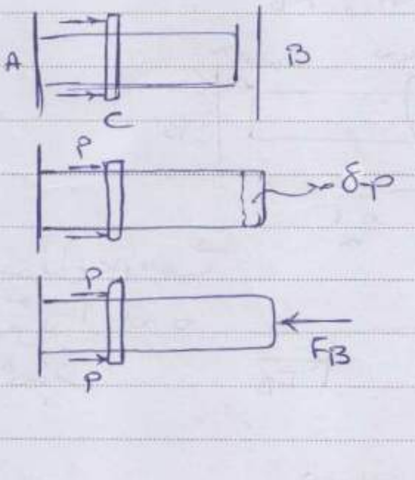


$$\delta = \frac{F l_1}{AE}$$

$$\delta = \frac{P(l_1+l_2)}{AE}$$

$$\frac{F l_1}{AE} = \frac{P(l_1+l_2)}{AE}$$

ه
اوردیم
مثال شکل



$$\delta_P = \frac{P L_{AC}}{AE} = 0.002\,\text{m}$$

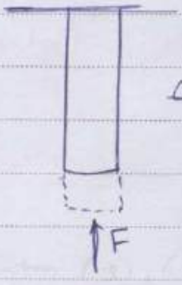
$$\delta_B = \frac{F_B L_{AB}}{AE}$$

$$\delta_P - \delta_B = 0.001\,\text{m}$$

$$\Rightarrow F_A \checkmark ,\ F_B \checkmark$$

$$\delta_T = \alpha(\Delta T) l$$

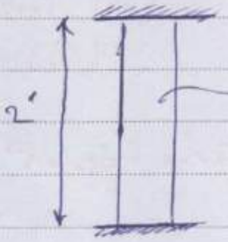
$$\delta_F = \frac{F l}{AE}$$



$$\alpha(\Delta T) l = \frac{F l}{AE}$$

$$\Rightarrow \boxed{F = \alpha \Delta T A E}$$

حل:



→ XS = $(0.5)^2$

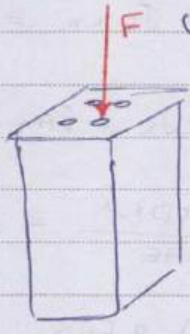
$T_1 = 60$ °F $T_2 = 120$ °F

$$F = 6.6\times10^{-6}\,\frac{1}{°F}\,(120-60)°F\,(0.5)^2 \times 29\times10^3$$

$$= 2.87 \text{ kip}$$

$$\sigma = \frac{2.87}{(0.5)^2} = 11.5 \text{ ksi}$$

composites)



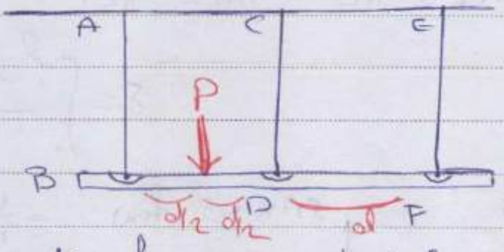
(1) $F = f_B + f_s$ $\delta_B = \delta_s$

$\Downarrow$

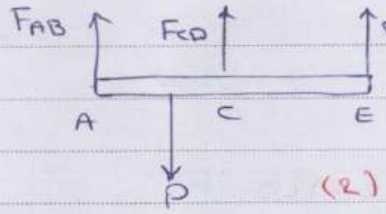
(2) $\frac{f_B \cancel{l}}{A_B E_B} = \frac{f_s \cancel{l}}{A_s E_s}$

(1), (2) $\Rightarrow f_B$ ✓, f_s ✓

حل:

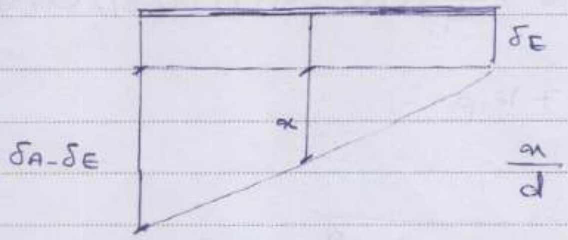


bars mode of same meterial with same XS
find the forceS in each bar



(1) $\sum M_A = 0 \Longrightarrow F_{EF}(2d) + F_{CD}(d) - P(d/2) = 0$

(2) $\sum F_y = 0 \Longrightarrow F_{AB} + F_{CD} + F_{EF} = P$



$$\frac{x}{d} = \frac{\delta_A - \delta_E}{2d} \Longrightarrow x = 1/2\,(\delta_A - \delta_E)$$

$$\delta_C = \delta_E + x = \frac{\delta_A + \delta_E}{2}$$

$$\Longrightarrow 2\delta_C = \delta_A + \delta_E$$

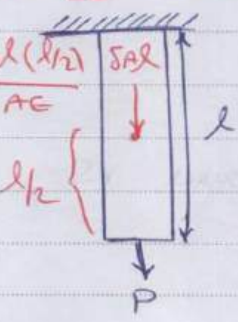
$$\Longrightarrow \frac{2F_{CD} \times l}{AE} = \frac{F_{AB} \times l}{AE} + \frac{F_{EF}\, l}{AE}$$

$$\Longrightarrow (3)\ 2F_{CD} = F_{AB} + F_{EF}$$

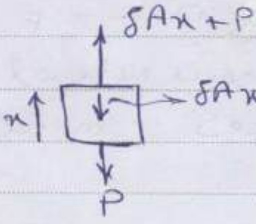
$F_{CD} = P/3$ $\quad F_{EF} = P/12$ $\quad F_{AB} = 7/12\, P$

$\delta = \frac{\delta l^2}{2E}$

$\delta = \frac{\delta A l (l/2)}{AE}$



$$\delta = \int_0^l \frac{f(x)\,dx}{AE}$$



$$= \frac{1}{AE}\int_0^l (\delta A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\delta A x^2}{2} + Px \Big|_0^l\right)$$

$$= \frac{\delta l^2}{2E} + \frac{Pl}{AE}$$

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حل تمرین مقاومت 1

فصل 1: 18 _ 43 _ 65 _ 33 _ 37

Sec1 ⇒ 2-15, 2-19, 2-21, 2-23, 2-30

Sec2 ⇒ 2-37, 2-41, 2-43, 2-45, 2-46

Sec3 ⇒ 2-49, 2-55, 2-60

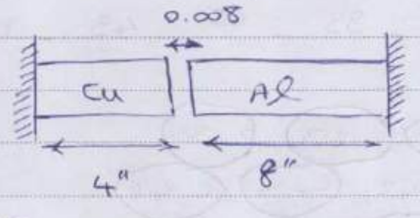
Sec4 ⇒ 2-73, 2-74, 2-83, 2-84

Sec5 ⇒ 1-9, 1-15, 1-20, 1-28

Sec6 ⇒ 2-66, 2-75, 2-76

Sec7 ⇒ 3-11, 3-21, 3-43, 3-50, 3-61, 3-63

یکشنبه ۱۷ آذر میان‌ترم سر کلاس



$T_1 = 60\,°F$

$d = 1.25''$

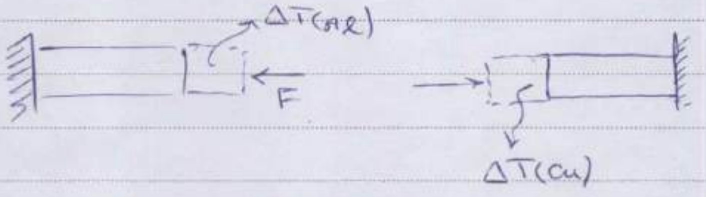
$$\begin{cases} \alpha_{Al} = 13\times10^{-6}\ 1/F \\ E_{Al} = 10\times10^{3}\ ksi \end{cases}$$

$$\begin{cases} \alpha_{Cu} = 9.4\times10^{-6}\ 1/F \\ E_{Cu} = 18\times10^{3}\ ksi \end{cases}$$

① $T_2 = ?$ تا gap بسته شود

② $T_2 = 200°F \Rightarrow$ تنش $= ?$

③ $T_2 = 300°F \Rightarrow L(Al) = ?$



① $\delta_T = 0.008 = \alpha_{Al}\underbrace{\Delta T}_{(T_2-60)} L_{Al} + \alpha_{Cu}\Delta T L_{Cu} \Rightarrow T_2 = 116\,°F$

② $0.008 = (\delta T)_{Cu} - (\delta F)_{\underset{Cu}{\cancel{Al}}} + (\delta T)_{Al} - \underbrace{(\delta F)_{Al}}_{(\delta F)_{Cu}}$

$$= 9.4\times10^{-6}(200-60)4 - \frac{F(4)}{\frac{\pi}{4}(1.25)^2\, 18\times10^3}$$

$$+ 13\times10^{-6}(200-60)8 - \frac{F(8)}{\frac{\pi}{4}(1.25)^2(10\times10^3)}$$

$\Rightarrow F = 14.195\ kip$

$$\sigma_{Al} = \sigma_{Cu} = \frac{14.195}{\frac{\pi}{4}(1.25)^2} = 11.6\ ksi$$

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③ $T_c=300$ عملیات مشابه ② برای $\Rightarrow F = 31.194\ \text{kip}$

$$\delta_{Al} = (\delta_T)_{Al} - (\delta_F)_{Al} = 0.00426''$$

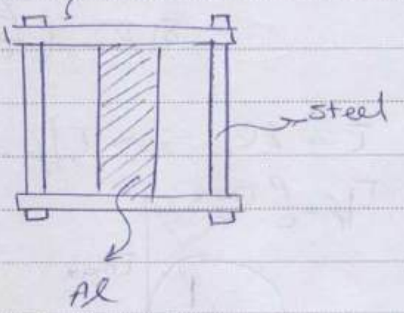
$$L_{Al} = 8 + 0.00426 = 8.00426''$$

this figure makes no sense.

شکل :

$$\begin{cases} d_{Al} = 50\ \text{mm} \\ d_{st} = 10\ \text{mm} \end{cases}$$

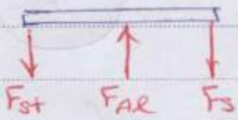
$$\begin{cases} l_{Al} = 100\ \text{mm} \\ l_{st} = 150\ \text{mm} \end{cases}$$



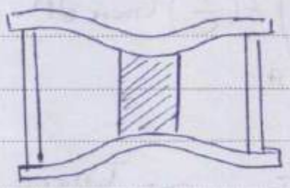
$$\begin{cases} \alpha_{st} = 17\times10^{-6}\ 1/^\circ C \\ \alpha_{Al} = 26\times10^{-6}\ 1/^\circ C \end{cases}$$

$$\begin{cases} E_{st} = 193\times10^{9}\ Pa \\ E_{Al} = 44.7\times10^{9}\ Pa \end{cases}$$

$\Delta T = 110$



$$\Rightarrow \begin{cases} 2F_{st} = F_{Al} \\ \delta_{Al} = \delta_{st} \end{cases}$$



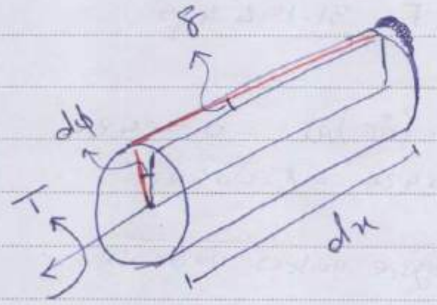
$$(\delta_T + \delta_F)_{st} = (\delta_T - \delta_F)_{Al}$$

$$17\times10^{-6}\times0.15\times110 + \frac{F(0.15)}{193(10^9)(2)\pi/4\,(0.01)^2} = 26\times10^{-6}\times0.1\times110 - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2}$$

$$\Rightarrow F = 904\ N$$

77-60-21-17 ②

فصل 3 :

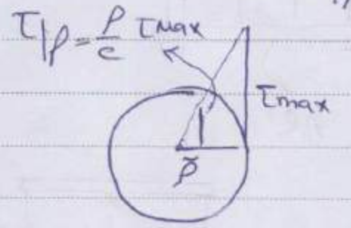


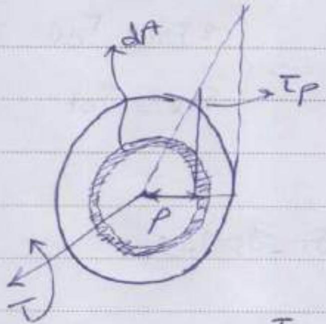
$c\,d\phi = \gamma\,dx$

$\Rightarrow \gamma = \frac{c\,d\phi}{dx}$

$\gamma|_\rho = \left(\frac{\rho}{c}\right)\gamma_{max}$

$\tau = \gamma G \Rightarrow \tau|_\rho = \frac{\rho}{c}\,\tau_{max}$





$T = \int_A \rho(\tau_\rho dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{max}\,dA$

$= \frac{\tau_{max}}{c}\int_A \rho^2 dA = \frac{\tau_{Max}}{c}J \Rightarrow \tau_{Max} = \frac{Tc}{J}$

$\tau_\rho = \frac{T\rho}{J}$

$J = I_x + I_y = \int \underbrace{(x^2+y^2)}_{\rho^2} dA$

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برای سطح دایره‌ای $J = \int_A \rho^2 dA = \int_0^c \rho^2(2\pi\rho\, d\rho) = 2\pi \int_0^c \rho^3 d\rho$

$$\Rightarrow \boxed{J = \frac{\pi}{2} c^4}$$

برای دایره توخالی $J = 2\pi \int_{R_i}^{R_o} \rho^3 d\rho = \boxed{\frac{\pi}{2}(R_o^4 - R_i^4)}$

$$c\, d\phi = \gamma\, dx \Rightarrow \rho\, d\phi = \gamma_\rho\, dx$$

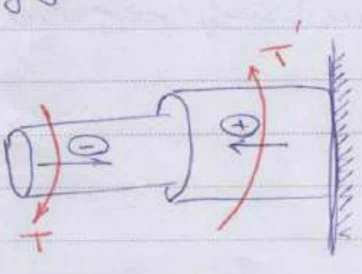
$$\Rightarrow d\phi = \frac{\gamma_\rho}{\rho}\, dx$$

$$\tau = \gamma G \Rightarrow \gamma = \frac{\tau}{G}$$

$$\tau_\rho = \frac{T\rho}{J} \Rightarrow \gamma_\rho = \frac{T\rho}{JG}$$

$$\Rightarrow d\phi = \frac{T\cancel{\rho}}{JG\cancel{\rho}}\, dx \Rightarrow \boxed{\phi = \int_0^l \frac{T}{JG}\, dx}$$

$$\Rightarrow \phi = \frac{Tl}{JG}$$



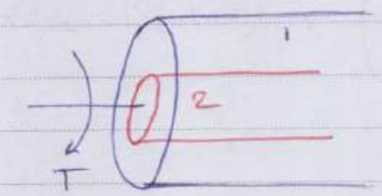
$$\phi = \sum_{i=1}^{n} \frac{T_i l_i}{J_i G_i}$$

PAPCO

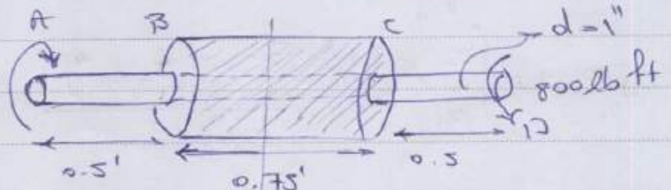
composites

$T = T_1 + T_2$

$\phi_1 = \phi_2$



مثال :
800 lbft



$800 \times 12 = T_t + T_{sh}$

$\phi_t = \phi_{sh}$

$$\frac{T_t(0.75\times12)}{\frac{\pi}{2}(1.5^4-1.25^4)G} = \frac{T_{sh}(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$$\rightarrow \begin{cases} T_t = 9376.42 \text{ lb-in} \\ T_{sh} = 223.58 \text{ lb-in} \end{cases}$$

$$\phi_{A/D} = \phi_{A/B} + \phi_{B/C} + \phi_{C/D} = \sum \frac{TL}{GJ}$$

$$= 2\times\frac{800\times12\times0.5\times12}{\frac{\pi}{2}(0.5)^4(11\times10^6)} + \frac{223.58(0.75)12}{\frac{\pi}{2}(0.5)^4(11\times10^6)^2}$$

برای ϕ_{BC} هم می‌توان چرخش پوسته را درنظر گرفت هم چرخش میله چون فرض کرده‌ایم این دو با هم می‌چرخند.

