دانشگاه صنعتی نوشیروانی بابل

تحلیل سازه ۲

دکتر حر خسروی

ویدئو کامل جلسات تحلیل سازه مطابق با این جزوه، در کانال یوتیوب به آدرس زیر قابل دسترسی می باشد.

@daneshesaze

مهر ۱۴۰۳

Video 1

Slope-Deflection 1

Wednesday, February 14, 2024 8:27

تحلیل سازه ها II

سرفصل مطالب

① روش شیب - افت

② روش توزیع لنگر (کراس)

③ خط تأثیر سازه های نامعین

④ تحلیل سازه های متقارن

⑤ تحلیل ماتریسی سازه ها

⑥ تحلیل تقریبی

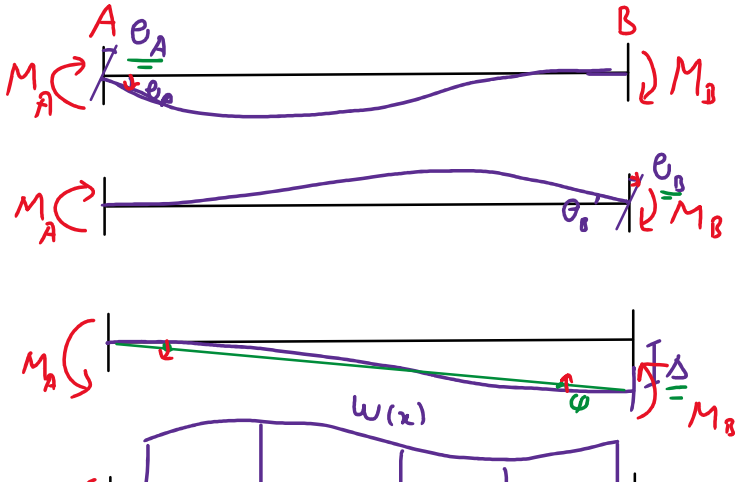
فصل اول :

روش شیب - افت

روش شیب - افت یک روش کمتی برای تحلیل سازه های نامعین است که برای اولین بار توسط پروفسور G. A. Maney در سال 1915 ارائه شد. این روش اساس تریا روش سختی برای تیرها و قاب ها بوده و مبنای روش های دیگر مانند روش توزیع لنگر، روش کانی و تحلیل ماتریسی سازه ها می باشد.

اثبات رابطه شیب - افت

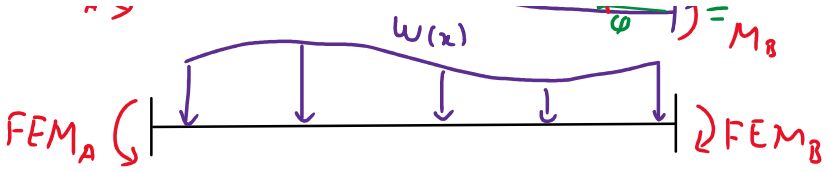
در رابطه شیب - افت نشان می دهیم که لنگرهای انتهایی یک تیر ناشی از چه عاملی می باشند:



① چرخش انتهای نزدیک تیر (θ_A)

② چرخش انتهای دور تیر (θ_B)

③ تغییر مکان نسبی دو سر تیر (Δ)
($\psi = \frac{\Delta}{L}$) دوران عضو

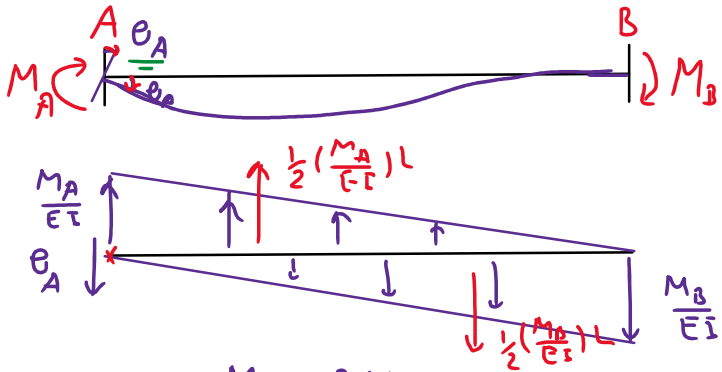


دوران عضو ($\psi = \frac{\Delta}{L}$)

① بردارهای خارجی (FEM)
Fixed End Moment

* در روش شیب ـ انحنا، لنگرها و چرخش‌ها در جهت ساعتگرد مثبت فرض می‌شود.

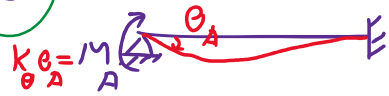
① چرخش انتهای نزدیک



$F = K\Delta$

k_θ

$M_A = \left(\frac{4EI}{L}\right)\theta_A$

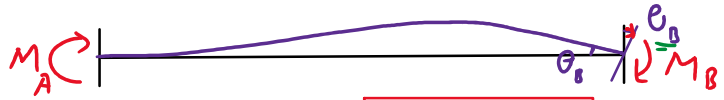


$\sum M_A = 0 \rightarrow M_A = 2M_B$

$\sum F_y = 0 \rightarrow \frac{1}{2}\frac{M_A L}{EI} - \frac{1}{2}\frac{\frac{M_A}{2}L}{EI} = \theta_A \rightarrow \theta_A = \frac{1}{4}\frac{M_A L}{EI}$

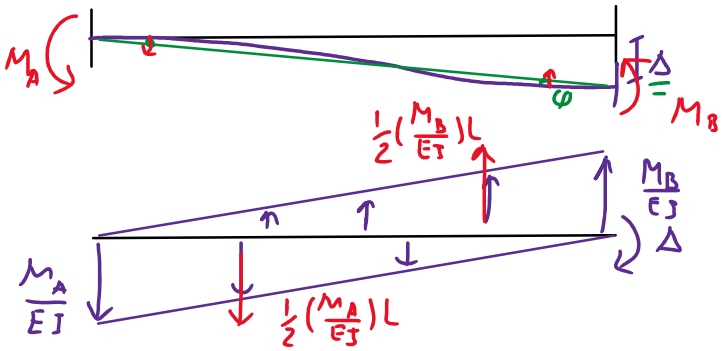
$M_A = \frac{4EI}{L}\theta_A$ $\qquad$ $M_B = \frac{2EI}{L}\theta_A$

② چرخش انتهای دور



$M_A = \frac{2EI}{L}\theta_B$ $\qquad$ $M_B = \frac{4EI}{L}\theta_B$

③ تغییرمکان نسبی دو سر تیر

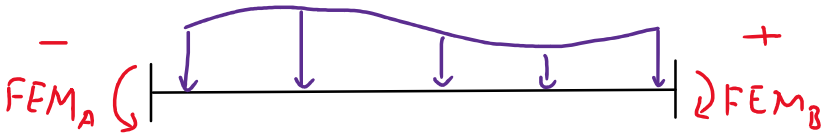


$M_A = \frac{4EI}{L}\psi + \frac{2EI}{L}\psi = \frac{6EI}{L}\psi = \frac{6EI}{L^2}\Delta$

$\sum F_y = 0 \rightarrow M_A = M_B$

$\sum M_B = 0 \rightarrow \Delta = \frac{1}{2}\left(\frac{M_A}{EI}\right)L\left(\frac{L}{3}\right) = \frac{1}{6}\frac{M_A L^2}{EI}$

$$M_A = M_B = -\frac{6EI}{L^2}\Delta$$

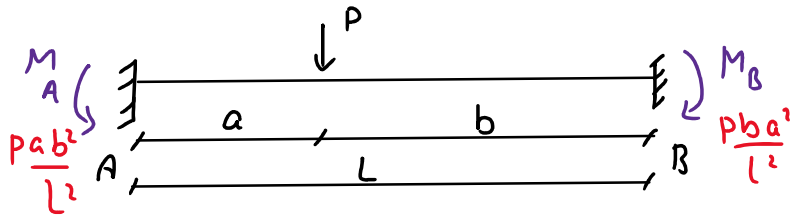


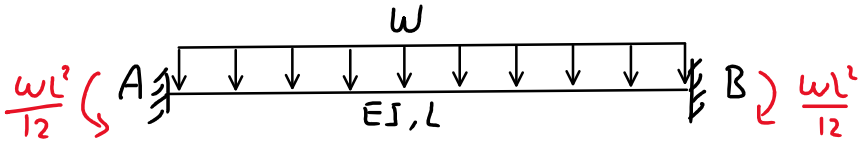
② در اثر بار خارجی

$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI}{L^2}\Delta + FEM_{AB}$$

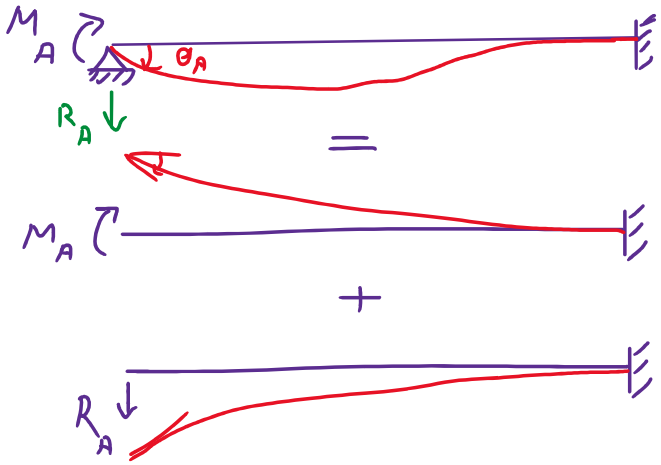
$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\frac{\Delta}{L}\right) + FEM_{AB}$$