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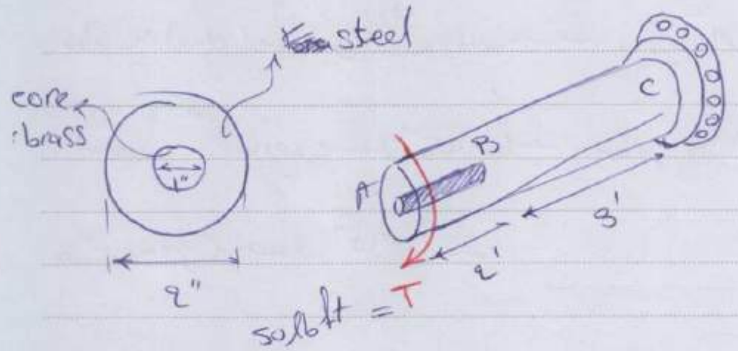
مثال :



50 lbft = T

$$\tau_{max}\Big|_{brass} = \frac{T_{br}\,0.5}{J_{br}} \qquad T_{br} + T_{st} = 50 \quad (1)$$

$$\phi_{A/B} = \frac{T_{br}(2)(12)}{\frac{\pi}{2}(0.5)^4(5.6)(10^6)} = \frac{T_{st}(2)(12)}{\frac{\pi}{2}(1^4-0.5^4)11.5(10^6)} \quad (2)$$

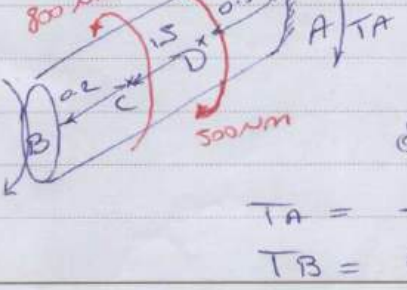
$$\Rightarrow (1),(2) \Rightarrow T_{st} = 48.42$$
$$T_{br} = 1.57$$

$$\phi = \sum \frac{TL}{JG} = 0.116°$$

τ_{max} برای brasl فقط در سطح آن اتفاق می‌افتد اما برای steel ممکن است در سطح مشترک AB باشد یا BC. AB، T کمتر دارد و BC، T بیشتر دارد

مثال: τ_{max} کجاست؟

* استاتیکی نامعین *



$$-T_B + 800 - 500 - T_A = 0$$

$$\phi_{B/A} = 0$$

$$T_A = -345\ \text{Nm}$$
$$T_B = 645\ \text{Nm}$$

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مواد ductile از نظر تنش برشی ضعیف هستند بنابراین اگر به یک ماده ductile نیروی
کششی وارد شود ماده در صفحه 45° می‌شکند. مواد brittle از نظر تنش عمودی ضعیف
هستند بنابراین در صفحه 90° می‌شکند

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$\overset{\text{کار}}{dW} = T d\theta$ work = torque x angle of twist

$P = \frac{dW}{dt} = T\frac{d\theta}{dt} = T\underset{\text{سرعت زاویه ای}}{\omega}$ power = $\frac{\text{work}}{\text{time}}$

$\boxed{P = T\omega}$ → rad/sec SI: 1 watt = 1 Nm/sec

watt Nm

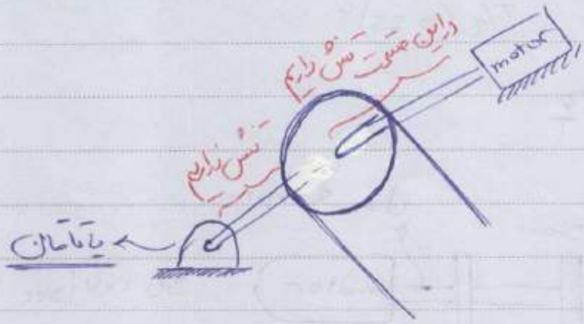
British: $P = $ ft.lb/sec
$T = $ ft.lb

1 hp = 550 ft.lb/sec

مثال: یک شفت توپر برای انتقال قدرت 5 hp با دور $\omega = 175$ rpm

round per minute

$\tau_{all} = 14.5$ ksi ، قطر شفت چقدر است.



$P = T\omega$ $\omega = 175\,(2\pi)\frac{1}{60} = 18.33$ rad/sec

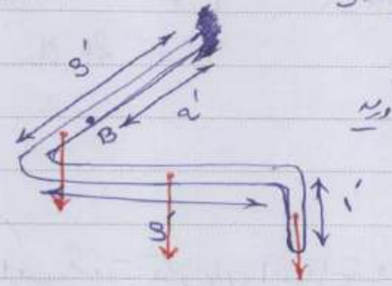
$P = 5(550) = 2750$ ft.lb/sec

$T = \frac{P}{\omega} = 150.1$ ft.lb

$$\tau_{all} = \frac{Tc}{J} \Rightarrow \frac{\overset{\frac{\pi}{2}c^4}{J}}{c} = \frac{T}{\tau_{all}} \Rightarrow c = \left(\frac{2T}{\pi \tau_{all}}\right)^{1/3}$$

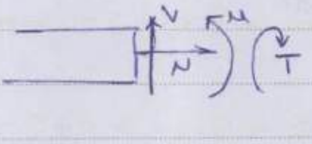
$$\Rightarrow c = \left(\frac{2 \times 150.1 \times 12}{\pi (14500)}\right)^{1/3} \Rightarrow c = 0.429'' \Rightarrow d = 0.858''$$

مثال: میله شکل زیر قطر 0.5" و وزن 5 lb/ft را دارا است. ماکزیمم تنش برشی را در مقطع B بدست آورید



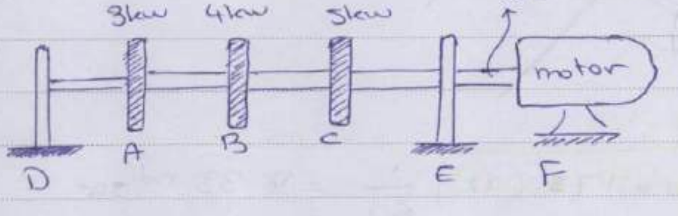
$$T = \Sigma M = 15(1.5) + 5(3) = 37.5 \ lb\,ft = 450 \ lb\,in$$

$$\tau_{max} = \frac{Tc}{J} = \frac{450(0.25)}{\pi/2\,(0.25)^4} = 18.3 \ ksi$$



مثال:

50 rev/sec



$$\omega = 50(2\pi) = 100\pi \ rad/sec$$

$$T_{CF} = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.2 \ Nm$$

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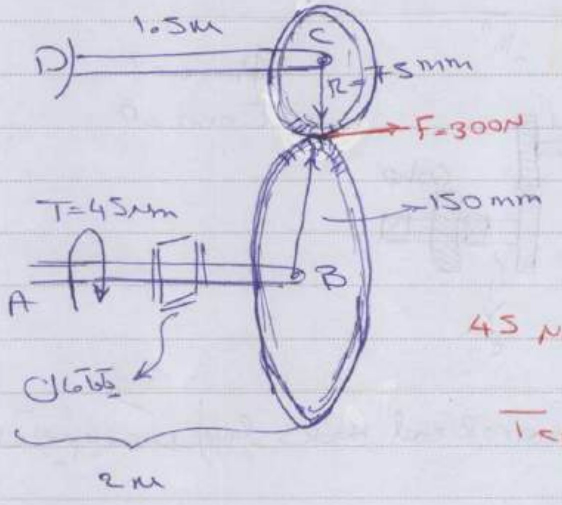
$$T_{AB} = \frac{3(10^9)}{100\pi} =$$

$$T_{BC} = \frac{7(10^3)}{100\pi} =$$

$$\tau_{max} = \tau_{CF} = \frac{Tc}{J}$$

$$\phi_{DF} = \sum\phi = \underbrace{\phi_{AB}}_{\frac{T_{AB}L}{JG}} + \phi_{BC} + \phi_{CF}$$



G = 80 GPa ; [illegible]
d = 20mm
Q: twist A = ?

45 Nm = F × 0.15 m ⟹ F = 300N

$T_{CD} = 300 \times 0.075 = 22.5\,\mathrm{Nm}$

$$\phi_{C/D} = \frac{22.5 \times 1.5}{\frac{\pi}{2}(0.01)^4 (80 \times 10^9)} = 0.0269 \text{ rad}$$

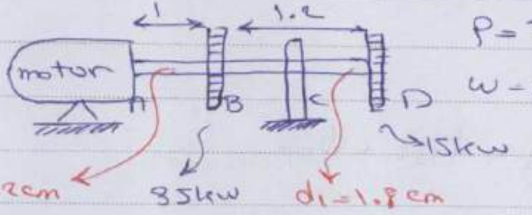
$$\phi_B R_B = \phi_C R_C$$

$$\phi_B (1.5) = 0.0269 \times 0.75 \Rightarrow \phi_{B/D} = 0.0134 \text{ rad}$$

$$\phi_{A/B} = \frac{45\times 2}{\pi/2\,(0.01)^4\,(80\times 10^9)} = 0.0716\ \text{rad}$$

$$\phi_{A/D} = \phi_{A/B} + \phi_{B/D} = 0.085\ \text{rad}$$

مثال:



$P = 50\ kw$

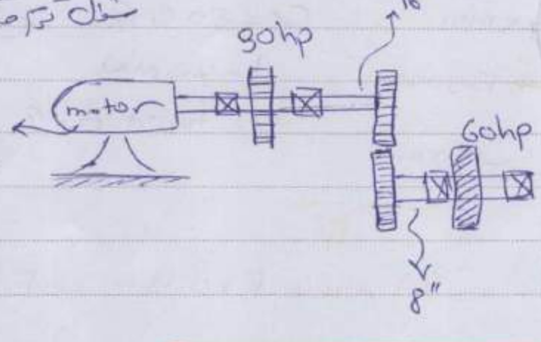
$\omega = 62.8\ rad/sec$

τ_{all}

$d = ?$

$95+15 \nleq 50$ ✓ موتورمان بسوزد!

شفت ترکیبی

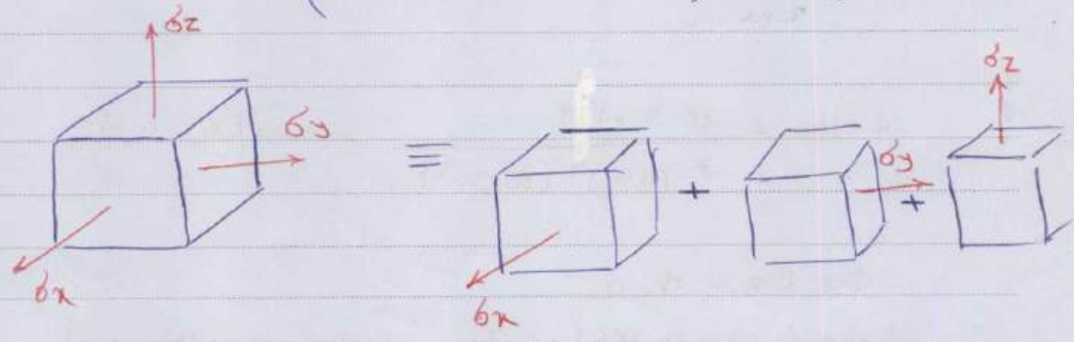


$\phi_{max} = ?$

$\tau_{max} = ?$

---

* قانون تعمیم یافته هوک (generalized Hook's law) *

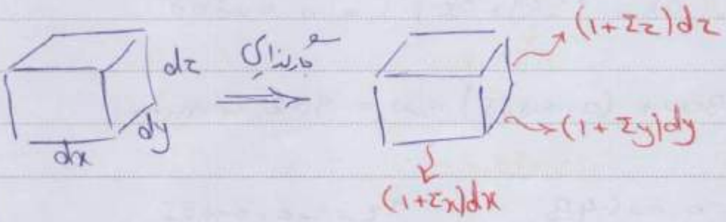


$$\varepsilon_x = \varepsilon'_x + \varepsilon''_x + \varepsilon'''_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$



تغییر حجم $\delta V = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)\,dx\,dy\,dz - dx\,dy\,dz$

$$\Rightarrow \delta V \simeq (\varepsilon_x + \varepsilon_y + \varepsilon_z)\underbrace{dx\,dy\,dz}_{dV}$$

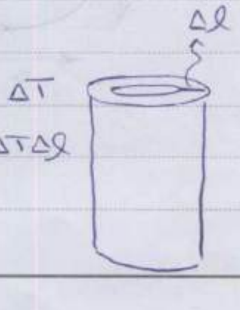
$$e = \frac{\delta V}{\Delta V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

که تغییر حجم به حجم اولیه.

با استفاده از قوانین تعمیم یافته هوک متناظر بر حسب تنش بصورت زیر می توان نوشت:

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$G = \frac{E}{(1+\nu)2}$$



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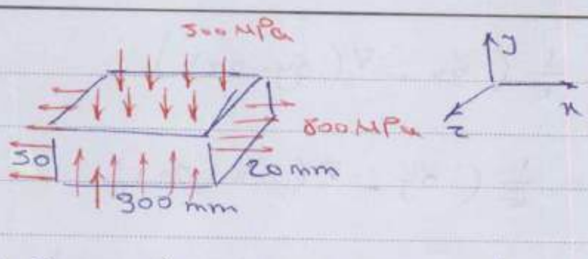
مثال:

$E = 120\ GPa$
$\nu = 0.34$



$\sigma_x = 800\ MPa\ ,\ \sigma_y = -500\ MPa\ ,\ \sigma_z = 0\ ,\ \tau = 0$

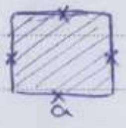
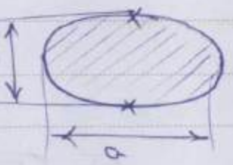
$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) = 0.00808$

طول جدید $= 300 + (0.00808)\,300 = 302.4\ mm$

$\varepsilon_y = -0.00643 \qquad \varepsilon_z = -0.00085$

عرض جدید $= 19.98\ mm$ ارتفاع جدید $= 49.68\ mm$

Tmax نقاط x مشخص شده‌اند

	τ_{max}	ϕ
	$\frac{4.81T}{a^3}$	$\frac{7.1Tl}{a^4G}$
	$\frac{20T}{a^3}$	$\frac{46Tl}{a^4G}$
	$\frac{2T}{\pi ab^2}$	$\frac{(a^2+b^2)Tl}{\pi(a^3b^3)G}$

$\tau_{max} = \frac{Tc}{J} \qquad \phi = \frac{Tl}{JG} \quad ; \quad J = \frac{1}{2}\pi c^4$

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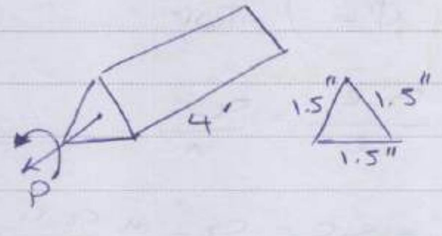
مثال:



$\tau_{all} = 8\,ksi$

$\phi_{All} = 0.02\,rad$

$T_{\text{مجاز}} = ?$

$\tau_{all} = \frac{20T}{a^3} \Rightarrow 8\times10^3 = \frac{20T}{(1.5)^3}$

مقایسه با مقطع هم وزن دایروی؟

$\Rightarrow T = 1350\ lb\,in$

$\phi = \frac{46Tl}{a^4 G} = \frac{46T(4\times12)}{(1.5)^4(3.7\times10^6)} \Rightarrow T = 170\ lb\text{-}in$

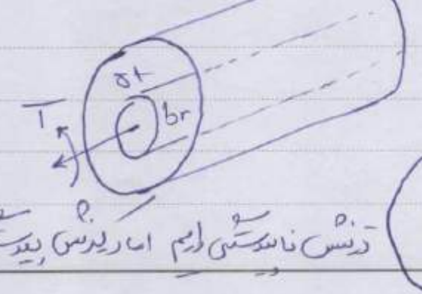
$\Rightarrow T_{max} = 170$

$\rho V_c = \rho V_t \Rightarrow V_c = V_t \Rightarrow \cancel{l}A_c = \cancel{l}A_t \Rightarrow A_c = A_t$

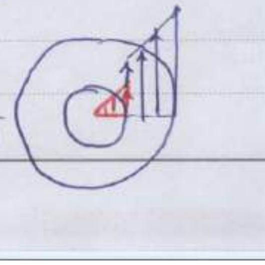
$\Rightarrow \pi R^2 = \frac{1}{2}(1.5)(1.5)\sin 60 \Rightarrow R = 0.577''$

$\tau_{all} = \frac{TR}{J} \Rightarrow 8\times10^3 = \frac{T(0.577)}{\frac{\pi}{2}(0.577)^4} \Rightarrow T = 2170\ lb\,in$

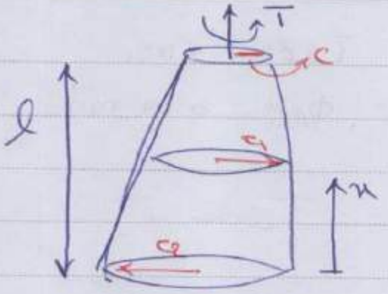
$\phi_{All} = \frac{Tl}{JG} \Rightarrow T = 233\ lb\,in$



$$\left.\begin{array}{l} T = T_{st} + T_{Br} \\ \phi_{st} = \phi_{Br} \end{array}\right\} \Rightarrow \begin{array}{l} T_{Br}\,✓ \\ T_{st}\,✓ \end{array}$$



تنش ناپیوستگی داریم اما کرنش پیوستگی داریم



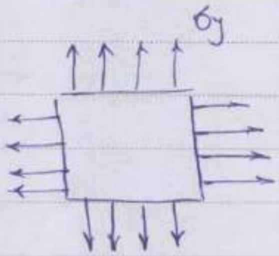
$$\phi = \int_0^l \frac{T\,dx}{J(x)\,G}$$

$$\frac{c_2 - c_1}{l} = \frac{c_2 - c}{x}$$

$$\Longrightarrow c = c_2 - x\,\frac{c_2 - c_1}{l}$$

$$J(x) = \frac{\pi}{2}\left(c_2 - x\,\frac{c_2 - c_1}{l}\right)^4$$

ⓒ 73



$$\sigma_x = E\left(\frac{\varepsilon_x + \nu\varepsilon_y}{1 - \nu^2}\right) \qquad \sigma_y = E\left(\frac{\nu\varepsilon_x + \varepsilon_y}{1 - \nu^2}\right)$$

$$\sigma_x = A\varepsilon_x + B\varepsilon_y$$
$$\sigma_y = C\varepsilon_x + D\varepsilon_y$$

$$\sigma_y = 0 \Longrightarrow \varepsilon_x = \frac{\sigma_x}{E}, \quad \varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\sigma_x = 0 \Longrightarrow \varepsilon_y = \frac{\sigma_y}{E}, \quad \varepsilon_x = -\nu\frac{\sigma_y}{E}$$

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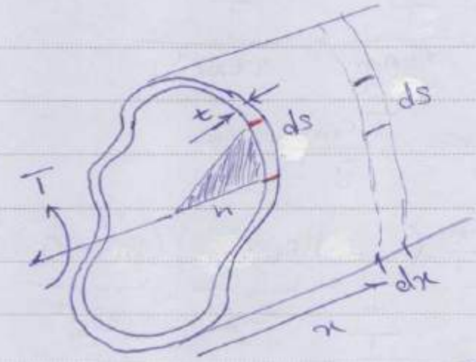
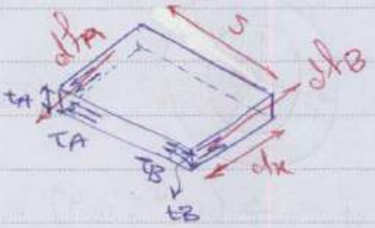




$$\tau_A (t_A dx) = \tau_B (t_B dx)$$

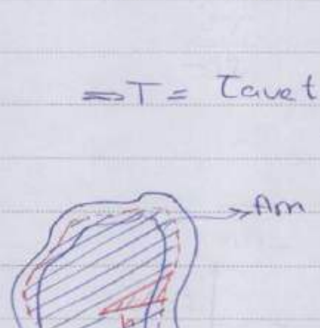
$$\Rightarrow \tau_A t_A = \tau_B t_B$$

P
جریان برشی $= q = \tau_{av} t = \text{const}$

$$dF = \tau_{ave}\, dA = \tau_{ave}\, (t\,ds)$$

$$dT = h(dF) = h(\tau_{ave}\, t\,ds) \Rightarrow T = \oint \underbrace{h \tau_{ave} t}_{q = \text{const}} ds$$

$$\Rightarrow T = \tau_{ave} t \oint h\,ds = 2\tau_{ave} t \oint dA_m = 2\tau_{ave} t A_m$$

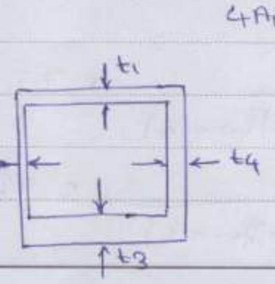


$$dA_m = \tfrac{1}{2} h\,ds$$

$$\Rightarrow \tau_{ave} = \frac{T}{2tA_m}$$

$$q = \frac{T}{2A_m}$$

P
[illegible] $\Rightarrow \phi = \dfrac{TL}{4A_m^2 G} \oint \dfrac{ds}{t}$



یافتن ϕ ، در هر یک از 4 قسمت t ثابت است ،
پس 4 انتگرال جداگانه می‌گیریم

$$\tau_{ave} = \frac{T}{2tA_m} = \frac{T}{2t\pi r_m^2}$$

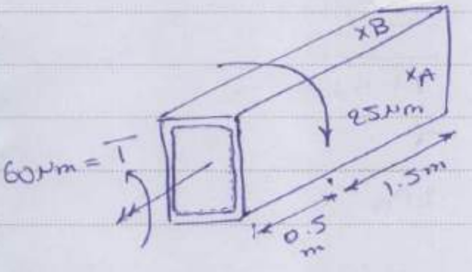
$$\rightarrow \tau_{ave} = \frac{T r_m}{J} \rightsquigarrow$$

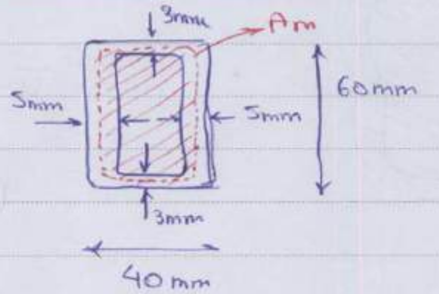
$$J = \frac{\pi}{2}(r_b^4 - r_a^4) = \frac{\pi}{2}\underbrace{(r_b - r_a)}_{t}\underbrace{(r_b + r_a)}_{2r_m} \cdot \underbrace{(r_a^2 + r_b)^2}_{2r_m^2} = 2\pi r_m^3 t$$

$$\rightsquigarrow \tau_{ave} = \frac{T r_m}{2\pi r_m^3 t} = \frac{T}{2t\pi r_m^2}$$

$$\phi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{\underset{\text{ثابت}}{\boxed{t}}} = \frac{TL}{4(\pi r_m^2)^2 G t}\overset{2\pi r_m}{\boxed{\oint ds}}$$

$$\Rightarrow \phi = \frac{TL}{\underbrace{(2\pi r_m^3 t)}_{J} G} = \frac{TL}{JG}$$





τ_A, τ_B ?
total ϕ

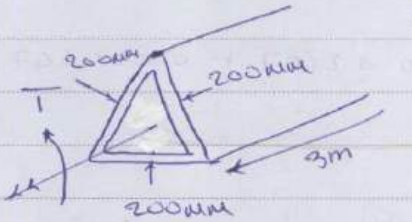
$$\tau_A = \frac{T}{2t_A A_m} \qquad A_m = (0.057)(0.035) = 0.002\ m^2$$

$$\tau_A = \frac{35}{2(0.005)(0.002)} = 1.75\ MPa$$

$$\tau_B = \frac{35}{2(0.003)(0.002)} = 2.92\ MPa$$

$$\phi = \sum \frac{TL}{4A_m^2 G} \oint \frac{ds}{t} = \frac{60 \times (0.5)}{4(0.002)^2 \times 35(10^9)} \left[2\frac{57}{5} + 2\frac{35}{3} \right]$$

$$+ \frac{35(1.5)}{4(0.002)^2 \times 35(10^9)} \left[2\frac{57}{5} + 2\frac{35}{3} \right] = 6.29\,(10^{-3})\ \text{rad}$$



$\tau_{all} = 90\,MPa$

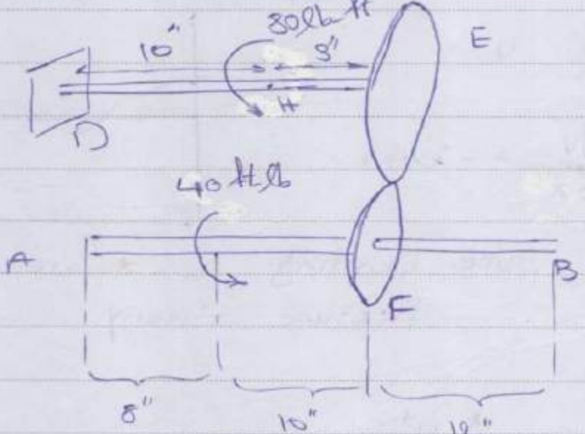
mode of 5mm plate

$\phi_{max} = 2 \times 10^{-3}$ rad

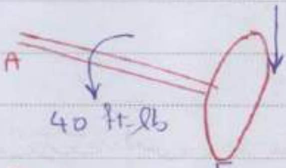
$T_{max} = ?$

$$\left.\begin{aligned} T_1 &= 15.6\,kNm \\ T_2 &= 500\,Nm \end{aligned}\right\} T_{max} = 500\,Nm$$

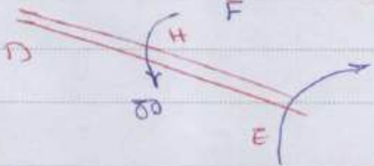
سو
Ⓐ + بولر
پرویس



$\phi_A)_{total} = ?$



$$F = \frac{40}{0.3333} = 120\ lb$$



$T_{HE} = 120 \times 5 = -60$

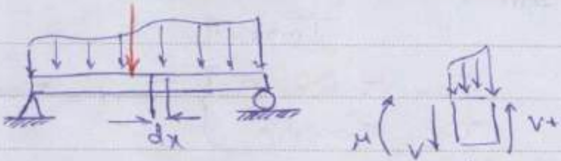
$T_{DH} = 80 - 60 = 20$

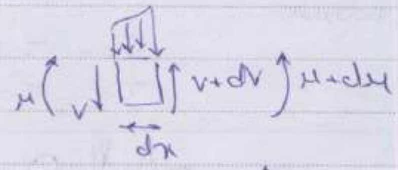
$$\phi_E = \frac{-60(12\times30)+20(12\times10)}{\frac{\pi}{2}(0.5)^4(11\times10^6)} = 0.01778 \text{ rad "cw"}$$

$$\phi_F = \frac{6}{4}\phi_E = 0.02667 \text{ (ccw)}$$

$$\phi_{A/F} = \frac{-40(12)(10)}{\frac{\pi}{2}(0.5)^4(11\times10^6)} = -0.004445 \text{ (ccw)}$$

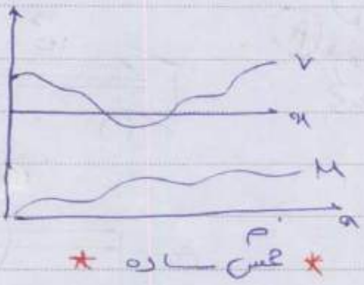
$$\phi_A\big)_{total} = \phi_F + \phi_{A/F} = 0.02667 + 0.004447 = \ldots$$





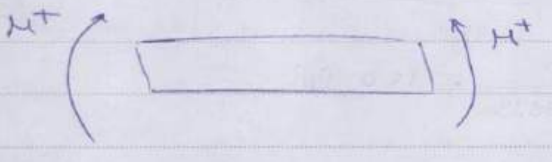
معادلات تعادل ⟵ برای المان

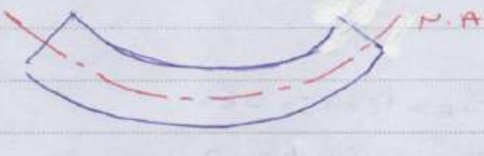
$$\begin{cases} \dfrac{dM}{dx} = V \\ \dfrac{dV}{dx} = -w(x) \end{cases}$$



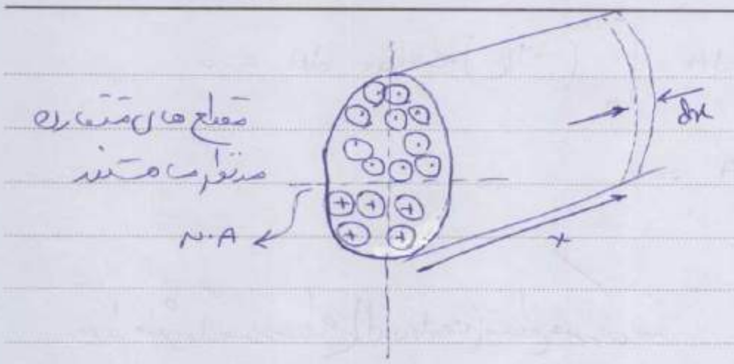
pure bending ★ خمش ساده ★

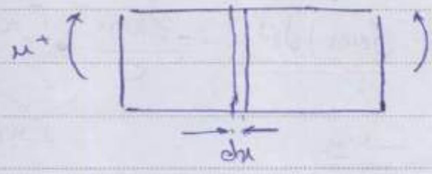
flexture theory

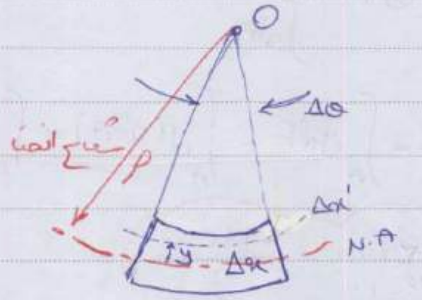




N.A ⟵ محور خنثی / نه تحت فشار است نه کشش





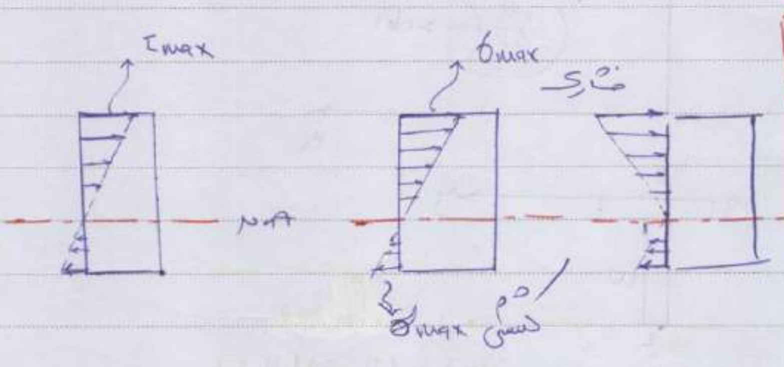


$$\varepsilon_y = \lim_{\Delta x \to 0} \frac{\Delta x' - \Delta x}{\Delta x}$$

$$\Delta x = \rho \Delta \theta \quad , \quad \Delta x' = (\rho - y)\Delta \theta$$

$$\varepsilon_y = \lim_{\Delta x \to 0} \frac{(\rho - y)\Delta \theta - \rho \Delta \theta}{\rho \Delta \theta} \longrightarrow \boxed{\varepsilon_y = \frac{-y}{\rho}}$$

$$\boxed{\varepsilon_y = \frac{y}{c}\,\varepsilon_{max}}$$



$$\sigma = E\varepsilon$$

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$$\sum F = 0 \Rightarrow \int_A \sigma \, dA = \int_A \left(-\frac{y}{c}\right) \sigma_{max} \, dA = 0$$

$$\Rightarrow \frac{\sigma_{max}}{c} \int_A y \, dA = 0$$

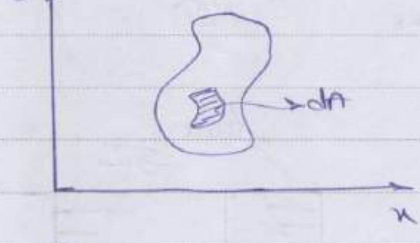
$$\bar{y} = \frac{\int y \, dA}{\int dA}$$

خط خنثی از مرکز سطح (centroid) مقطع عبور می کند

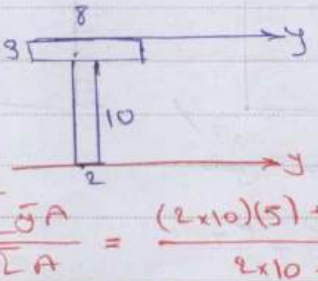
$$M = \int_A y \, dF = \int_A y(\sigma dA) = \int_A y\left(\frac{-y}{c}\sigma_{max}\right) dA = -\frac{\sigma_{max}}{c} \cdot \underbrace{\int y^2 dA}_{I_{xx}}$$

$$\boxed{\sigma_{max} = \frac{-Mc}{\underset{\text{نسبت به محور خنثی}}{I}}} \qquad \sigma_y = \frac{-My}{I}$$

$$\bar{y} = \frac{\int y \, dA}{\int dA} = \frac{\sum \bar{y} A}{\sum A} \quad , \quad I_{xx} = \int y^2 dA$$

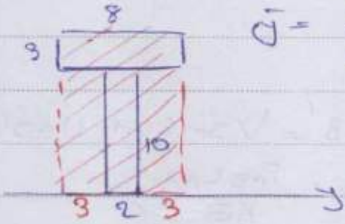


مثال:



$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{(2\times10)(5) + (3\times8)(11.5)}{2\times10 + 3\times8} = 8.55 \quad \text{از پایین}$$

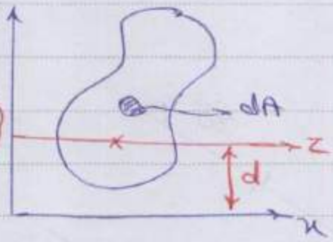
$$\bar{y} = \frac{(3\times8)(-1.5) + (10\times2)(-8)}{3\times8 + 10\times2} = -4.45 \quad \text{از بالا}$$



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(13\times8)(6.5) - 2(10\times3)(5)}{13\times8 - 2\times(10\times3)} = 8.55$$