رابطه شیب - انت





$\delta_A = 0$



$$\frac{M_A L^2}{2EI} - \frac{R_A L^3}{3EI} = 0 \rightarrow R_A = \frac{3}{2}\frac{M}{L}$$

$$\frac{M_A L}{2EI} - \frac{K_A L}{3EI} = 0 \rightarrow K_A = \frac{3}{2}\frac{I}{L}$$

$$\theta_A = \frac{M_A L}{EI} - \left(\frac{3}{2}\frac{M_A}{L}\right)\frac{L^2}{2EI} = \frac{1}{4}\frac{M_A L}{EI} \rightarrow \boxed{M_A = \frac{4EI}{L}\theta_A}$$

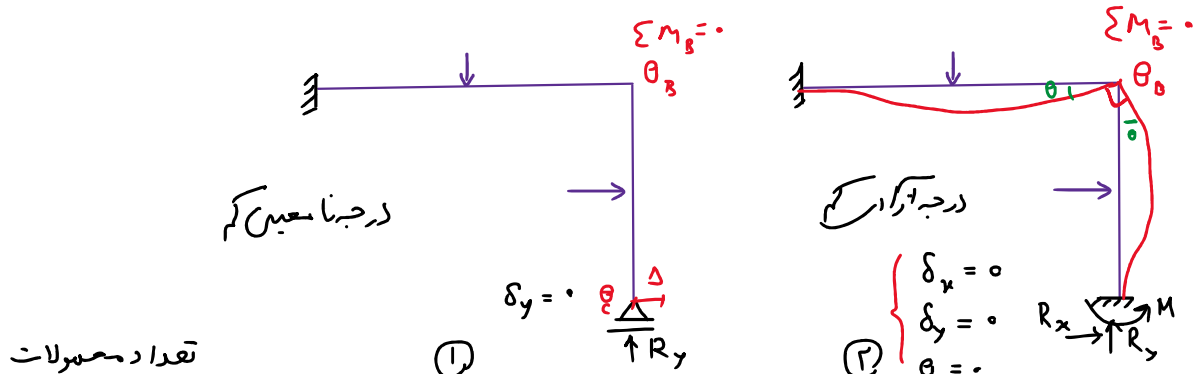
Video 2

Slope-Deflection 2

Monday, February 19, 2024 8:25

یادآوری روش های تحلیل سازه های نامعین

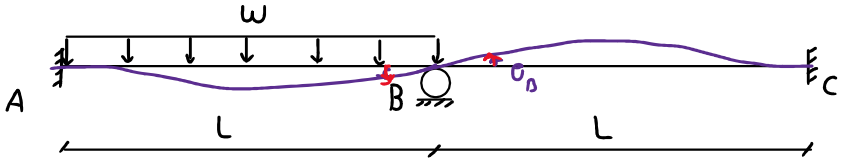
	مجهولات	معادلات
روش نیروی (نرم)	نیرو (M, F)	تغییرمکان (θ, δ)
روش تغییرمکان (سخت)	تغییرمکان (θ, δ)	نیرو (M, F)



تعداد مجهولات	①	②
M, F	۱	۳
θ, Δ	۳	۱
روش تحلیل	نرم	سخت

مثال: تیر شکل زیر را تحلیل نموده و دیاگرام لنگر و برش آن را رسم نمایید.

EI: Const.



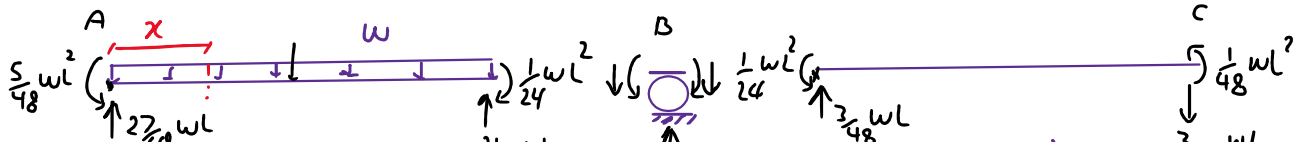
مجهول: θ_B معادله $\Sigma M_B = 0 \rightarrow M_{BA} + M_{BC} = 0$

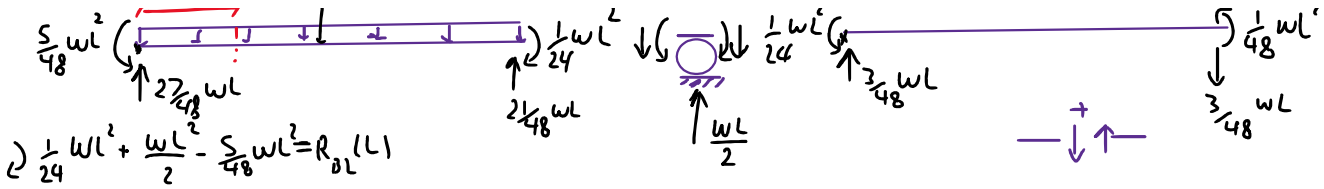
$$\begin{cases} M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - 3\frac{\Delta}{L}) + \frac{wL^2}{12} \\ M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 3\frac{\Delta}{L}) \end{cases}$$

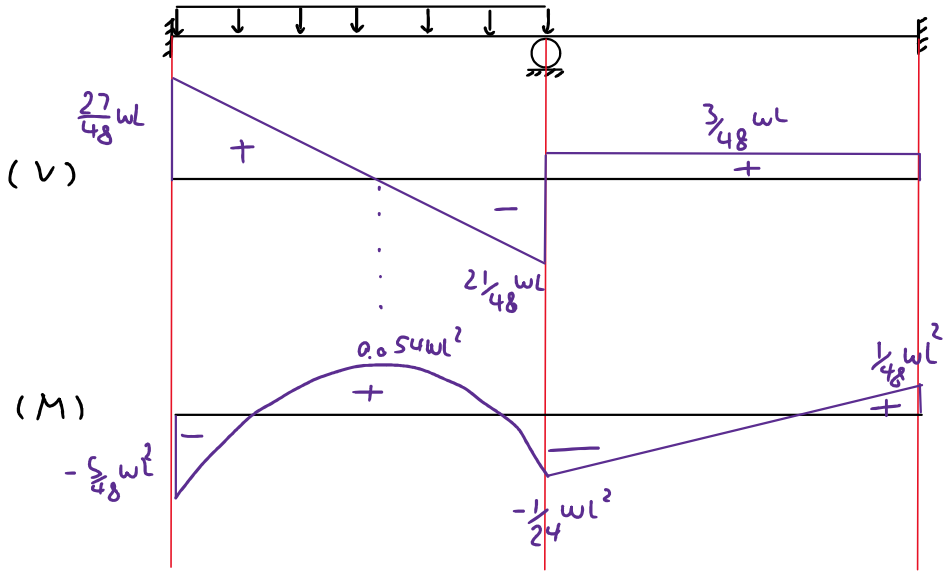
$$M_{BA} + M_{BC} = 0 \rightarrow \frac{8EI}{L}\theta_B + \frac{wL^2}{12} = 0 \rightarrow \boxed{\theta_B = -\frac{wL^3}{96EI}}$$

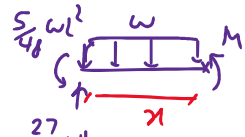
$$M_{AB} = \frac{2EI}{L}(\theta_B) - \frac{wL^2}{12} = -\frac{5}{48}wL^2 \qquad M_{BC} = \frac{2EI}{L}(2\theta_B) = -\frac{1}{24}wL^2$$

$$M_{BA} = \frac{2EI}{L}(2\theta_B) + \frac{wL^2}{12} = \frac{1}{24}wL^2 \qquad M_{CB} = \frac{2EI}{L}(\theta_B) = -\frac{1}{48}wL^2$$









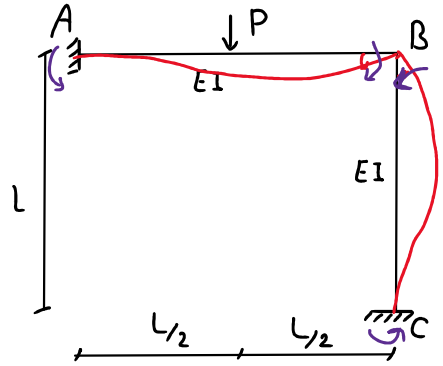
$\Sigma F_y = 0$

$\frac{27}{48}wL - wx = 0$

$x = \frac{27}{48}L$

$M = -\frac{5}{48}wL^2 - \frac{w}{2}\left(\frac{27}{48}L\right)^2 + \frac{27}{48}wL\left(\frac{27}{48}L\right) = 0.054wL^2$

مثال: لنگرهای انتهایی تیر دستون را به دست آورید.

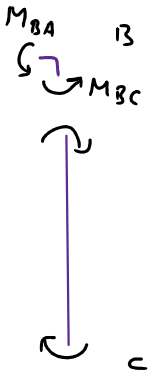


مجهول: θ_B معادله: $\Sigma M_B = 0$

$$\begin{cases} M_{BA} = \frac{2EI}{L}(2\theta_B) + \frac{PL}{8} \\ M_{BC} = \frac{2EI}{L}(2\theta_B) \end{cases}$$

$M_{BA} + M_{BC} = 0 \rightarrow \frac{8EI}{L}\theta_B + \frac{PL}{8} = 0$

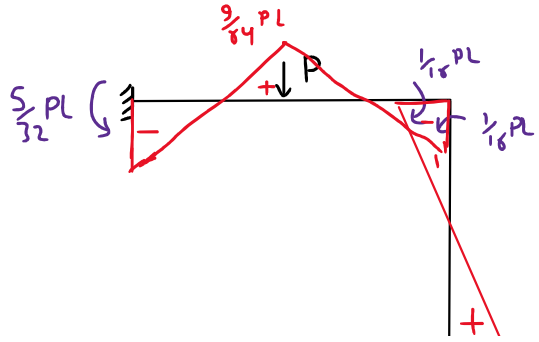
$\theta_B = -\frac{PL^2}{64EI}$

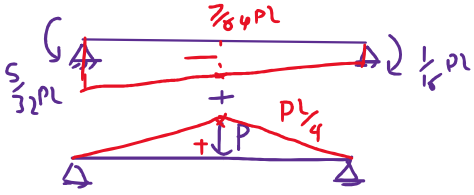


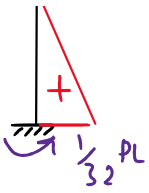
$$\begin{cases} M_{AB} = \frac{-5}{32}PL \\ M_{BA} = \frac{1}{16}PL \\ M_{BC} = \frac{-1}{16}PL \\ M_{CB} = -\frac{1}{32}PL \end{cases}$$

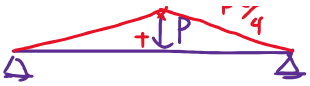
$M_{AB} = \frac{2EI}{L}(\theta_B) - \frac{PL}{8}$

$M_{CB} = \frac{2EI}{L}(\theta_B)$









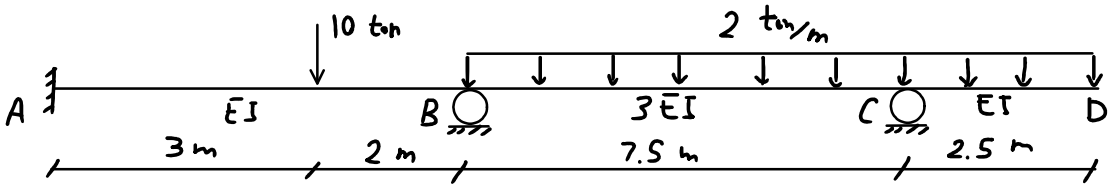
$\frac{PL}{4} - \frac{7}{64}PL = \frac{9}{64}PL$

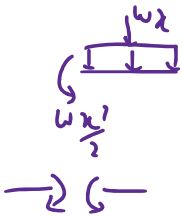
Video 3

Slope-Deflection 3

Monday, February 19, 2024 8:26

مثال: تیر شکل زیر را تحلیل کنید.





مجهول: θ_B , θ_C معادله: $\sum M_B = 0$, $\sum M_C = 0$

$$\begin{cases} M_{BA} = \frac{2EI}{5}(2\theta_B) + \frac{10(2)(3^2)}{5^2} = 0.8EI\theta_B + 7.2 \\ M_{BC} = \frac{2(3EI)}{7.5}(2\theta_B + \theta_C) - \frac{2(7.5^2)}{12} = 1.6EI\theta_B + 0.8EI\theta_C - 9.375 \end{cases} \rightarrow M_{BA} + M_{BC} = 0$$

$$\begin{cases} M_{CB} = \frac{2(3EI)}{7.5}(2\theta_C + \theta_B) + \frac{2(7.5^2)}{12} = 0.8EI\theta_B + 1.6EI\theta_C + 9.375 \\ M_{CD} = -\frac{2(2.5^2)}{2} = -6.25 \end{cases} \rightarrow M_{CB} + M_{CD} = 0$$

$$\begin{cases} 2.4EI\theta_B + 0.8EI\theta_C - 2.175 = 0 \\ 0.8EI\theta_B + 1.6EI\theta_C + 3.125 = 0 \end{cases}$$

حل دو معادله دو مجهول

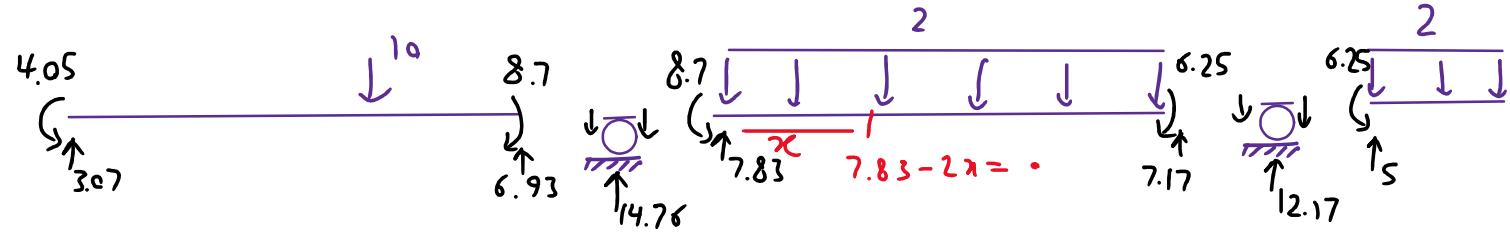
$EI\theta_B = 1.87$

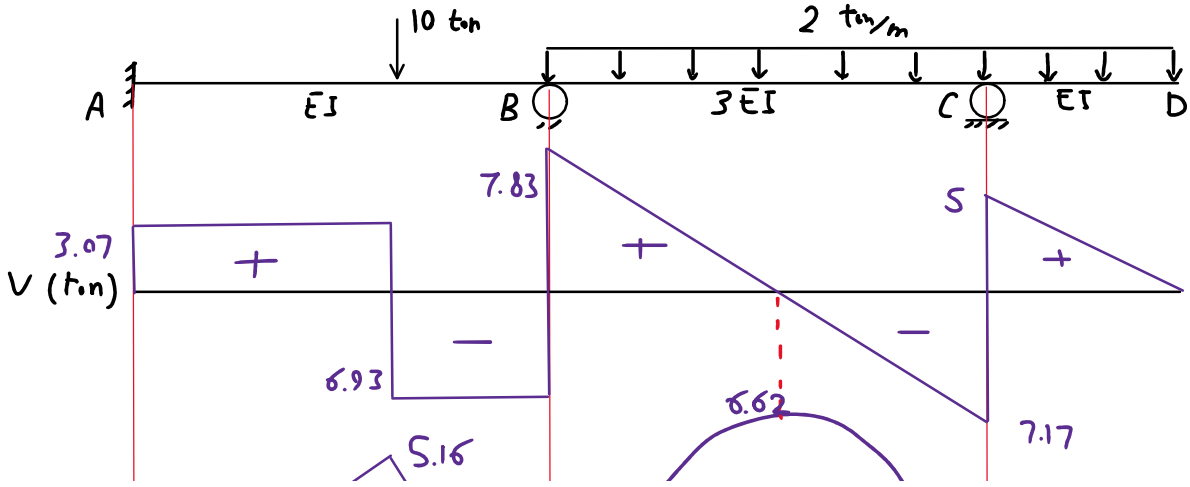
$EI\theta_C = -2.895$

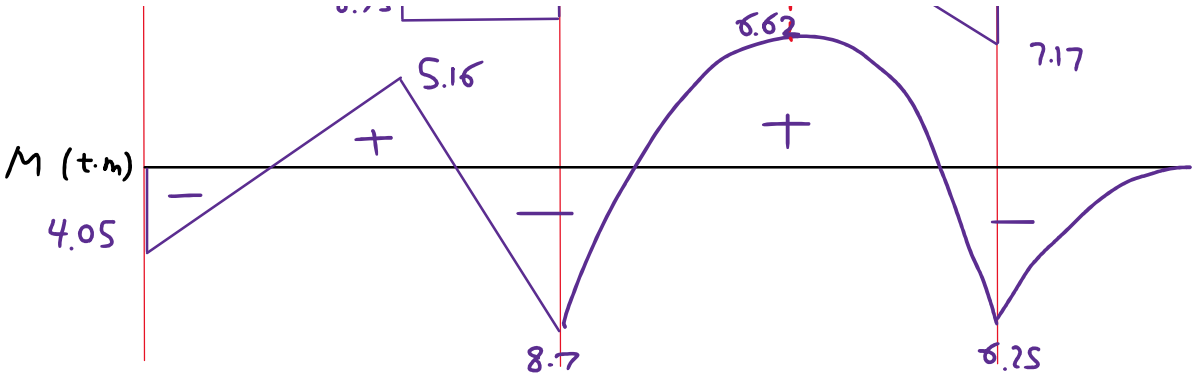
$M_{AB} = -4.05$ $M_{BA} = 8.7$ $M_{CB} = 6.25$