محور z باید از مرکز سطح بگذرد

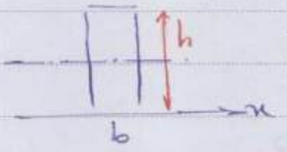
$$I_x = I_z + Ad^2$$



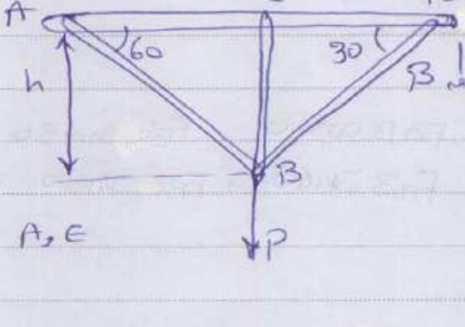
$$I_x = \int_A y^2\,dA$$

$$I_y = \int_A x^2\,dA$$

$$J = \int_A r^2\,dA = \int_A (x^2+y^2)\,dA = I_x + I_y$$

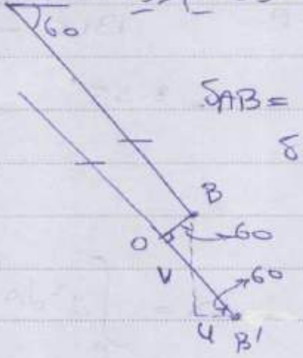


$$I_{N.A} = \tfrac{1}{12}bh^3 \rightarrow I_x = \tfrac{1}{12}bh^3 + (bh)(h/2)^2 = \tfrac{1}{3}bh^3$$



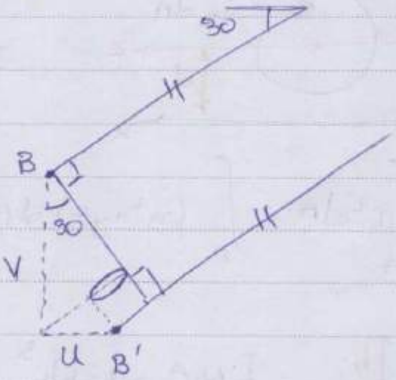
جابجایی افقی و عمودی نقطه B

فرض کنیم زاویه 60° و 60° باقی بماند



$$\delta_{AB} = OB' = V\sin 60 + U\cos 60$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{AE}.$$



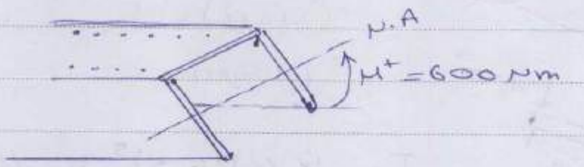
$$\delta_{BD} = V\sin 30 - u\cos 30 = \frac{F_{BD} L_{BD}}{AE}$$

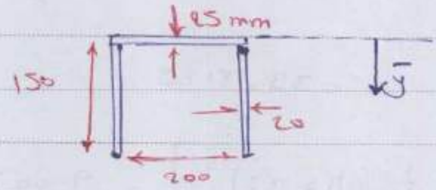
$$\delta_{BC} = V = \frac{F_{BC} L_{BC}}{AE}$$

$$\sum F_x = 0 \Rightarrow F_{AB}\cos 60 = F_{BD}\sin 30$$

$$\sum F_y = 0 \Rightarrow F_{AB}\sin 60 + F_{BD}\sin 30 + F_{BC} = P$$

مثال: برآیند نیروی وارد بر بال فوقانی را محاسبه کنید





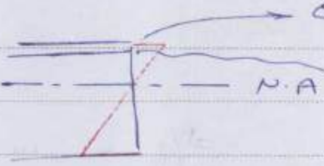
$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(0.24 \times 0.025)(0.0125) + 2(0.15 \times 0.02)(0.025 + 0.075)}{(0.24 \times 0.025) + 2(0.15 \times 0.02)}$$

$$= 0.05625\ m$$

$$I = \sum\left(\frac{bh^3}{12} + Ad^2\right) = 1/12(0.24)(0.025)^3 + (0.24)(0.025)(0.05625 - 0.0125)^2$$

$$+ 2 \times \frac{1}{12}(0.02)(0.15)^3 + 2(0.15)(0.02)(0.04375)^2$$

$$= 34.53 \times 10^{-6}\ m^4$$

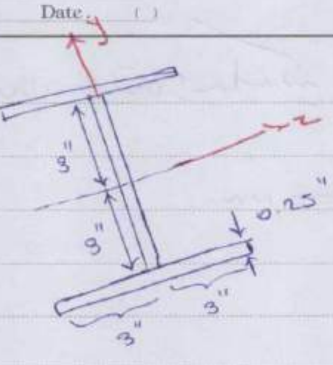


$$\sigma_{max} = \frac{600(0.05625)}{34.53 \times 10^{-6}} = 0.977\ MPa$$

$$\sigma_{min} = \frac{600(0.175 - 0.05625)}{34.53 \times 10^{-6}} = 0.593$$

$$F = \frac{(0.977 + 0.593)}{2}(0.025 \times 0.24) = 4.56\ kN$$

مثال



$\sigma_{all} = 24$ ksi

$(M_{all})_z$

$(M_{all})_y$

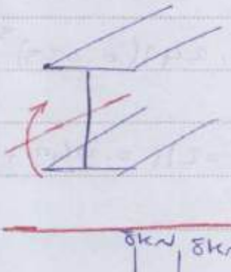
$I_z = \frac{1}{12}(6)(6.5)^3 - \frac{1}{12}(6-0.25)6^3$

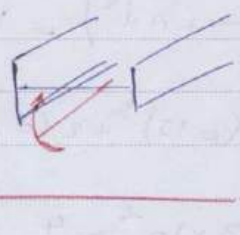
$= 33.8125$

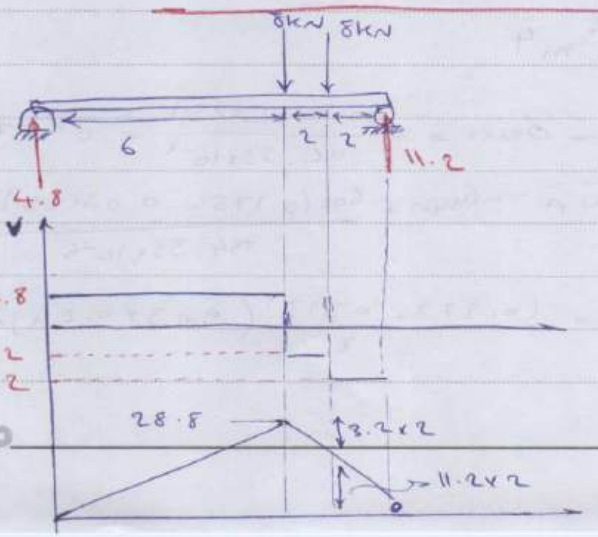
$I_y = 2\left(\frac{1}{12}(0.25)(6)^3 + \frac{1}{12}(6)(0.25)^3\right] = 9.0078$

$(M_{all})_z = \frac{\sigma_{all} I_z}{c} = \frac{24(33.8125)}{3.25} = 249.7$ kip-in

$(M_{all})_y = \frac{\sigma_{all} I_y}{c} = \frac{24(9.0078)}{3} = 72.0625$ kip-in

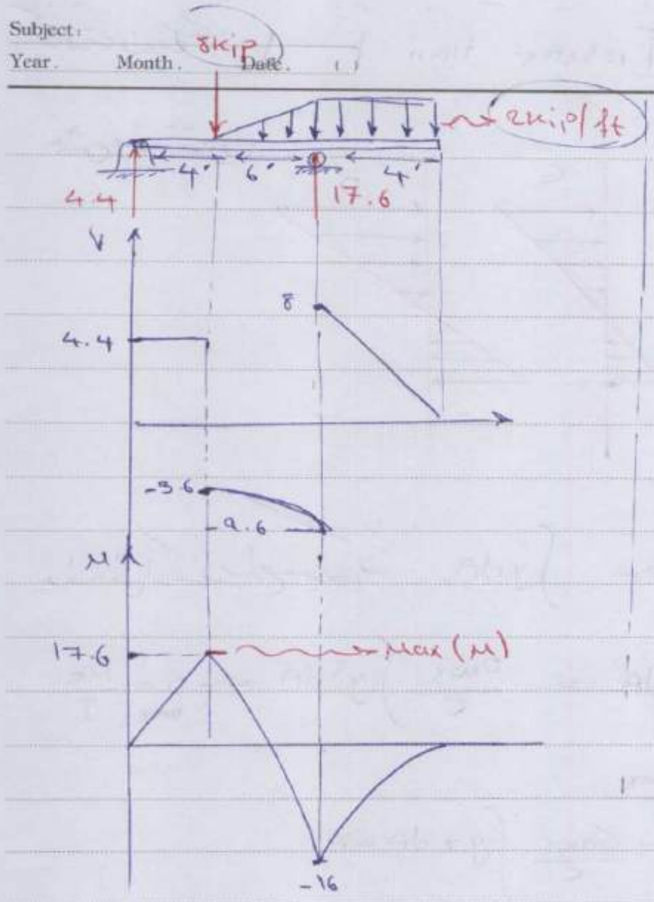


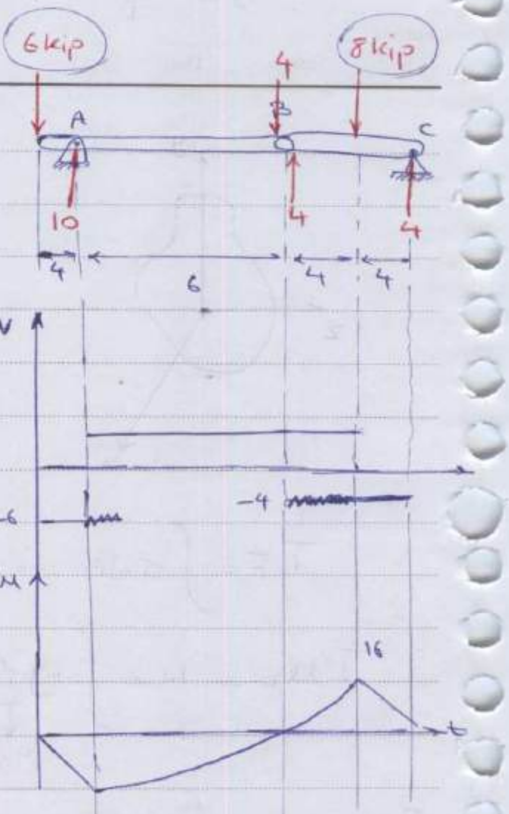


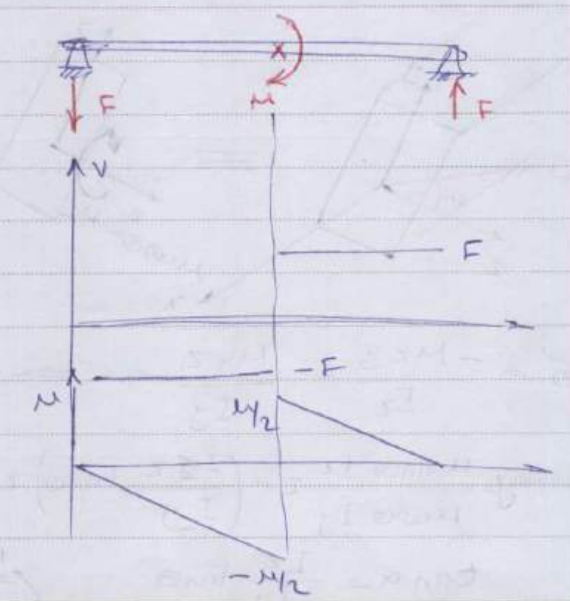


$\frac{dV}{dx} = -w$

$\frac{dM}{dx} = V$


8kip
2kip/ft
4'
6'
4'
4.4
17.6
V
8
4.4
-3.6
-9.6
M
17.6
Max (M)
-16


6kip
4
8kip
A
B
C
10
4
4
4
6
4
4
V
-6
-4
M
16
t


F
M
F
V
F
-F
M
M/2
-M/2

$\sigma = \frac{My}{I}$ —— باید حالتی را ترسیم کنیم که I بیشتر باشد پس بیشتر آن را به صورت کاملتری وارد کرد.

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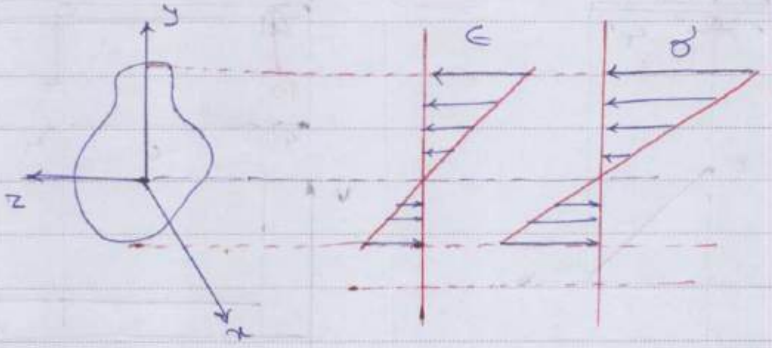
I is better than ├──┤

خمش غیرمتقارن



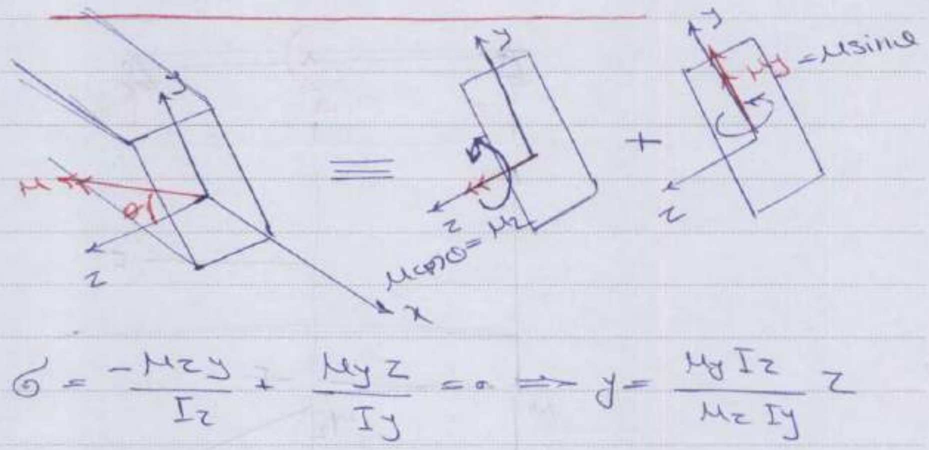
$$\Sigma F = \int \sigma\, dA = 0 \Rightarrow \int y\, dA$$

خط خنثی از مرکز سطح عبور می‌کند

$$\Sigma M_z = M = \int -y\underset{\downarrow\ \frac{y\sigma_{max}}{c}}{\sigma}\, dA = \frac{\sigma_{max}}{c}\int y^2\, dA \Rightarrow \sigma_{max} = \frac{Mc}{I}$$

$$\Sigma M_y = 0 = -\int z\sigma\, dA = -\frac{\sigma_{max}}{c}\int yz\, dA = 0$$

مثال:



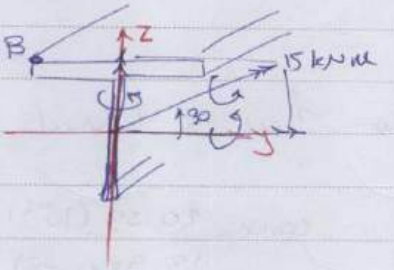
$$\sigma = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} = 0 \Rightarrow y = \frac{M_y I_z}{M_z I_y} z$$

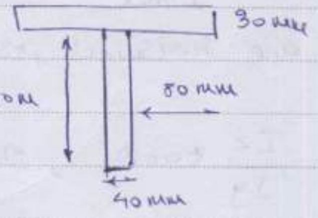
$$\Rightarrow y = \frac{M\sin\theta\, I_z}{M\cos\theta\, I_y} z = \left(\frac{I_z}{I_y}\tan\theta\right) z$$

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

شیب خط خنثی از مرکز سطح

حل:





$M_y = 15\cos 30 = 12.99\ kNm$

$M_z = 15\sin 30 = 7.5\ kNm$

$$\bar{z} = \frac{\sum \bar{z}A}{\sum A} = \frac{0.05(0.1)(0.04) + 0.115(0.03)(0.2)}{(0.1)(0.04) + (0.03)(0.2)}$$

$\rightarrow \bar{z} = 0.089\ m$ از پایین

$$I_z = \frac{1}{12}(0.1)(0.04)^3 + \frac{1}{12}(0.03)(0.1)^3 = 20.53(10^{-6})\ m^4$$

$$I_y = \left[\frac{1}{12}(0.04)(0.1)^3 + (0.1)(0.04)(0.089-0.05)^2\right] + \left[\frac{1}{12}(0.2)(0.03)^3 + (0.2)(0.03)(0.115-0.087)^2\right]$$

$$= 19.92(10^{-6})\ m^4$$

$$\sigma_B = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{(7.5)(-0.1)}{20.53\times10^{-6}} + \frac{12.99(0.041)}{19.92\times10^{-6}} = 74.8\ MPa$$

$$\sigma_C = \frac{-7.5(0.02)}{20.53(10^{-6})} + \frac{12.99(-0.089)}{19.92(10^{-6})} = -90.3\ MPa$$

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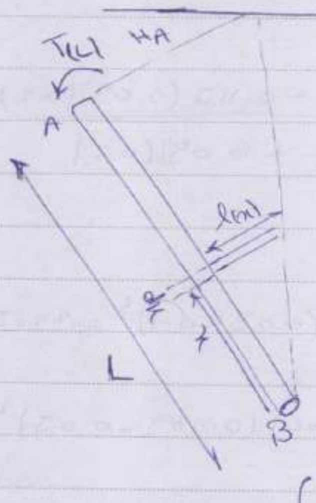
y represents I_{min}

z " I_{max}

θ & α are measured positive from +z towards +y axis.