$M_{BC} = -8.7$ $M_{CD} = -6.25$

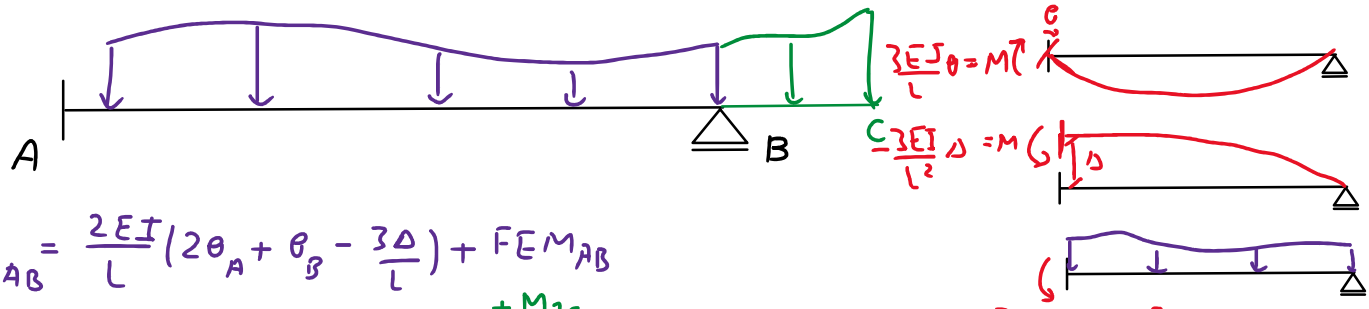
$M_{AB} = \frac{2EI}{5}(\theta_B) - \frac{10(3)(2^2)}{5^2} = 0.4(1.87) - 4.8$







رابطه شیب- انت اصلاح شده برای عضو یک سر مفصل



① $M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - \frac{3\Delta}{L}) + FEM_{AB}$

② $M_{BA} = \frac{2EI}{L}(2\theta_B + \theta_A - \frac{3\Delta}{L}) + FEM_{BA} + M_{BC} = 0 \rightarrow M_{BA} + M_{BC} = 0$

$M = FEM_{AB} - \frac{1}{2}FEM_{BA}$

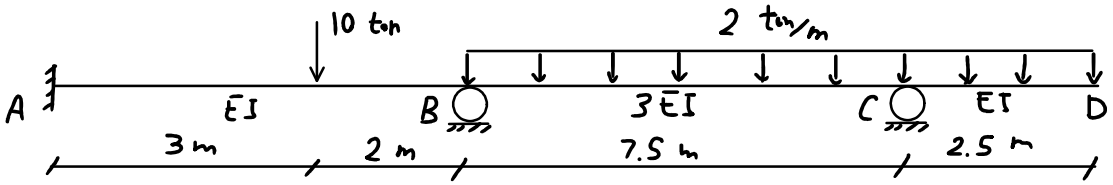
* $\theta_B = \frac{-\theta_A}{2} + \frac{3}{2}\frac{\Delta}{L} - FEM_{BA}(\frac{L}{4EI})$

$M_{AB} = \frac{2EI}{L}(2\theta_A + [-\frac{\theta_A}{2} + \frac{3}{2}\frac{\Delta}{L} - FEM_{BA}\frac{L}{4EI}] - \frac{3\Delta}{L}) + FEM_{AB}$

$$M_{AB} = \frac{3EI}{L}(\theta_A - \frac{\Delta}{L}) + (FEM_{AB} - \frac{1}{2}FEM_{BA})$$

$FEM_{BA} + M_{BC}$

مثال: تیر شکل زیر را تحلیل کنید.



مجهول: θ_B معادله: $\Sigma M_B = 0$

$$\begin{cases} M_{BA} = \frac{2EI}{5}(2\theta_B) + \frac{10(2)(3^2)}{5^2} = 0.8EI\theta_B + 7.2 \\ M_{BC} = \frac{3(3EI)}{7.5}(\theta_B) + (-9.375) - \frac{1}{2}(9.375 - 6.25) = 1.2EI\theta_B - 10.94 \end{cases}$$

$M_{BA} + M_{BC} = 0$

$2EI\theta_B - 3.74 = 0 \rightarrow$ $EI\theta_B = 1.87$

$M_{BA} = 8.7$ t.m $\quad M_{AB} = -4.05$

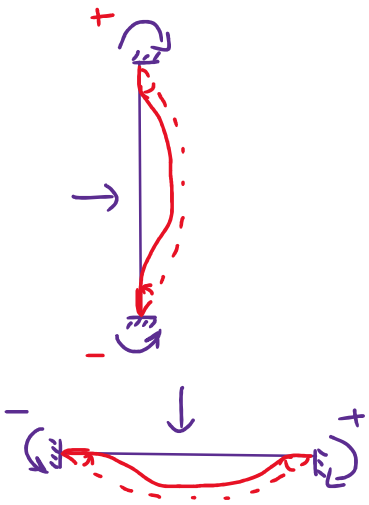
$M_{BC} = -8.7$ t.m

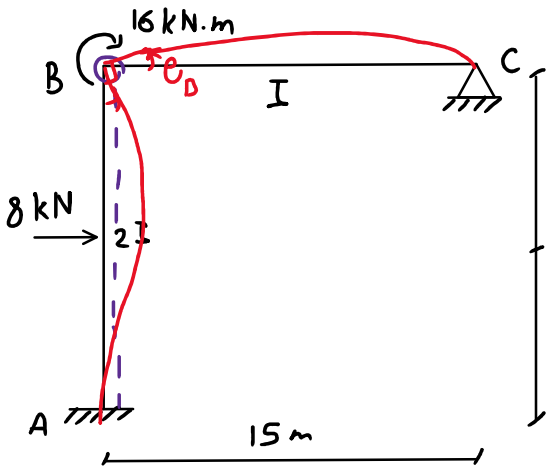
Video 4

Slope-Deflection 4

Monday, February 19, 2024 8:26

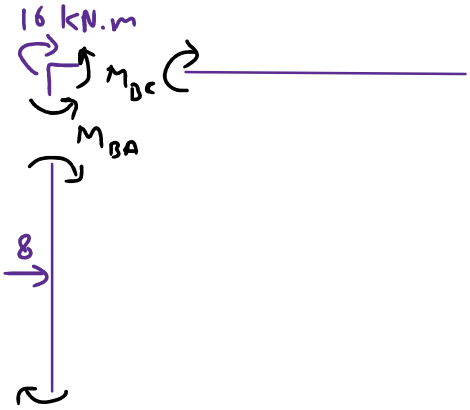
مثال: لنگرهای M_A و M_B را در تاب شکل زیر به دست آورید.





مجهول: θ_B

معادله: $\Sigma M_B = 0$



$$\begin{cases} M_{BA} = \frac{2(2EI)}{20}(2\theta_B) + \frac{8(20)}{8} = 0.4EI\theta_B + 20 \\ M_{BC} = \frac{3EI}{15}(\theta_B) = 0.2EI\theta_B \end{cases}$$

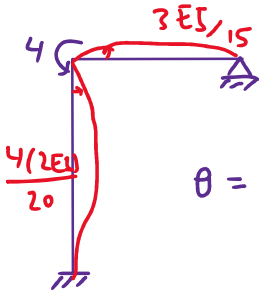
$$\rightarrow M_{BA} + M_{BC} - 16 = 0$$

$$0.6EI\theta_B + 20 - 16 = 0 \rightarrow \boxed{EI\theta_B = -6.6}$$

$$M_{BC} = 0.2(-6.6) = -1.33 \text{ kN.m}$$

$$M_{BA} + (-1.33) - 16 = 0 \rightarrow \boxed{M_{BA} = 17.33}$$

$$M_{AB} = \frac{2(2EI)}{20}(\theta_B) - 20 = -21.33$$

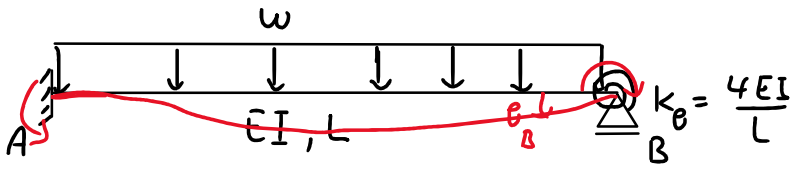


$$\theta = \frac{M}{\Sigma k_\theta} = \frac{4}{\left(\frac{3}{15} + \frac{8}{20}\right)EI} = 6.66$$

مثال: لنگرهای دو سر تیر را به دست آورید.

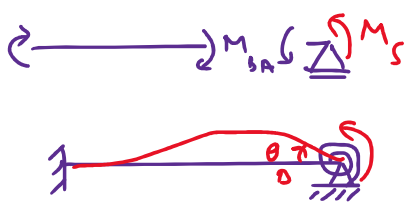
مجهول: θ_B

معادله: $\Sigma M_B = 0$



$$\begin{cases} M_{BA} = \frac{2EI}{L}(2\theta_B) + \frac{wL^2}{12} \\ M_S = k_\theta\theta_B = \frac{4EI}{L}\theta_B \end{cases}$$

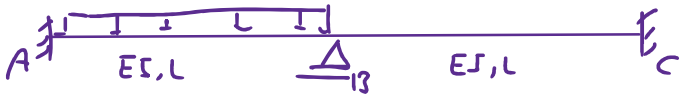
$$\rightarrow M_{BA} + M_S = 0$$



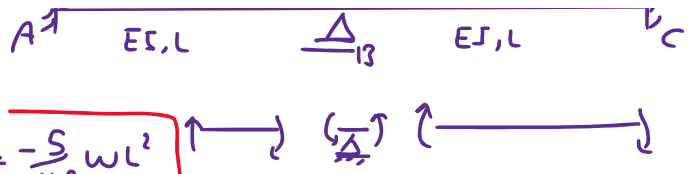
$$M_S = +k_\theta\theta_B$$

$$\frac{8EI}{L}\theta_B + \frac{wL^2}{12} = 0 \rightarrow \boxed{\theta_B = \frac{-wL^2}{96EI}}$$

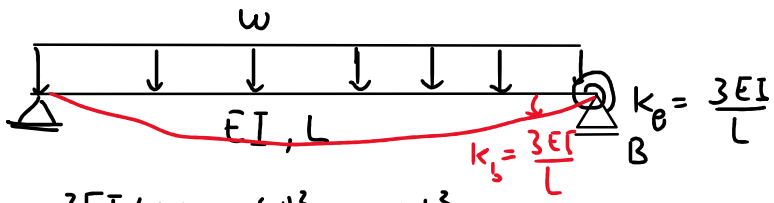
$$M_S = \frac{4EI}{L}\theta_B = \frac{wL^2}{24}$$



$$M_S = \frac{4EI}{L}\theta_B = \frac{wL^2}{24}$$

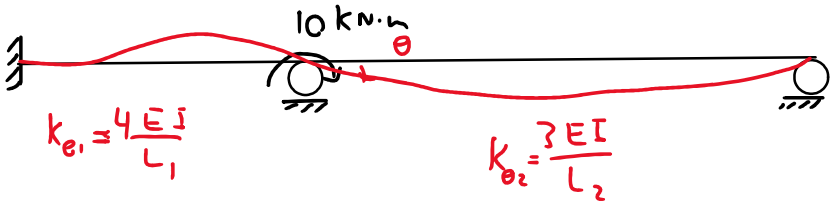


$$M_{AB} = \frac{2EI}{L}(\theta_B) - \frac{wL^2}{12} = -\frac{1}{48}wL^2 - \frac{1}{12}wL^2 = \frac{-5}{48}wL^2$$



$$\begin{cases} M_{BA} = \frac{3EI}{L}(\theta_B) + \frac{wL^2}{12} - \frac{1}{2}\left(\frac{-wL^2}{12}\right) \\ M_S = \frac{3EI}{L}\theta_B \end{cases} \rightarrow \frac{6EI}{L}\theta_B + \frac{wL^2}{8} = 0 \rightarrow \theta_B = \frac{wL^2}{48EI}$$

$$M_S = \frac{wL^2}{16}$$



$$\theta = \frac{M}{\Sigma k_\theta}$$

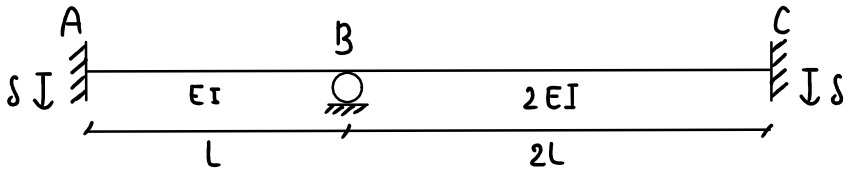
Video 5

Slope-Deflection 5

Monday, February 19, 2024 8:26

نشست و دوران تکیه‌گاهی

مثال: در تیر شکل زیر، تحت نشست تکیه‌گاهی نشان داده شده، M_{AB} را به دست آورید.



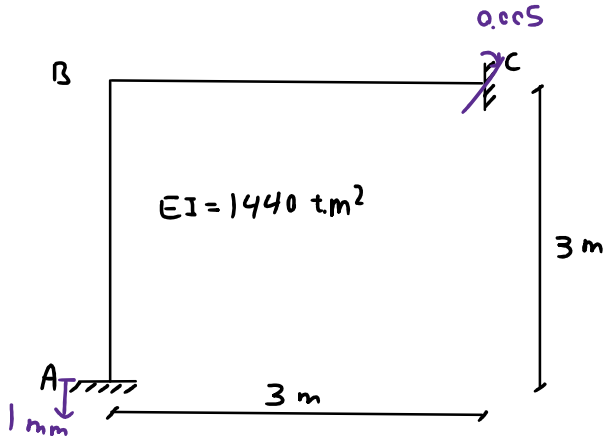
مجهول: θ_B

معادله: $\sum M_B = 0$

$$\begin{cases} M_{BA} = \frac{2EI}{L}\left(2\theta_B + \frac{3\delta}{L}\right) \\ M_{BC} = \frac{2(2EI)}{2L}\left(2\theta_B - \frac{3\delta}{2L}\right) \end{cases} \rightarrow \frac{8EI}{L}\theta_B + \frac{3EI}{L^2}\delta = 0 \rightarrow \boxed{\theta_B = \frac{-3}{8}\frac{\delta}{L}}$$

$$M_{AB} = \frac{2EI}{L}\left(\theta_B + \frac{3\delta}{L}\right) = \frac{21}{4}\frac{EI}{L^2}\delta \qquad \left(-\frac{3}{8}\frac{\delta}{L}\right)$$

مثال: در قاب شکل زیر تکیه‌گاه A به اندازه 1mm نشست و تکیه‌گاه C به اندازه 0.005 rad دوران ساعتگرد دارد. M_{CB} را محاسبه کنید.



مجهول: θ_B

معادله: $\sum M_B = 0$

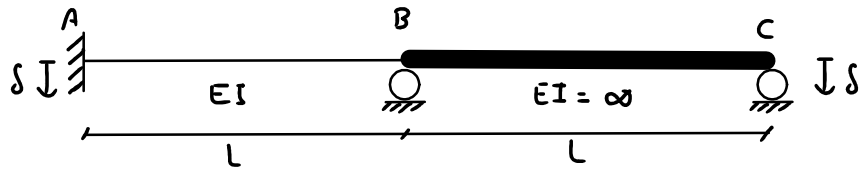
$\rightarrow M_{BA} + M_{BC} = 0 \rightarrow \theta_B$

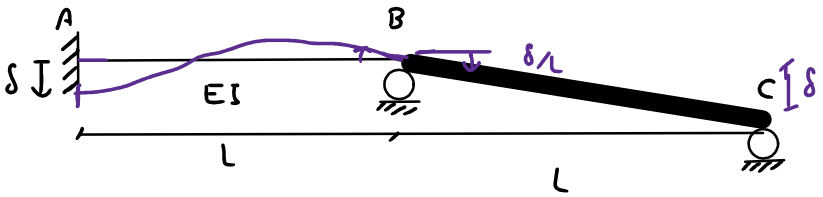
$$\begin{cases} M_{BC} = \frac{2EI}{3}\left(2\theta_B + 0.005 + \frac{3\times 0.001}{3}\right) \\ M_{BA} = \frac{2EI}{3}\left(2\theta_B\right) \end{cases} \rightarrow \frac{2EI}{3}\left(4\theta_B + 0.006\right) = 0 \rightarrow \boxed{\theta_B = -0.0015}$$

$$M_{CB} = \frac{2EI}{3}\left(2\times 0.005 - 0.0015 + 3\times\frac{0.001}{3}\right) = 9.12\ t.m \qquad (EI = 1440)$$

مثال: عکس‌العمل تکیه‌گاه B را محاسبه کنید.

مثال: عکس‌العمل تکیه C را محاسبه کنید.

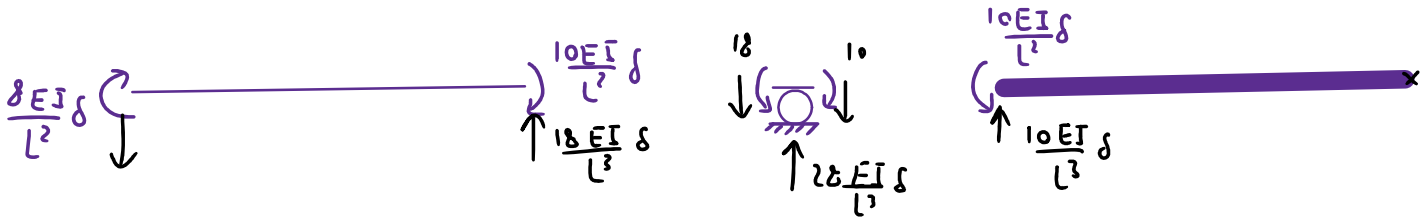




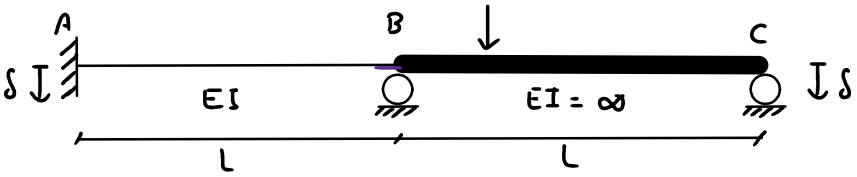
$$M_{AB} = \frac{2EI}{L}\left(\theta_B - 3\frac{\delta}{L}\right) = \frac{2EI}{L}\left(\frac{\delta}{L} + 3\frac{\delta}{L}\right) = \frac{8EI}{L^2}\delta$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B - 3\frac{\delta}{L}\right) = \frac{2EI}{L}\left(2\frac{\delta}{L} + 3\frac{\delta}{L}\right) = \frac{10EI}{L^2}\delta$$

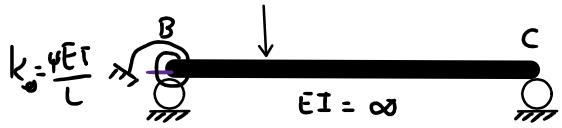
$$R_B = 28\frac{EI}{L^3}\delta$$



$EI_{BC} = x\,EI$



$$\begin{cases} M_{BA} = \frac{2EI}{L}\left(2\theta_B + 3\frac{\delta}{L}\right) \\ M_{BC} = \frac{3xEI}{L}\left(\theta_B - \frac{\delta}{L}\right) \end{cases} \rightarrow \left(\frac{4EI}{L} + \frac{3xEI}{L}\right)\theta_B = \frac{EI}{L}\delta$$



Video 6

Slope-Deflection 6

Saturday, March 9, 2024 9:35

تحلیل سازه دارای درجه آزادی تغییرمکان (Δ)

درجات آزادی عبارتند از تعداد مجهولات θ و Δ برای حل معادلات شیب-افت.