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta \quad ; \theta = 60 \qquad \tan\alpha = \frac{20.33(10^{-6})}{13.92(10^{-6})}\tan 60$$

$$\Rightarrow \alpha = 68.6$$



ضخامت تیره — جرم واحد حجم 9-4-14

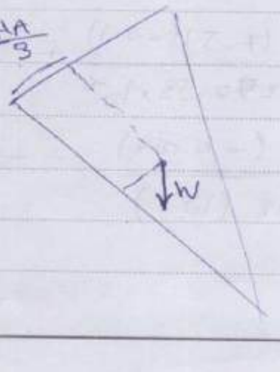
$$T(x) = \underset{k}{\boxed{t\rho g}}\; l(x)\left(\frac{l(x)}{2}\right)dx$$

$$l(x) = \frac{H_A x}{L}$$

$$W = \int_0^L k\,l(x)\,dx \Rightarrow k = \frac{2W}{LH_A}$$

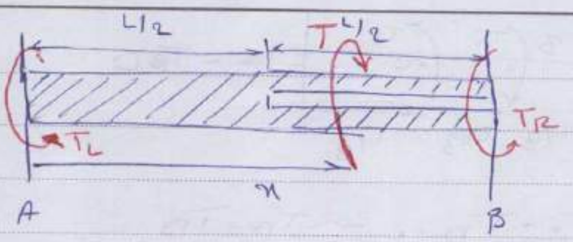
$$T(L) = \int_0^L \frac{kH_A^2}{2L^2}x^2\,dx \Rightarrow T(L) = \frac{H_A W}{3}$$

$$\Rightarrow T_{max} = \ldots$$



PAPCO

3-8-7



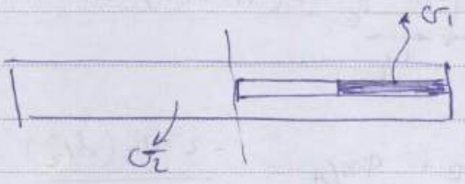
$$T_R + T_L = T \xrightarrow{T_R = T_L} T_R = T_L = T/2$$

$$\phi_R = \frac{T_R(L-x)}{GJ_H}$$

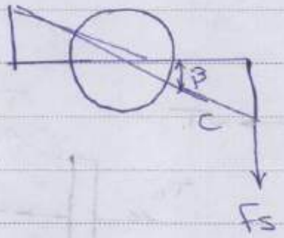
$$\Rightarrow \phi_R = \phi_L$$

$$\phi_L = \frac{T_L(x-L/2)}{GJ_H} + \frac{T_L\, L/2}{GJ_N}$$

$$\Rightarrow \frac{T/2\,(L-x)}{J_H} - \frac{T(x-L/2)}{J_H} = \frac{\frac{T}{4}L}{J_N}$$



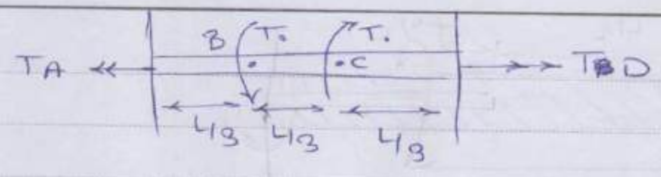
3-8-8



$$\left.\begin{array}{l} F_S = k(b - c\beta) \Rightarrow T_S = 2k(b-\beta c)c \\ \beta = \dfrac{T_S L}{G I_P} \end{array}\right\}$$

$$\Rightarrow \beta \checkmark$$

3-82



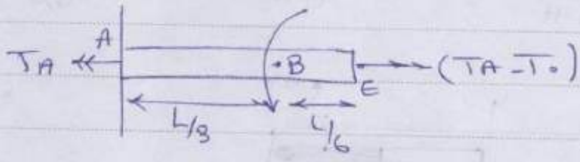
$-T_A+T_0-T_0+T_D=0 \Rightarrow T_A=T_D$

$\phi_{AD}=0 \Rightarrow \phi_{A/B}+\phi_{B/C}+\phi_{C/D}=0$

$$\Rightarrow \frac{T_A(l/3)}{JG}+\frac{(T_A-T_0)\,l/3}{JG}+\frac{(T_D)\,l/3}{JG}=0$$

$$\Rightarrow T_A=T_D=T_0/3$$

$$\phi_{max}=\phi_{AB}=\frac{\frac{T_0}{3}(l/3)}{JG}$$

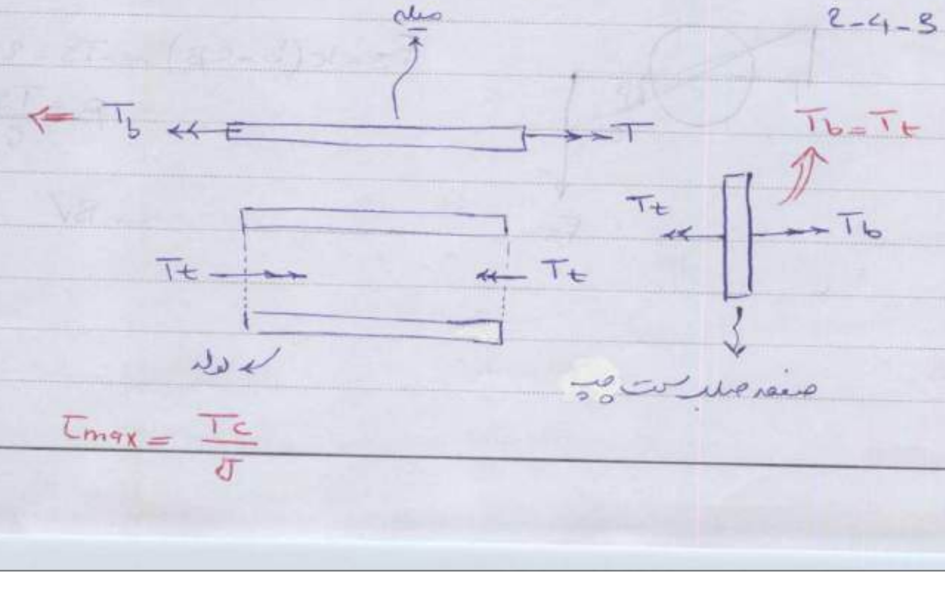


$(T_A-T_0)=-2T_0/3$

$$\phi_{E/A}=\phi_{E/B}+\phi_{B/A}=\frac{-2T_0/3\,(l/6)}{JG}+\frac{T_0/3\,(l/3)}{JG}=0$$

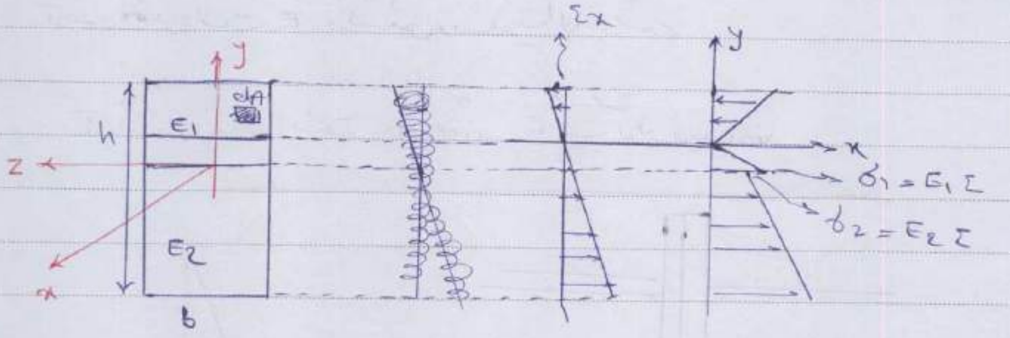
2-4-3

$T_b=T$



$T_b=T_t$

$$\tau_{max}=\frac{Tc}{J}$$



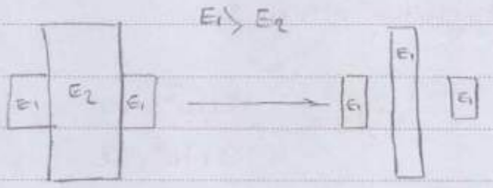
$E_1 > E_2$

$dA = dy\,dz$

$$\underbrace{\sigma_1 dA_1}_{dF_1} = (E_1 \varepsilon)\,dy\,dz = dF_2 = \sigma_2 dA_2 = (E_2 \varepsilon) n\,dy\,dz$$

$$\Rightarrow n = \frac{E_1}{E_2}$$

$y = \text{const}$ $\quad dy = \text{const}$ $\qquad dz_2 = n\,dz_1$



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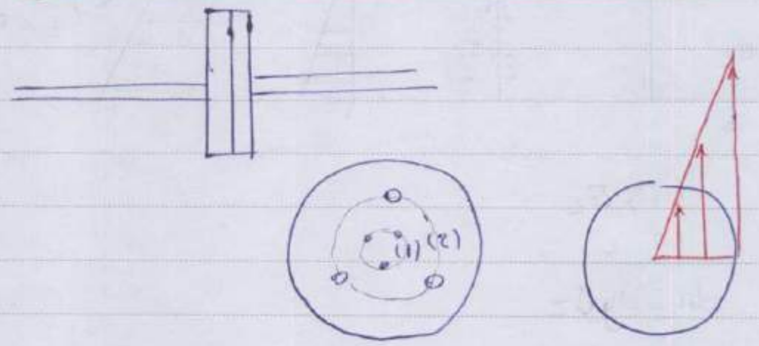
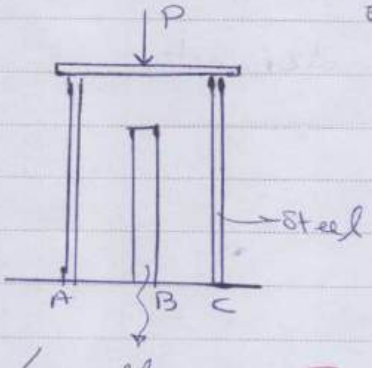
توجه: ممکن است E در کشش و فشار یکسان نباشد

I اتصال خاص نسبت به محور چرخش + پیچ پرچ فلنج ها



$$T = \left(\tau_{all} * A * \text{شعاع}\right) * \text{تعداد پیچ} + \left(\frac{r_1}{r_2}\tau_{all} * A * \text{شعاع}\right) * \text{تعداد پیچ}$$

(2) (1)



$E_s = 3 \times 10^6$ psi

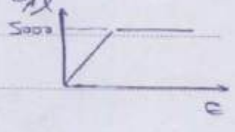
$l = 10''$, $A = 2\,in^2$

$E_{Al} = 10^7$ psi

$\sigma_y = 5000$ psi

$l = 9.995''$

$A = 4\,in^2$



Al الاستیک پلاستیک کامل

ⓐ gap = 0.005"

ⓐ P = 30 kips

ⓑ P = 200 kips

$$\Delta l_s = \frac{15 \times 10^3 \times 10}{3 \times 10^6 \times 2} = 0.025'' > 0.005''$$

gap بسته می شود ← نیرو به صفحه وارد می شود

PAPCO

$$\begin{cases} 2F_S + F_{Al} = P \quad (1) \\ \Delta_S = \Delta_{Al} + gap \, (2) \Rightarrow \dfrac{F_S(10)}{3\times10^6\times2} = \dfrac{\overset{9.995}{\cong}}{} \dfrac{F_{Al}(10)}{4\times10^7} + 0.005 \end{cases}$$

$(1),(2) \Rightarrow F_S = 5769.28$

$F_{Al} = 18462.54 \quad \sigma_{Al} = \dfrac{F_{Al}}{4} = 4615.63 < \sigma_y$

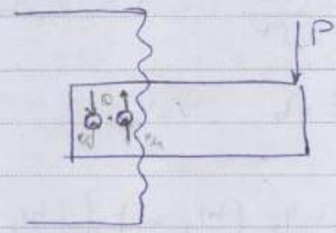
ⓑ $F_S = 25384.6 \quad F_{Al} = 199231.78$

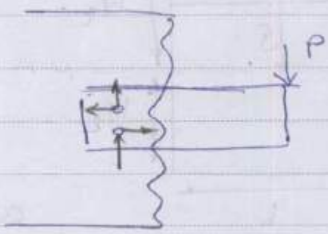
$\sigma_{Al} = 97307.94 > \sigma_y \rightarrow$ not accepted

چون آلومینیم بیشتر از σ_y تنش تحمل نمی‌کند

$\sigma_{Al})_{max} = 5000 \Rightarrow F_{Al})_{max}) = 5000\times4 = 20\,000 \text{ lb} = R_B$

$F_S = \dfrac{200\,000 - 20\,000}{2} = 90\,000 = R_A = R_C$





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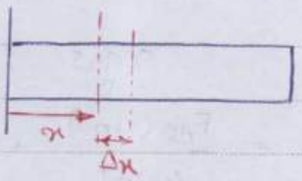
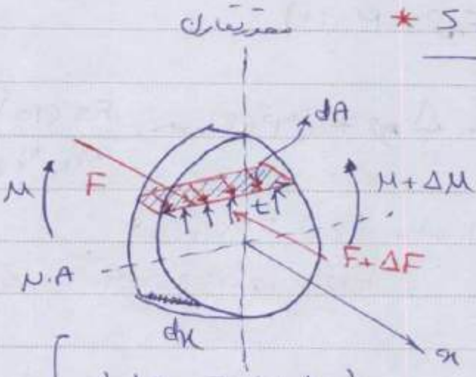
* فصل ۶ *



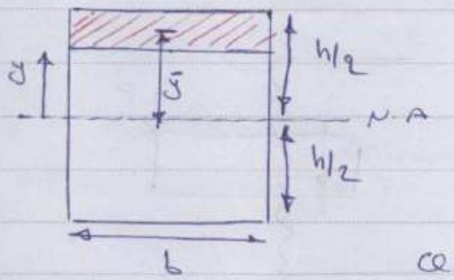


$$\sum F_x = 0 \Rightarrow \int_{A'} \sigma' dA - \int_{A'} \sigma dA - \tau (t dx) = 0$$

$$\Rightarrow \int_{A'} \left(\frac{M+dM}{I}\right) y dA - \int_{A'} \frac{M}{I} y dA - \tau t dx = 0$$

$$\Rightarrow \frac{dM}{I} \int_{A'} y dA = \tau t dx \Rightarrow \tau = \frac{1}{It}\left(\frac{dM}{dx}\right)\int_{A'} y dA$$

$$\Rightarrow \tau = \frac{VQ}{It} \quad ; \quad Q = \int_{A'} y dA \quad ; \quad V = \frac{dM}{dx}$$



$$\tau = \frac{VQ}{It} \qquad I = \frac{1}{12} b h^3$$

$$t = b \qquad V = V$$

$$Q = \underbrace{\left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right]}_{\bar{y}} \underbrace{\left(\frac{h}{2} - y\right) b}_{A}$$

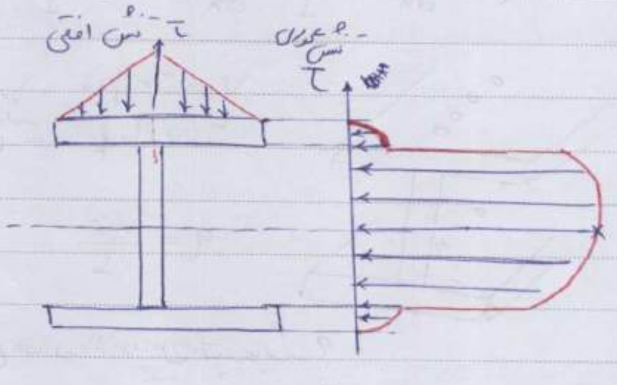
$$\Rightarrow Q = \frac{1}{2}\left(\frac{h^2}{4} - y^2\right) b$$

$$\tau = \frac{V\frac{1}{2}\left(\frac{h^2}{4}-y^2\right)b}{\frac{1}{12}bh^3 b} = \frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right)$$

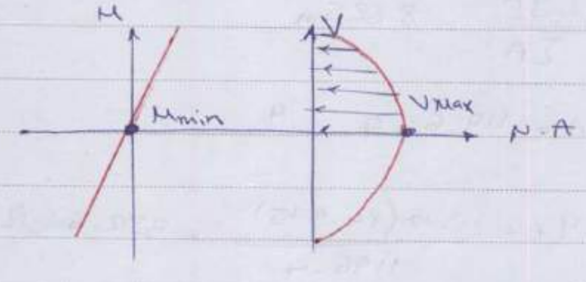
$$y = h/2 \longrightarrow \tau = 0 \qquad y = 0 \Longrightarrow \tau_{max} = \frac{6V}{4bh} = 1.5\frac{V}{A}$$

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right)$$

$$\int_{-h/2}^{h/2} \frac{6V}{bh^3}\left(\frac{h^2}{4}-y^2\right)\underbrace{b\,dy}_{dA} = \frac{6V}{h^3}\left[\frac{h^2}{4}y - \frac{y^3}{3}\Big|_{-h/2}^{h/2}\right] = V$$

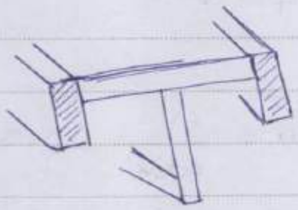


X Section مستطیلی

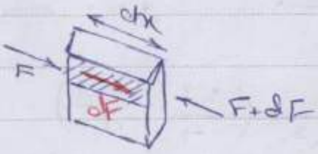


* تنش برشی ماکزیمم همیشه در سطح خنثی اتفاق می‌افتد *

Subject:
Year. Month. Date. ( )