دوران — تغییرمکان

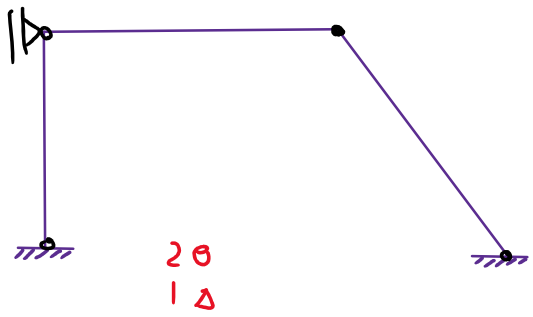
تعیین درجات آزادی Δ

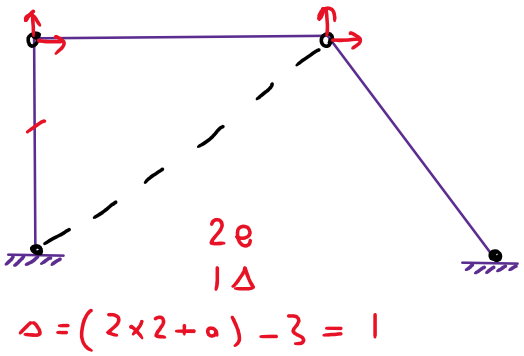
روش اول: ابتدا همه گره های سازه را تبدیل به مفصل می کنیم. در این صورت اگر سازه پایدار باشد، سازه اصلی دارای درجه آزادی Δ نیست. درصورت ناپایداری سازه، درجه آزادی Δ به تعداد حداقل تکیه گاه های ساده مجازی (یا میله مجازی) مورد نیاز برای پایدار کردن سازه است.

روش دوم: درصورتی که سازه مفصلی شده ناپایداری هندسی نداشته باشد، درجه آزادی Δ از رابطه زیر به دست می آید:

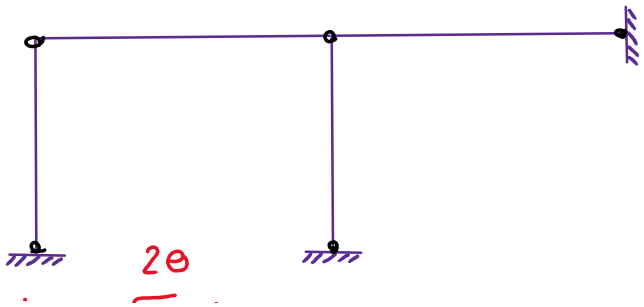
Δ = (2 × تعداد گره های داخلی + تعداد تکیه گاه ها) − (تعداد اعضا)

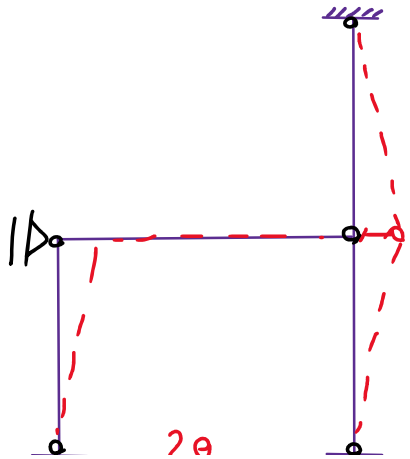
* در تعیین درجه آزادی Δ، از تغییر طول محوری اعضا صرف نظر می شود.