جریان برشی: نیروی برشی به ازای واحد طول $q = \frac{dF}{dx}$



$$\boxed{q = \frac{VQ}{I}}$$

$$dF = \sigma' \int_A dA = \frac{dM}{I}\int y\,dA = \frac{dM}{I} Q$$

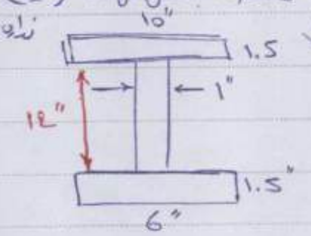
$$q = \frac{dF}{dx} = \frac{1}{I}\frac{dM}{dx} Q = \frac{VQ}{I}$$

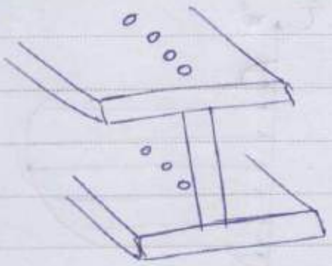
مثال:

$V = 5\text{kips}$

نیروی در هر میخ قابل تحمل ← 500 lbf

فاصله‌ها با شکل نمی‌خونند (امیدوارم اشکال نداره...!)





$$q = \frac{VQ}{I}$$

فاصله میخ‌ها در بالا و پایین چقدر باشد؟

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = 8.625\text{in}$$

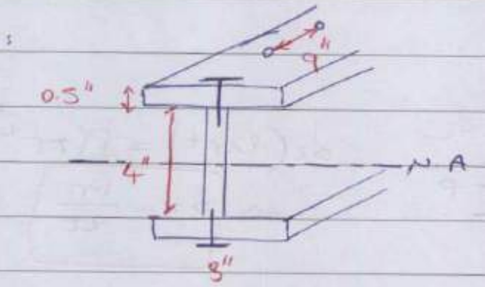
$$I_{N.A} = 119.6.4\ \text{in}^4$$

$$q_t = \frac{500(84.375)}{1196.4} = 352.6\ \text{lb/in} \Rightarrow S_t = \frac{500}{352.6} = 1.42$$

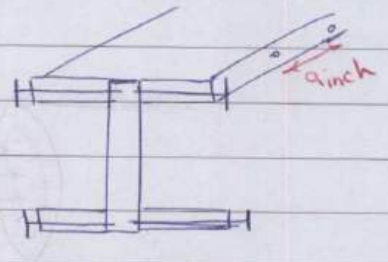
$$q_b = \frac{500(70.87)}{1196.4} = 296.19\ \text{lb/in} \Rightarrow S_b = \frac{500}{296.19} = 1.69$$

PAPCO

مثال:



Case I



Case II

ماکزیمم نقل "V" را در هر مورد بدست آورید. $40\,lb_f =$ قدرت هر میخ

$$I = \frac{1}{12}\times 3\times 5^3 - 2\times\frac{1}{12}\times 1\times 4^3 = 20.58\ in^4$$

$$\text{case I}: \quad q = \frac{VQ}{I} \Rightarrow \frac{40}{9} = \frac{V\,(3\times 0.5)\times(2.25)}{20.58} \Rightarrow V = 27.1\ lb$$

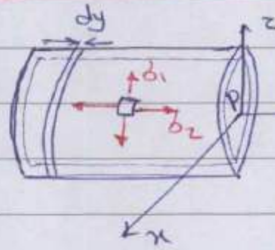
$$\text{case II}: \quad q = \frac{VQ}{I} \Rightarrow \frac{40}{9} = \frac{V\,(1\times 0.5)\times(2.25)}{20.58}$$

مخازن جدار نازک:

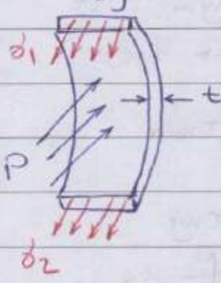
$\sigma_2 =$ تنش طولی

$\sigma_1 =$ تنش محیطی

تنش "hoop"

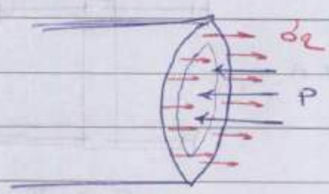


$$\frac{r}{t} \geq 10$$



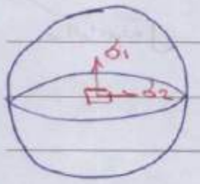
$$\Sigma F_x = 0 \Rightarrow 2(\sigma_1)(t\,dy) = P(2r\,dy)\ P$$

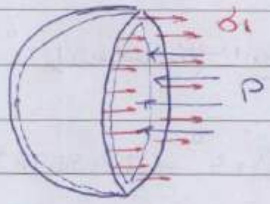
$$\Rightarrow \boxed{\sigma_1 = \frac{Pr}{t}}$$



$$\sigma_2(2\pi r t) = P(\pi r^2)$$

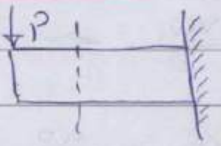
$$\Rightarrow \boxed{\sigma_2 = \frac{Pr}{2t}}$$

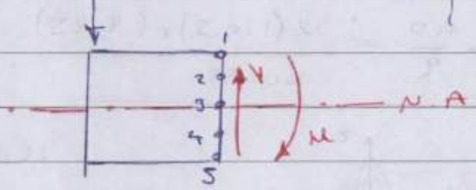


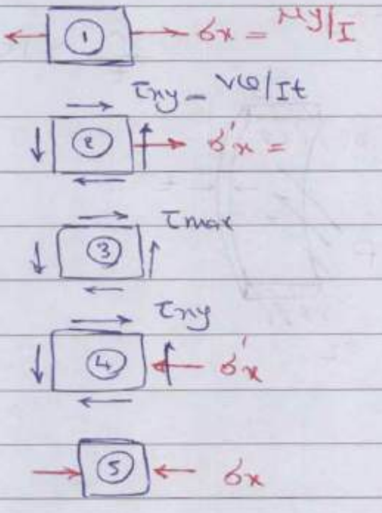


$$\sigma_1(2\pi r t) = P(\pi r^2)$$

$$\Rightarrow \boxed{\sigma_1 = \sigma_2 = \frac{Pr}{2t}}$$







$\sigma_x = My/I$

$\tau_{xy} = VQ/It$

$\sigma'_x =$

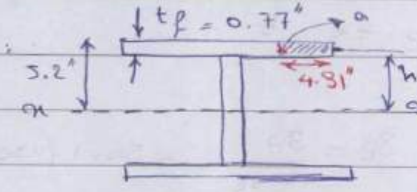
τ_{max}

τ_{xy}

σ'_x

σ_x

حل:



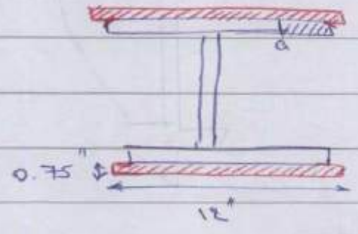
$I_{xx} = 394\ in^4$

$V = 50\ kips$

$\tau_a = \frac{VQ}{It}$ $h = 5.2 - \frac{0.77}{2} = 4.815"$

$Q = (4.31 \times 0.77) \times 4.815 = 19.98\ in^3$

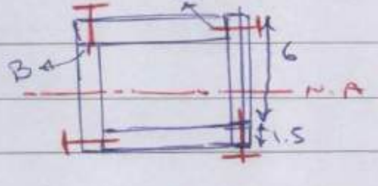
$\tau = \frac{50}{394} \times \frac{19.98}{0.77} = 2.63\ ksi$



$Q = 2\left(4.31 \times 0.77 \times 4.815\right) + \left(12 \times 0.75 \times 5.575\right)$
$= 82.1\ in^3$

$I = 394 + 2\left(\frac{1}{12} \times 12 \times (0.75)^3 + (12 \times 0.75)(5.575)^2\right) = 954\ in^4$

$\tau_a = \frac{50 \times 82.1}{954 \times \underbrace{(1.54)}_{2 \times 0.77}} = 2.79\ ksi$



قص C: 30 lb مسافه مسامير C = ?

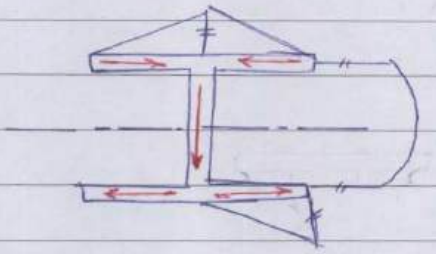
$I_{N.A} = \frac{1}{12}(7.5)^4 - \frac{1}{12}(4.5)^4 = 229.5\ in^4$

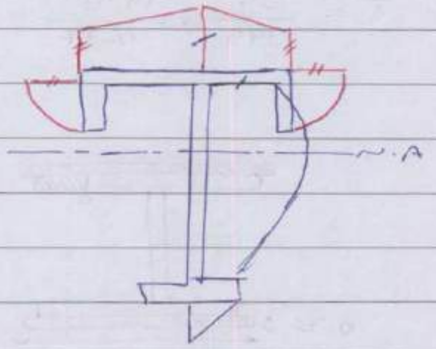
$Q_B = \left(\frac{4.5}{2} + 0.75\right)(1.5 \times 7.5) = 33.75\ in^3$

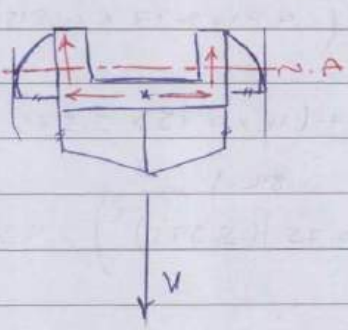
$q_B = \frac{VQ}{I} = 11.76\ lb/in$ $S_B = \frac{30}{\frac{11.76}{2}} = 50.1''$ (use $50''$)

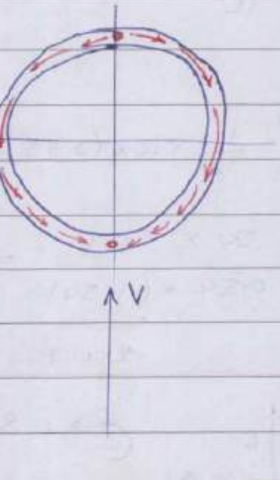
$q_c = \frac{VQ}{I} = \frac{80 \times 20.25}{229.5} = 7.059\ lb/in$

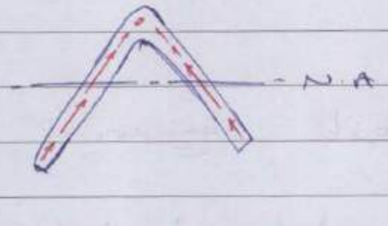
$Q_c = 3(4.5 \times 1.5) = 20.25$ $S_c = \frac{30}{(7.059)/2} = 8.5''$

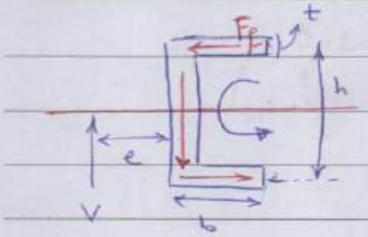




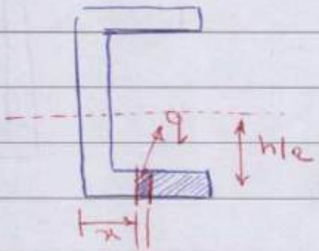








$Ve = F_f\, h$



$$F_f = \int_0^b q\,dx$$

$$q = \frac{VQ}{It}$$

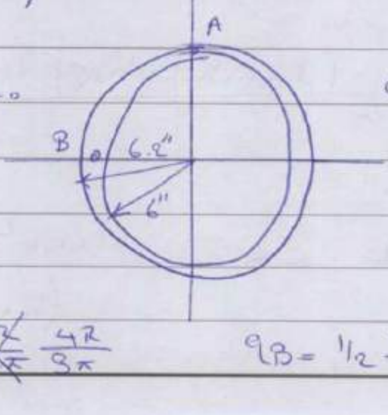
$$I = \frac{1}{12}th^3 + 2(bt)(h/2)^2 = \frac{th^2}{2}\left(h/6 + b\right)$$

$$q = \frac{VQ}{I} = \frac{V h/2\,(b-x)t}{\frac{th^2}{2}(h/6+b)} = \frac{V(b-x)}{h(h/6+b)}$$

$$F_q = \int_0^b q\,dx = \frac{V}{h(h/6+b)}\int_0^b (b-x)\,dx \longrightarrow F_q = \frac{Vb^2}{2h(h/6+b)}$$

$$Ve = F_f h \longrightarrow e = \frac{b^2}{h/3 + 2b}$$

$$I = \pi/4\,(6.2^4 - 6^4) = 45.4\pi \quad in^4$$

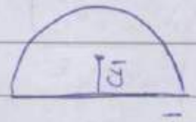


A [illegible] : $q_A = 0$, $\tau_A = 0$

$Q_A = 0$

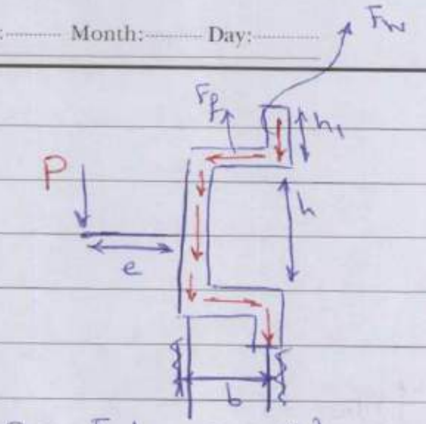
$$Q_B = \sum y'A' = \frac{4(6.2)}{3\pi} \times \frac{\pi(6.2)^2}{2} - \frac{4(6)}{3\pi}\left(\frac{\pi 6^2}{2}\right)$$

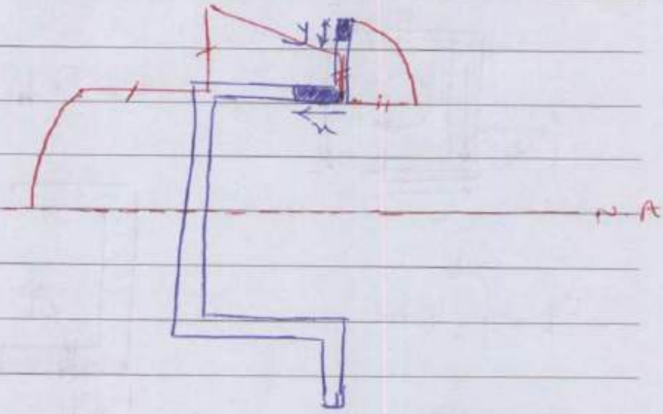
$$= 14.88 \text{ in}^4$$



$$\bar{y} = \cancel{\frac{3R}{4\pi}}\ \frac{4R}{3\pi}$$

$$q_B = 1/2\,\frac{VQ_B}{I} = 417 \text{ lb/in}$$





$P_e = F_f h - 2F_w (b)$

$I = \frac{1}{12} t(h+2h_1)^3 + 2(bt)(h/2)^2 = \frac{t}{12}\left((h+2h_1)^3 + 6bh^2\right)$

$Q_1 = (h/2 + h_1 - y/2)\,yt = t(h/2\,y + h_1 y - y^2/2)$

$Q_2 = (h/2 + h_1/2)(h_1 t) + h/2\,xt = t/2\left(h_1(h+h_1) + hx\right)$

$q_1 = \frac{VQ_1}{I} = Pt(h/2\,y + h_1 y - y^2/2)/I$

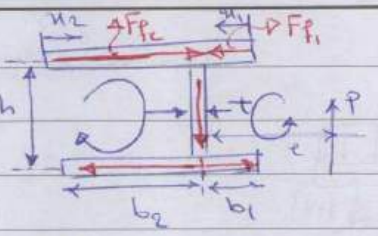
$q_2 = \frac{VQ_2}{I} = Pt\left(h_1(h+h_1) + hx\right)/2I$

$F_w = \int_0^{h_1} q_1\,dy = \frac{Pt}{I}\int_0^{h_1} (\qquad)\,dy = \frac{Pt}{12I}\left(3hh_1^2 + 4h_1^3\right)$

$F_f = \int_0^{b} q_2\,dx = \frac{Pt}{2I}\int_0^{b} \left(h_1(h+h_1) + hx\right) dx = \frac{Pt}{2I}\left(2bh_1(h+h_1) - b^2h\right)$

$Pe = F_f h - eF_w b \longrightarrow e = \frac{b(6h_1h^2 + 3bh^2 - 8h_1^3)}{(h+2h_1)^3 + 6bh^2}$

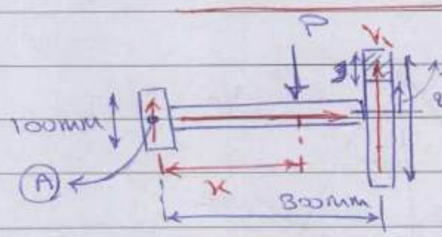
حل:

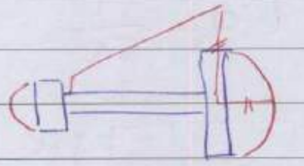


$Pe = (F_{f_2})h - (F_{f_1})h$

$I = \frac{th^2}{12}\left(h + 6(b_1 + b_2)\right)$

$Q_1 = \frac{bI}{2} x_1 \qquad Q_2 = \frac{bI}{2} x_2 \qquad e = \frac{3(b_2^2 - b_1^2)}{h + 6(h_1 + h_2)}$





$\sum M_A = 0 \Longrightarrow Pe = 300V_1$

$I = \frac{1}{12}t(0.1)^3 + \frac{1}{12}t(0.2)^3 = 0.75(10^{-3})t \ m^4$

$\bar{y}' = y + \frac{0.1 - y}{2} = \frac{1}{2}(y + 0.1)$

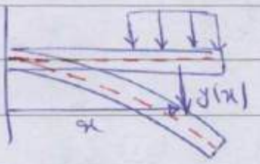
$Q = \bar{y}'A' = \frac{1}{2}(y + 0.1)(0.1 - y)t = \frac{t}{2}(0.01 - y^2)$

$q = \frac{VQ}{I} = 666.67P(0.01 - y^2)$

$V_1 = \int_{-0.1}^{0.1} q\,dy = 666.67P(0.01 - y^2)\,dy = 0.8889P$

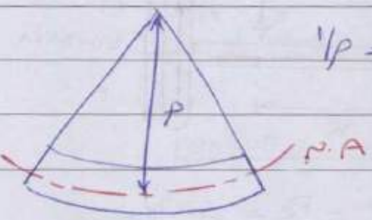
Diflection of beams / shafts — خیز تیرها / شفت‌ها

$$\begin{cases} \sigma = F/A \\ \delta l = \dfrac{FL}{AE} \end{cases}$$

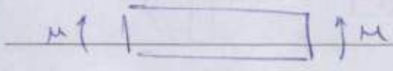


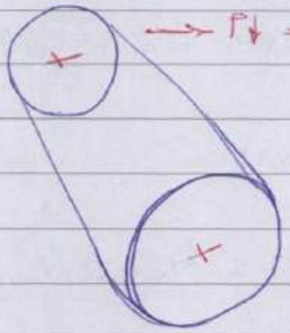
$$\begin{cases} \tau = \dfrac{T\rho}{J} \\ \phi = \dfrac{TL}{JG} \end{cases}$$

$$\sigma = \frac{My}{I}$$



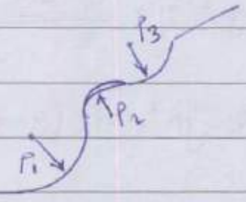
$$1/\rho = M/EI$$





→ P↓ → M↑

همان گشتاوری باید وارد شود تا به دور
پولی خم شود

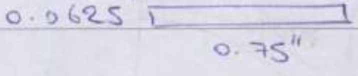


ρ ثابت ⟹ $1/\rho = \dfrac{M}{EI}$ ⟹ M ثابت

$\sigma = \dfrac{My}{I}$ ⟹ σ ثابت ⟹ بنابراین
پلاستیک می‌رسانند

$1/\rho = \frac{M}{EI}$

مثال: تسمه اره:



$\Rightarrow M = \frac{EI}{\rho}$

حول یک pulley با شعاع 12'

$\sigma = \frac{Mc}{I} = \frac{EIc}{I\rho} = \frac{cE}{\rho} = 29\times10^3 \times \frac{0.0625}{2\times12} = ۷۵٫۵\ \text{ksi}$

EI = ثابت ← در طول تیر

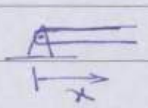
$$EI\frac{d^4y}{dx^4} = -w(x)$$

$$\xrightarrow{\text{انتگرال}} EI\frac{d^3y}{dx^3} = V(x) \xrightarrow{\text{انتگرال}} \boxed{EI\frac{d^2y}{dx^2} = M(x)}$$

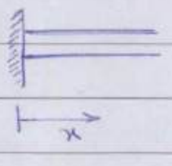
تقعر ⊙

مقعر به بالاست

انتگرال می‌خواهد (تیرهم که دو شرط دارد)



شرایط مرزی: $y(x=0) = 0$ و $\frac{d^2y}{dx^2} = M(x=0) = 0$

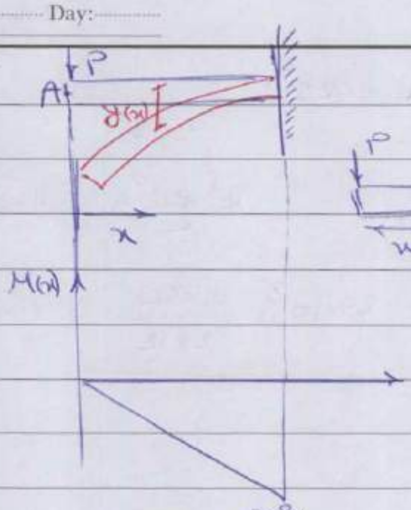


شرایط مرزی: $y(x=0) = 0$, $\frac{dy}{dx}(x=0) = 0$

$$\overset{M(x=0)}{\overline{y''}}(x=0) = 0$$

$$\underset{V(x=0)}{\underline{y'''}}(x=0) = 0$$

مثال:



$M(x) = -Px$

$EIy'' = M(x) = -Px$

$EIy' = -Px^2/2 + c_1$

$EIy = -Px^3/6 + c_1x + c_2$

$y(x=\ell) = 0 \Rightarrow c_2 = -P\ell^3/3$

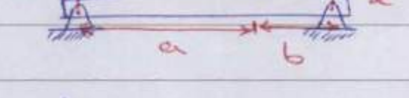
$y'(x=\ell) = 0 \Rightarrow c_1 = P\ell^2/2$

$\Rightarrow y = \frac{P}{6EI}\left(-x^3\cancel{/3} + 3\ell^2x - 2\ell^3\right)$

$y_{max} = y(x=0) = \frac{-P\boxed{\ell^3}}{3EI}$ ⟹ توان 3 ⟹ most imp

$y'(x=0) = \frac{P\ell^2}{2EI}$ $\theta_A = \tan^{-1}\left(y'(x=0)\right)$

مثال:



$EIy'' = M(x)$

$0 \leq x_1 \leq a \rightarrow EIy_1''(x_1) = M_1(x_1)$ $M_1(x_1) = \frac{Pb}{\ell}x_1$

$EIy_1'' = \frac{Pb}{\ell}x_1 \Rightarrow EIy_1 = \frac{Pb}{6\ell}x_1^3 + c_1x_1 + c_2$

$y_1(x_1=0) = 0$

$a \le x_2 \le l$ $\quad EIy_2'' = M_2(x_2)$ $\quad M_2 = \frac{Pb}{l}x_2 - P(x_2 - a)$

$\xrightarrow{b=(l-a)} M_2 = Pa(1 - x_2/l)$

$EIy_2'' = Pa(1 - x_2/l) \rightarrow EIy_2' = Pa\left(x_2 - \frac{x_2^2}{2l}\right) + c_3$

$\Rightarrow EIy_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^3}{6l}\right) + c_3x_2 + c_4$

$y_2(x_2 = l) = 0$

$y_1(x_1 = a) = y_2(x_2 = a)$ $\quad c_1 = \frac{-Pb}{6l}(l^2 - b^2)$ $\quad c_2 = 0$

$y_1'(x_1 = a) = y_2'(x_2 = a)$

$c_3 = \frac{-Pa}{6l}(2l^2 + a^2)$ $\quad c_4 = Pa^3/6$

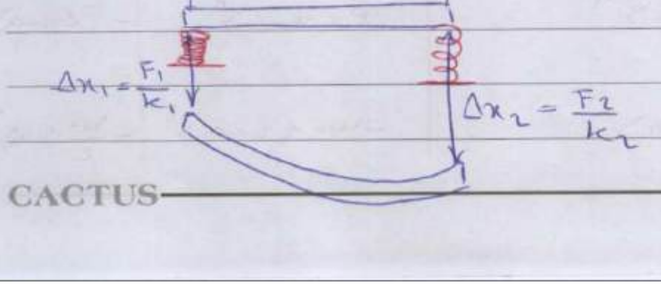
$\Rightarrow y_1 = \frac{Pb}{6EIl}\left(x_1^3 - (l^2 - b^2)x_1\right)$

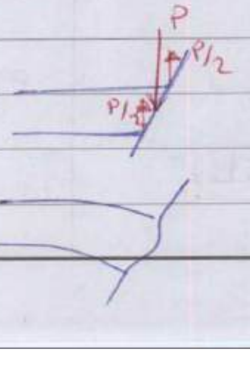
$y_2 = \frac{Pa}{6EIl}\left(3x_2^2 l - x_2^3 - (2l^2 + a^2) + a^2 l\right)$

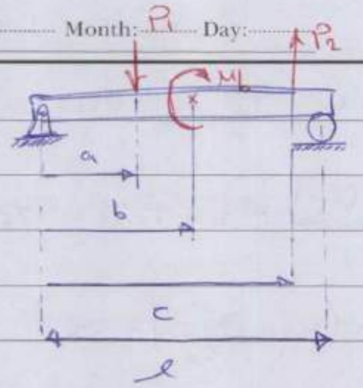
برای یافتن MAX خیز باید معادلات $\underbrace{y_1' = 0}_{\text{جواب باید}}$ و $\underbrace{y_2' = 0}_{\text{جواب باید}}$ را حل کرد

جواب باید → $0 < x_1 < a$

جواب باید ↓ $a < x_2 < l$







* برای تیر شکل *

$$M = R_1 x \qquad 0 \leqslant x \leqslant a$$

$$M = R_1 x - P_1(x-a) \qquad a \leqslant x \leqslant b$$

$$M = R_1 x - P_1(x-a) + M_b \qquad b \leqslant x \leqslant c$$

$$M = R_1 x - P_1(x-a) + M_b + P_2(x-c)$$

$$c \leqslant x \leqslant l$$

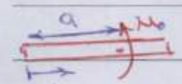
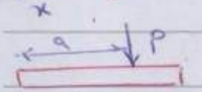
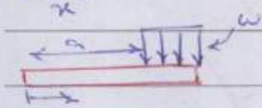
$$\langle x-a\rangle^n = \begin{cases} 0 & 0 < x < a \\ (x-a)^n & a < x > \infty \end{cases} \qquad n \geqslant 0$$

$$\langle x-a\rangle^n = \begin{cases} 0 & x < a \\ (x-a)^n & x \geqslant a \end{cases} \qquad n \geqslant 0$$

$$\int \langle x-a\rangle^n \, dx = \frac{\langle x-a\rangle^{n+1}}{n+1} + C \qquad n \geqslant 0$$

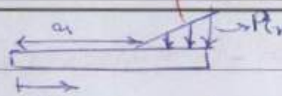
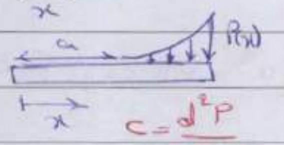
$$\int \langle x-a\rangle^n \, dx = \langle x-a\rangle^{n+1} \qquad n = -1, -2$$

$\langle x-a\rangle^{-2}$ و $\langle x-a\rangle^{-1}$ هردو تابع منفردند یعنی در $x = a$ بی‌نهایت می‌گردند.

Loading	Loading function $w(x)$	Shear $V = -\int w(x)dx$	$M = \int V dx$
	$M_0 \langle x-a\rangle^{-2}$	$-M_0 \langle x-a\rangle^{-1}$	$-M_0 \langle x-a\rangle^{0}$
	$P \langle x-a\rangle^{-1}$	$-P \langle x-a\rangle^{0}$	$-P \langle x-a\rangle^{1}$
	$w_0 \langle x-a\rangle^{0}$	$-w_0 \langle x-a\rangle^{1}$	$-\frac{w_0}{2} \langle x-a\rangle^{2}$

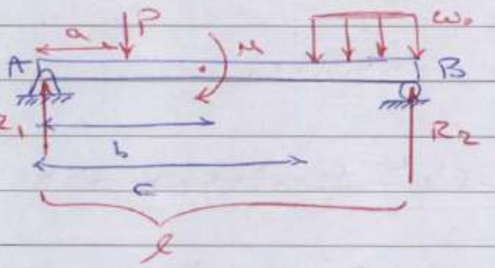
شیب خط $m = \frac{dP}{dx}$

Year: Month: Day: Subject:

	$w(x)$	$V(x)$	$M(x)$
	$m\langle x-a\rangle^{1}$	$-\frac{m}{2}\langle x-a\rangle^{2}$	$-\frac{m}{6}\langle x-a\rangle^{3}$
	$\frac{c}{2}\langle x-a\rangle^{2}$	$-\frac{c}{6}\langle x-a\rangle^{3}$	$-\frac{c}{24}\langle x-a\rangle^{4}$

$c = \frac{d^2P}{dx^2}$

مثال :

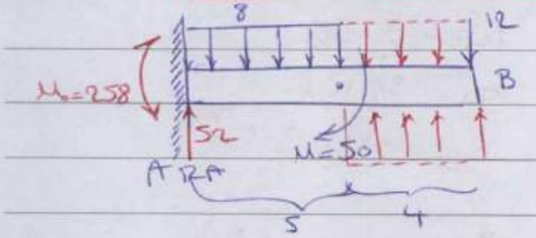


$$\omega(x) = -R_1\langle x-0\rangle^{-1} + P\langle x-a\rangle^{-1} - M_0\langle x-b\rangle^{-2} + \omega_0\langle x-c\rangle^{0}$$

اگر عکس‌العمل R1 را دارد ادامه میدهیم (مانند مثال بالا) دو ثابت اول انتگرال صفر است.

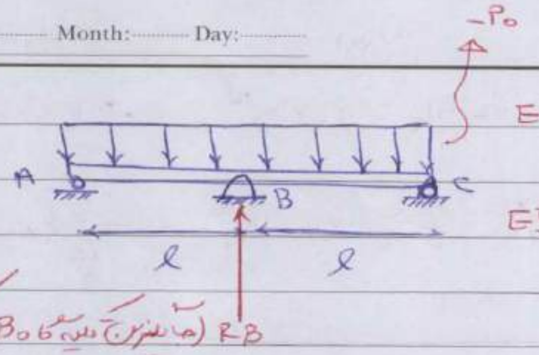
A در ثابت اول همان R1 ، ثابت دوم به M مربوط می‌شود

مثال :



$$\omega(x) = \Big(-52\langle x-0\rangle^{-1} + 258\langle x-0\rangle^{-2} + 8\langle x-0\rangle^{0}$$
$$-8\langle x-5\rangle^{0} - 50\langle x-5\rangle^{-2}\Big) = EI\,y^{(4)}_{(x)}$$

* چون RA و M را داریم و w(x) را داریم پس دیگر دو ثابت اول انتگرال را نخواهیم داشت



$$EIy^{(4)} = \omega(x) = -P_0 + R_B\langle x-l\rangle^{-1}$$

$$EIy^{(3)} = -P_0\, x + R_B\langle x-l\rangle^{0} + c_1$$

RB (جانشین تکیه گاه B میانی)

$$EIy'' = \frac{-P_0}{2}x^2 + R_B\langle x-l\rangle + c_1x + c_2$$

$$EIy' = \frac{-P_0}{6}x^3 + \frac{R_B}{2}\langle x-l\rangle^2 + \frac{c_1}{2}x^2 + c_2x + c_3$$

$$EIy = \frac{-P_0}{24}x^4 + \frac{R_B}{6}\langle x-l\rangle^3 + \frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3x + c_4$$

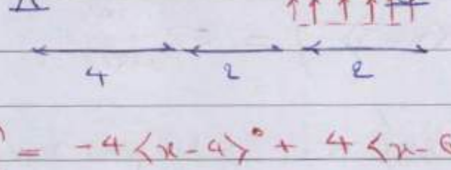
۵ مجهول c_1, c_2, c_3, c_4, R_B که به ۵ شرط ضروری لازم است:

$y(0) = y(l) = y(2l) = 0$
$EIy''(0) = EIy''(2l) = 0$

$c_4 = c_2 = 0$
$R_B = \frac{5}{4}P_0 l$
$c_1 = 3/8\, P_0 l$
$c_3 = -1/48\, P_0 l^3$

$$\Rightarrow y(x) = \frac{-P_0}{48EI}\left(2x^4 - 3lx^3 - 10l\langle x-l\rangle^3 \overset{\wedge}{} + l^3x\right)$$

مثال:



$$EIy^{(4)} = -4\langle x-4\rangle^0 + 4\langle x-6\rangle^0$$
$$EIy^{(3)} = -4\langle x-4\rangle^1 + 4\langle x-6\rangle^1 + c_1$$

$$EIy'' = -2\langle x-4\rangle^2 + 2\langle x-6\rangle^2 + c_1x + c_2$$

$$\left.\begin{array}{l} y''(0)=0 \\ y''(8)=0 \end{array}\right\} \Rightarrow \begin{array}{l} c_1=3 \\ c_2=0 \end{array}$$

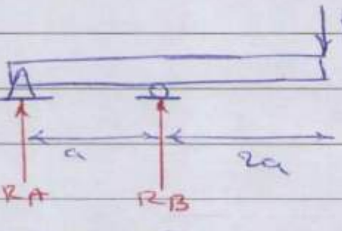
$$\rightarrow EIy'' = -2\langle x-4\rangle^2 + 2\langle x-6\rangle^2 + 3x$$

$$\rightarrow EIy' = -2/3\langle x-4\rangle^3 + 2/3\langle x-6\rangle^3 + 3/2\,x^2 + c_3$$

$$\rightarrow EIy = -1/6\langle x-4\rangle^4 + 1/6\langle x-6\rangle^4 + \frac{1}{2}x^3 + c_3x + c_4$$

$$y(0)=y(l)=0 \Rightarrow c_4=0,\ c_3=-27$$

:مثال



$$RA = -2P$$
$$RB = 3P$$

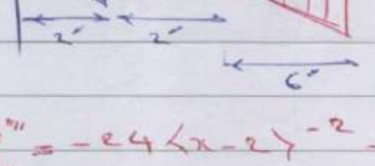
$$EIy'' = -2Px + 3P\langle x-a\rangle^1$$

$$EIy' = -Px^2 + 3P/2\langle x-a\rangle^2 + c_3$$

$$EIy = \frac{-Px^3}{3} + \frac{P}{2}\langle x-a\rangle^3 + c_3x + c_4$$

$$y(0)=y(a)=0 \rightarrow c_4=0$$
$$c_3 = Pa^2/3$$

:مثال



dP/dx

$$EIy'''' = -24\langle x-2\rangle^{-2} - 3\langle x-4\rangle^0 + 3/6\langle x-4\rangle^1$$

$$EIy''' = -24\langle x-2\rangle^{-1} - 3\langle x-4\rangle^1 + \frac{3}{12}\langle x-4\rangle^2 + c_1$$

$$\boxed{EIy'''(10)=0 \Rightarrow c_1 = 9^k}$$ مشکل پیش آمده!!!

$$EIy'' = -24\langle x-2\rangle^0 - 3/2\langle x-4\rangle^2 + 1/12\langle x-4\rangle^3 + 9x + c_2$$

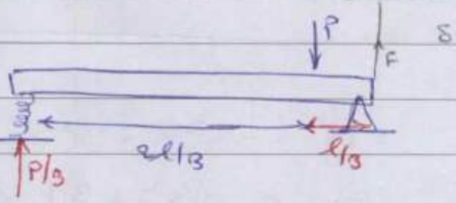
$EI\,y''(0)=0 \Rightarrow c_2=-30k$

$EI\,\theta(x) = -24\langle x-2\rangle^1 - \frac{1}{2}\langle x-4\rangle^3 + \frac{1}{48}\langle x-4\rangle^4 + \frac{9}{2}x^2$
$+30x + c_3$

$\theta(0)=0 \Rightarrow c_3=0$

$EI\,y = -12\langle x-2\rangle^2 - \frac{1}{8}\langle x-4\rangle^4 + \frac{1}{240}\langle x-4\rangle^5 + \frac{3}{2}x^3$
$-15x^2 + c_4$

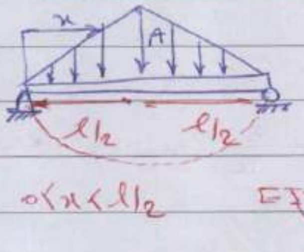
$y(0)=0 \Rightarrow c_4=0$



$\delta = \frac{Fl}{AE} = y(l)$

$EI\,y'' = Px/3 - P\langle x-2l/3\rangle^1$
$EI\,y' = Px^2/6 - P/2\,\langle x-2l/3\rangle^2 + c_1$
$EI\,y = \frac{Px^3}{18} - P/6\,\langle x-2l/3\rangle^3 + c_1x + c_2$

$$\begin{cases} y(0) = -\frac{P}{3k} \\ y(l) = 0 \end{cases} \longrightarrow \begin{array}{l} c_2 = -\frac{PEI}{3k} \\ c_1 = \frac{PEI}{3kl} - \frac{4Pl^2}{81} \end{array}$$



$0<x<l/2 \qquad EI\,y'' = \overset{\frac{Al}{4}}{R}x - \frac{Ax^3}{3l}$

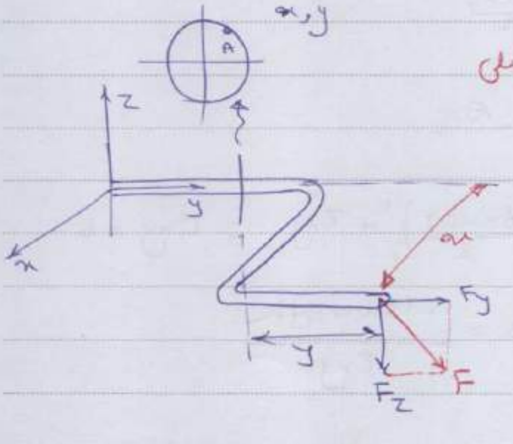
$EI\,y = \frac{A}{12EI}\left(l/2\,x^3 - \frac{x^5}{5l} + \frac{5l^3}{16}x\right)$

Stress Transformation. (تبدیل تنش)

$$\sigma_A = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} = \begin{pmatrix} \sigma_{*}^{(1)} & 0 \\ 0 & \sigma_{*}^{(2)} \end{pmatrix} = \begin{pmatrix} \frac{\sigma_x+\sigma_y}{2} & \tau_{max} \\ \tau_{max} & \frac{\sigma_x+\sigma_y}{2} \end{pmatrix}$$

x,y — x,y اصلی — $\bar{x}, \bar{y}$

* تنش های اصلی

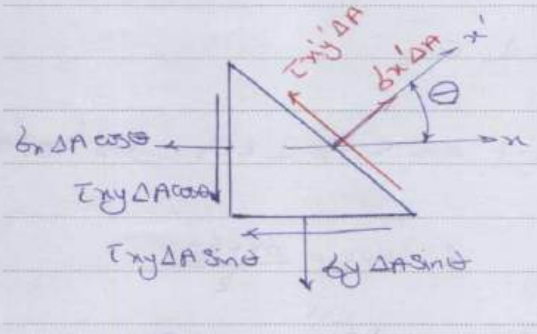


Fz(x) → پیچش می‌دهد

Fz(y) → M می‌دهد → خم می‌دهد

Fy → σ می‌دهد

Fz → برش می‌دهد



$$\sum F_{x'} = 0 \Rightarrow \sigma_{x'}\Delta A - \tau_{xy}\,\Delta A \sin\theta\cos\theta + \cdots$$

$$\cdots - \sigma_y \Delta A \sin^2\theta - \tau_{xy}\,\Delta A \cos\theta\sin\theta = 0$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos^{2}\theta + \tau_{xy}\sin 2\theta$$

$\sum F_{y'} = 0 \Rightarrow \tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$\theta \rightarrow \theta + \pi/2 \quad \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$\frac{d\sigma_{x'}}{d\theta} = 0 \Rightarrow \tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$

$2\theta_{p_1}$ "(180)" $2\theta_{p_2} \Rightarrow \theta_{p_1} \perp \theta_{p_2}$

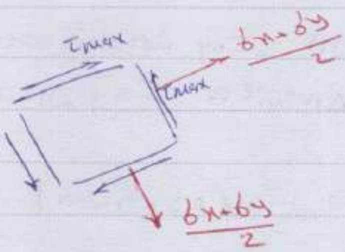
$\sigma^{(1),(2)} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \; ; \; \tau_{xy} = 0$

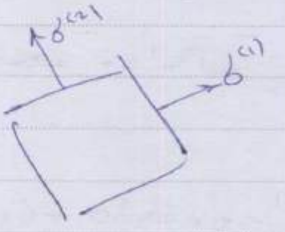
$\frac{d\tau_{x'y'}}{d\theta} = 0 \Rightarrow \tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$

$2\theta_{s_1}$ "(180)" $2\theta_{s_2} \Rightarrow \theta_{s_1} \perp \theta_{s_2}$

$(\tan\theta\, 2\theta_p) * (\tan 2\theta_s) = -1 \Rightarrow 2\theta_p \perp 2\theta_s \Rightarrow \theta_p$ "(45)" θ_s

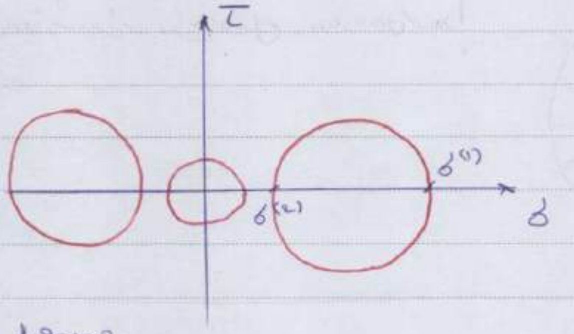
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \; ; \; \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$



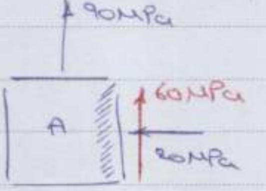


Subject:
Year. Month. Date. ( )

* دایره آمای مور *



مثال:



$\sigma_x = -20$

$\sigma_y = 90$

$\tau = 60$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} \longrightarrow 2\theta_{p_2} = -47.49$$

$$\Rightarrow \theta_{p_2} = -23.7$$

$$\Rightarrow \theta_{p_1} = 66.3$$

$$\sigma^{(1),(2)} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \begin{cases} \sigma^{(1)} = 116\,\text{MPa} \xrightarrow{\text{جایگذاری}} \text{(θp1 یا θp2) مال کدامین θ است} \\ \sigma^{(2)} = -46.4\,\text{MPa} \end{cases}$$

$$\tan 2\theta_s = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}} \qquad 2\theta_s = 42.5$$

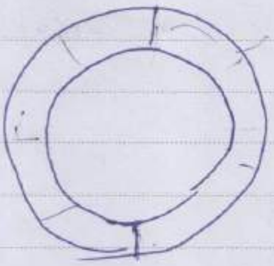
$$\theta_{s_1} = 21.8$$

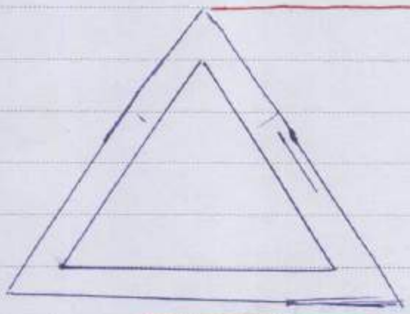
$$\theta_{s_2} = 111.8$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 81.4$$

$$\sigma = \begin{pmatrix} -20 & 60 \\ 60 & 90 \end{pmatrix} = \begin{pmatrix} 116 & 0 \\ 0 & -46.4 \end{pmatrix} = \begin{pmatrix} 35 & 81.4 \\ 81.4 & 35 \end{pmatrix}$$

* مقطع کامل، مقطع شیاردار، دایره کامل دوباره حل شود.
مثلث هم حل شود





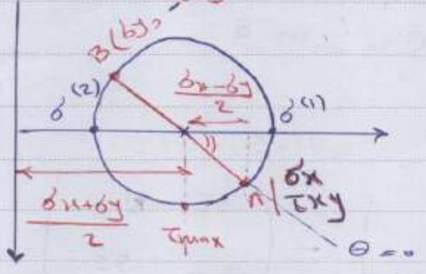
$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right) = \left(\frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)$$

$$(\tau_{x'y'}) = \left(-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)$$

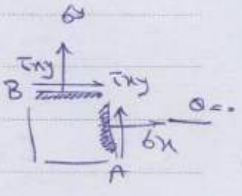
$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

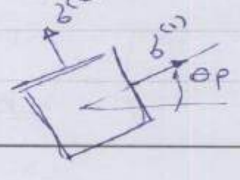
$$(x - a)^2 + (y-b)^2 = R^2$$

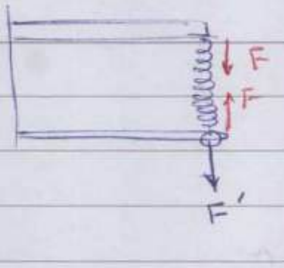
$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

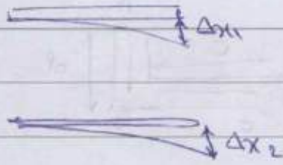


$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}}$$

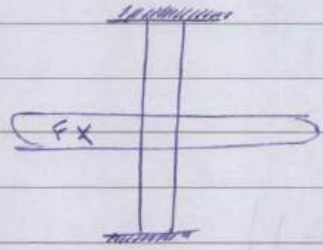


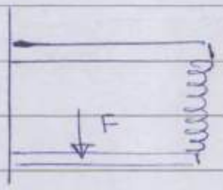


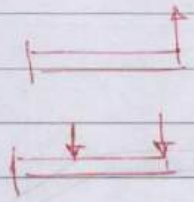


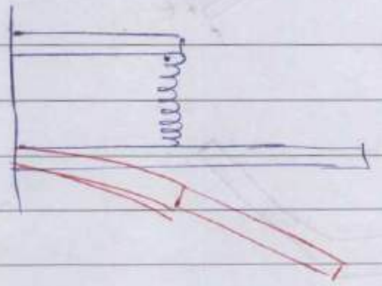


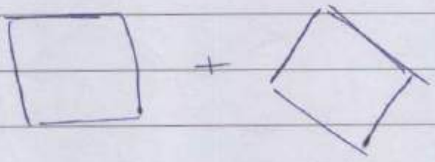
$|\Delta x_2 - \Delta x_1| = F/k$

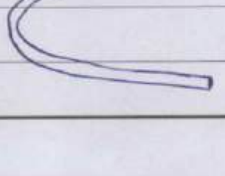


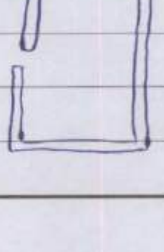


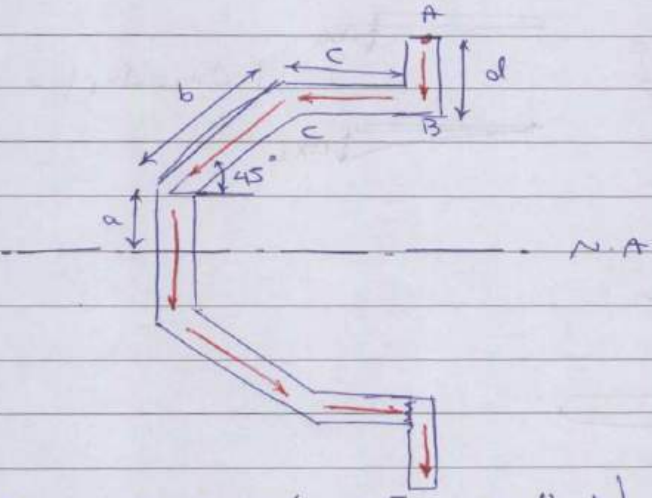






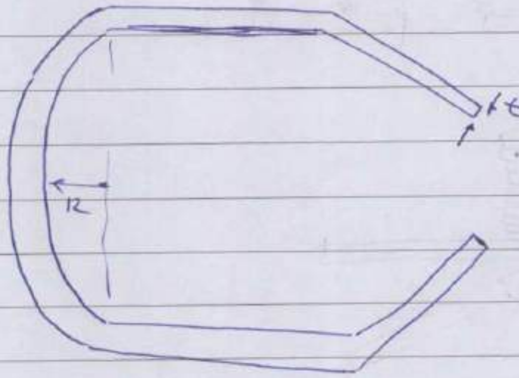


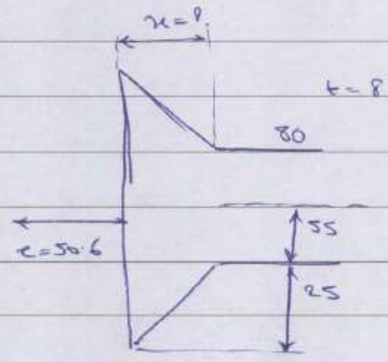


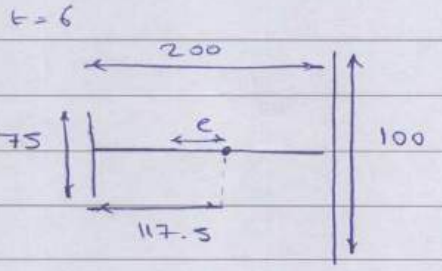


A-B ; $\tau_{xy} = \frac{V}{It}\left(\upsilon t \left(a + b\sqrt{2}/_2 + d - \frac{\upsilon}{2} \right) \right)$

B-C ; $\tau_{xz} =$



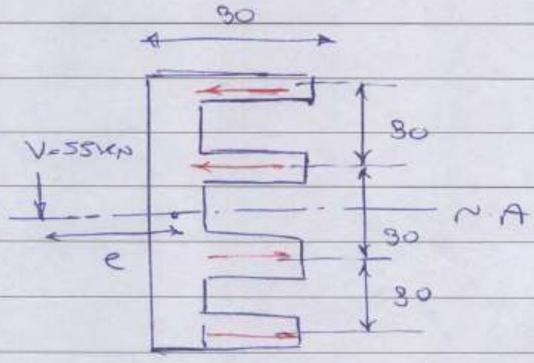




Iy = 1.902 x10^6

Iz = 1.526 x10^6

(e=81.6 maybe)



Iz = 0.9333x10^6

e = 9.12