Δ = (2×2+0) − 3 = 1





2θ

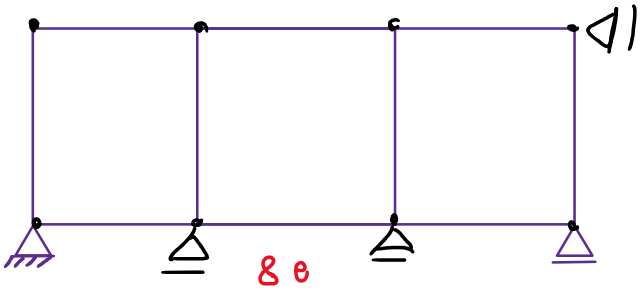
درجه آزادی ندارد

$\Delta = (2 \times 2 + 0) - 4 = 0$

2θ

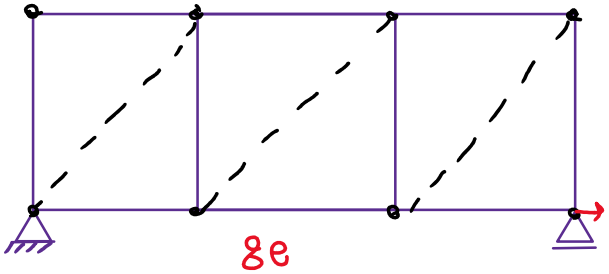
1Δ

$\Delta = (2 \times 2 + 0) - 4 = 0$ نابایداری هندسی



8θ

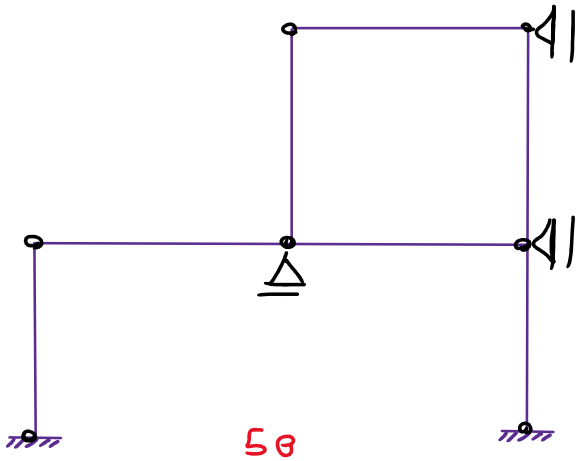
3Δ



8θ

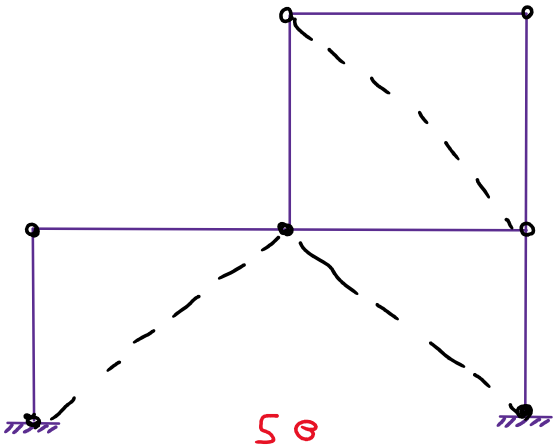
3Δ

$\Delta = (6 \times 2 + 1) - (10) = 3$



5θ

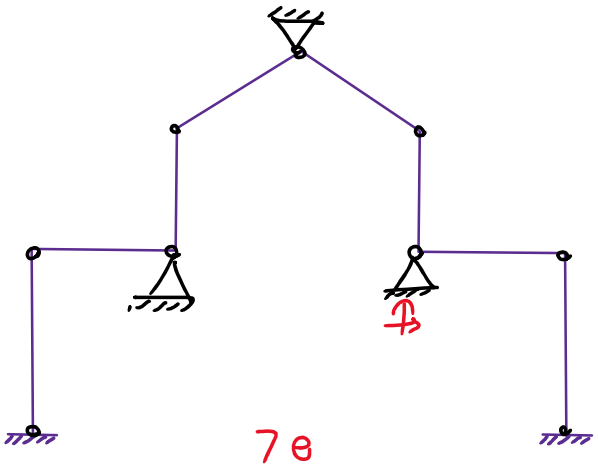
3Δ



5θ

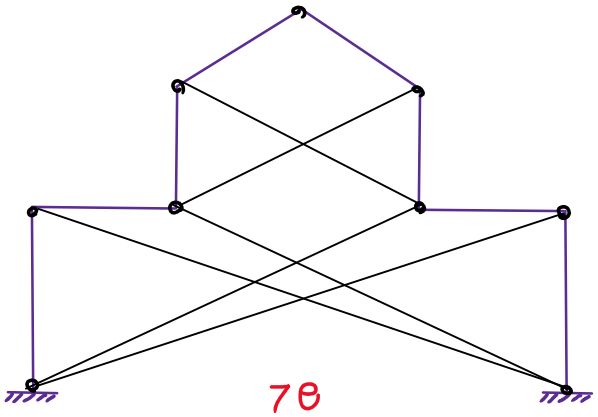
3Δ

$\Delta = (5 \times 2 + 0) - 7 = 3$



7θ

6Δ

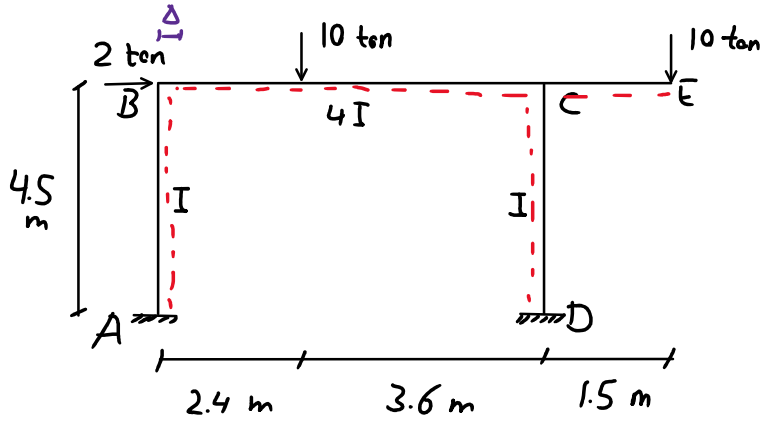


7θ

6Δ

$\Delta = (7 \times 2 + 0) - 8 = 6$

مثال: برای قاب نشان داده شده، لنگرهای انتهایی را محاسبه نموده و دیاگرام لنگر را رسم نمایید.



مجهول: θ_B ، θ_C ، Δ

معادله: $\Sigma M_B = 0$

$\Sigma M_C = 0$

نیز $\Sigma F_x = 0$ معادله برش

$$M_{AB} = \frac{2EI}{4.5}\left(\theta_B - \frac{3\Delta}{4.5}\right)$$

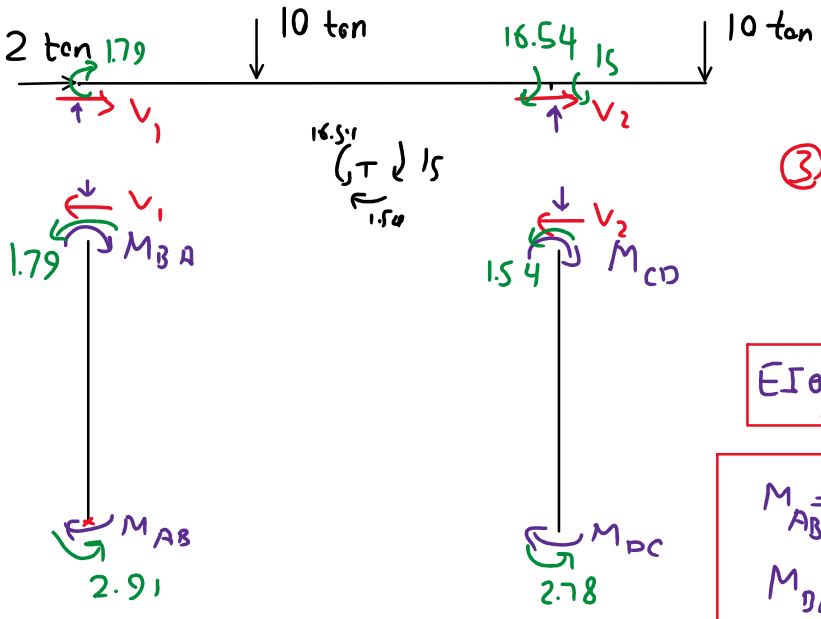
$$\begin{cases} M_{BA} = \frac{2EI}{4.5}\left(2\theta_B - \frac{3\Delta}{4.5}\right) \\ M_{BC} = \frac{2(4EI)}{6}\left(2\theta_B + \theta_C\right) - \frac{10\times 2.4\times 3.6^2}{6^2} \end{cases}$$

$$\rightarrow (1)\quad M_{BA} + M_{BC} = 0$$

$$\begin{cases} M_{CB} = \frac{2(4EI)}{6}\left(2\theta_C + \theta_B\right) + \frac{10\times 3.6\times 2.4^2}{6^2} \\ M_{CD} = \frac{2EI}{4.5}\left(2\theta_C - \frac{3\Delta}{4.5}\right) \\ M_{CE} = -15 \end{cases}$$

$$\rightarrow (2)\quad M_{CB} + M_{CD} + M_{CE} = 0$$

$$M_{DC} = \frac{2EI}{4.5}\left(\theta_C - \frac{3\Delta}{4.5}\right)$$



$$\begin{cases} V_1 = \frac{M_{AB} + M_{BA}}{4.5} \\ V_2 = \frac{M_{CD} + M_{DC}}{4.5} \end{cases}$$

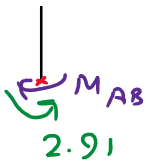
$$V_1 + V_2 + 2 = 0$$

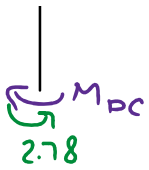
$$(3)\quad M_{AB} + M_{BA} + M_{CD} + M_{DC} + 9 = 0$$

حل ۳ معادله ۳ مجهول:

$$EI\theta_B = 2.51\ ,\ EI\theta_C = -2.78\ ,\ EI\Delta = 13.57$$

$M_{AB} = -2.91$	$M_{BC} = 1.79$	$M_{CD} = -1.54$
$M_{BA} = -1.79$	$M_{CD} = 16.54$	$M_{DC} = -2.78$

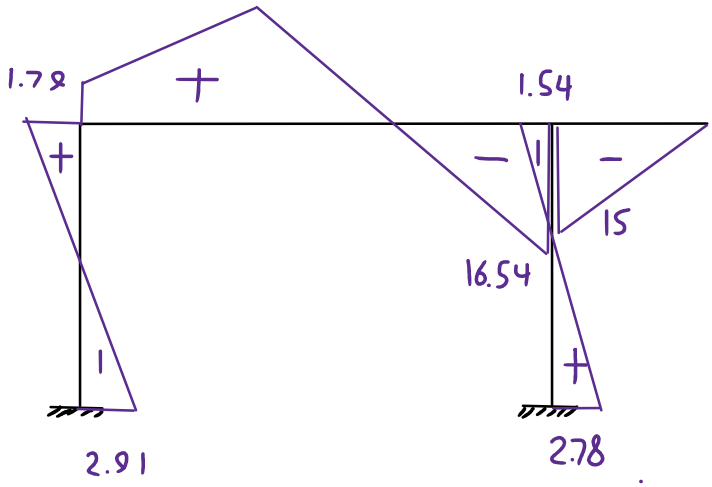




$M_{AB} = -2.91$ $M_{BC} = 1.79$ $M_{CD} = -1.54$

$M_{DA} = -1.79$ $M_{CD} = 16.54$ $M_{DC} = -2.78$

(M)



$1.79 + 16.54 - 10 \times 3.6 + V \times 6 = 0$

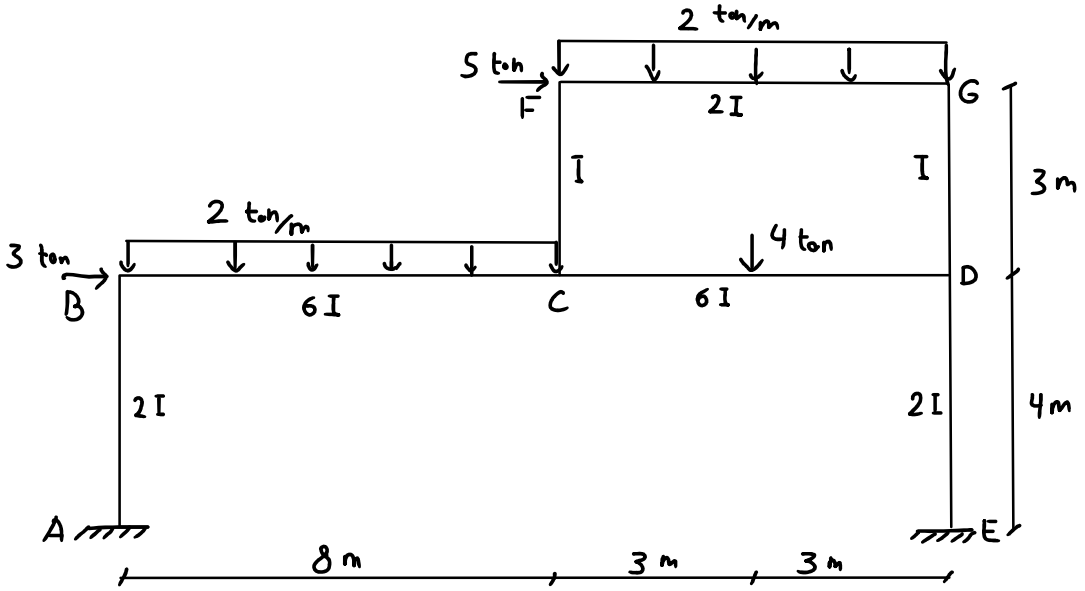
$M = 1.79 + 2.945 \times 2.4 = 8.86$

Video 7

Slope-Deflection 7

16:02 Wednesday, March 13, 2024

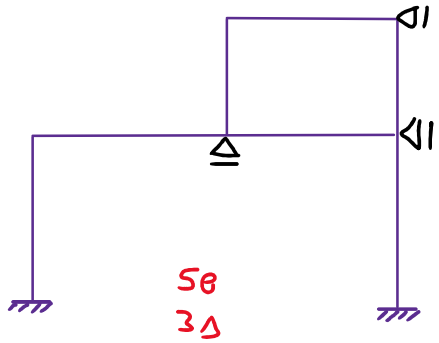
مثال: قاب شکل زیر را به روش شیب - انحنا تحلیل کنید.



$\frac{WL^2}{12} = \frac{2\times 6^2}{12} = 6$

$\frac{WL^2}{12} = \frac{2\times 8^2}{12} = 10.67$

$\frac{PL}{8} = \frac{4\times 6}{8} = 3$



$M_{AB} = \frac{2(2EI)}{4}\left(\theta_B - 3\frac{\Delta_2}{4}\right)$

$M_{BA} = \frac{2(2EI)}{4}\left(2\theta_B - 3\frac{\Delta_2}{4}\right)$

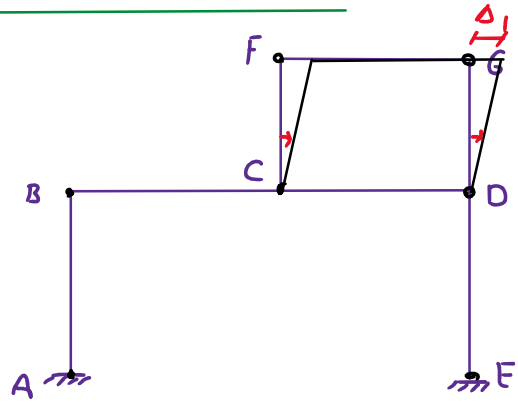
$M_{ED} = \frac{2(2EI)}{4}\left(\theta_D - 3\frac{\Delta_2}{4}\right)$

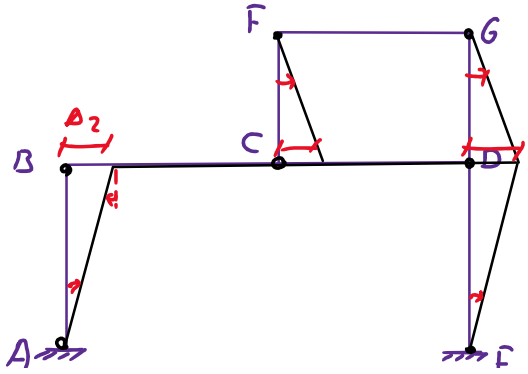
$M_{DE} = \frac{2(2EI)}{4}\left(2\theta_D - 3\frac{\Delta_2}{4}\right)$

$M_{CF} = \frac{2(EI)}{3}\left(2\theta_C + \theta_F - 3\frac{\Delta_1}{3} + 3\frac{\Delta_2}{3}\right)$

$M_{FC} = \frac{2EI}{3}\left(2\theta_F + \theta_C - 3\frac{\Delta_1}{3} + 3\frac{\Delta_2}{3}\right)$

$M_{DG} = \frac{2EI}{3}\left(2\theta_D + \theta_G - 3\frac{\Delta_1}{3} + 3\frac{\Delta_2}{3}\right)$





$$M_{DG} = \frac{2EI}{3}\left(2\theta_D + \theta_G - 3\frac{\Delta_1}{3} + 3\frac{\Delta_2}{3}\right)$$

$$M_{GD} = \frac{2EI}{3}\left(2\theta_G + \theta_D - 3\frac{\Delta_1}{3} + 3\frac{\Delta_2}{3}\right)$$

$$M_{BC} = \frac{2(6EI)}{8}\left(2\theta_B + \theta_C - 3\frac{\Delta_3}{8}\right) - 10.67$$

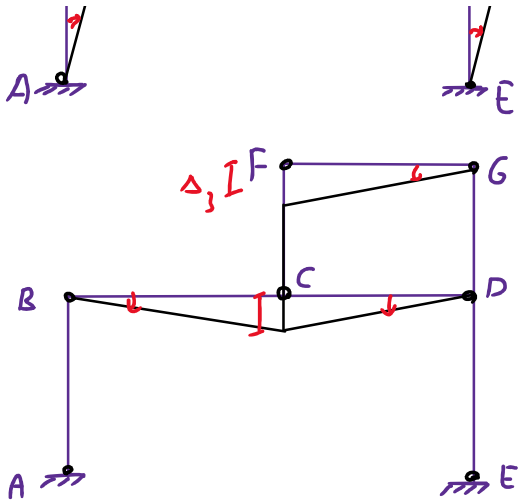
$$M_{CB} = \frac{2(6EI)}{8}\left(2\theta_C + \theta_B - 3\frac{\Delta_3}{8}\right) + 10.67$$

$$M_{CD} = \frac{2(6EI)}{6}\left(2\theta_C + \theta_D + 3\frac{\Delta_3}{6}\right) - 3$$

$$M_{DC} = \frac{2(6EI)}{6}\left(2\theta_D + \theta_C + 3\frac{\Delta_3}{6}\right) + 3$$

$$M_{FG} = \frac{2(2EI)}{6}\left(2\theta_F + \theta_G + 3\frac{\Delta_3}{6}\right) - 6$$

$$M_{GF} = \frac{2(2EI)}{6}\left(2\theta_G + \theta_F + 3\frac{\Delta_3}{6}\right) + 6$$



① $M_{BA} + M_{BC} = 0$

② $M_{CB} + M_{CD} + M_{CF} = 0$

③ $M_{DE} + M_{DC} + M_{DG} = 0$

④ $M_{FC} + M_{FG} = 0$

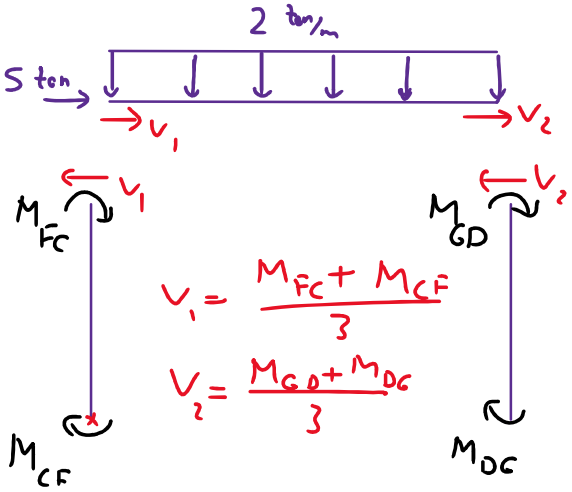
⑤ $M_{GD} + M_{GF} = 0$

⑥ $M_{FC} + M_{CF} + M_{GD} + M_{DG} + 15 = 0$

⑦ $3(M_{AB} + M_{DA} + M_{DE} + M_{ED}) - 4(M_{FC} + M_{CF} + M_{GD} + M_{DG}) + 36 = 0$

⑧ $4(M_{FG} + M_{GF} + M_{CD} + M_{DC}) - 3(M_{CB} + M_{BC}) - 320 = 0$

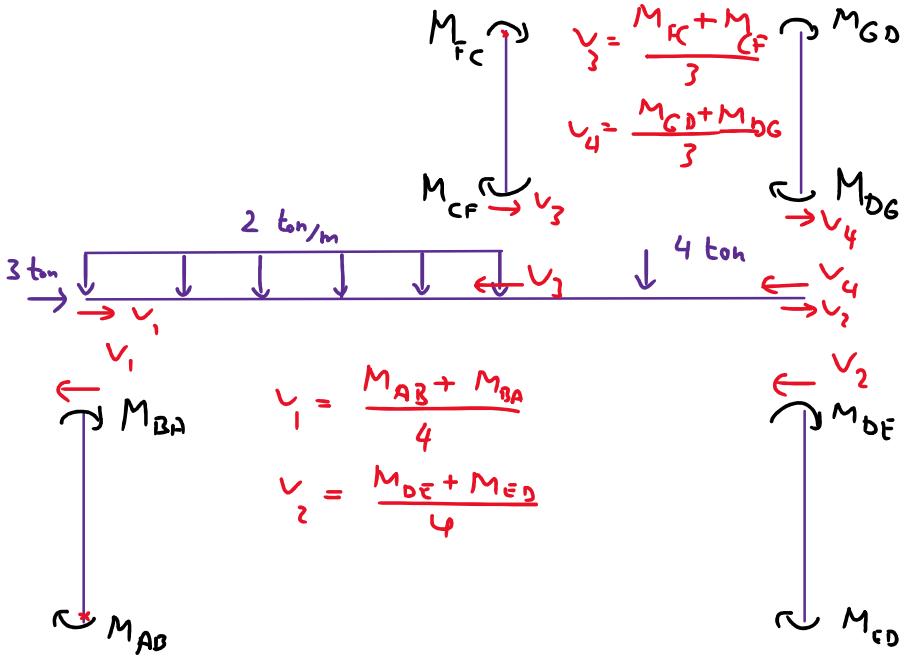
2 unknowns



$$V_1 = \frac{M_{FC} + M_{CF}}{3}$$

$$V_2 = \frac{M_{GD} + M_{DG}}{3}$$

⑥ $V_1 + V_2 + 5 = 0$



$$V_3 = \frac{M_{FC} + M_{CF}}{3}$$

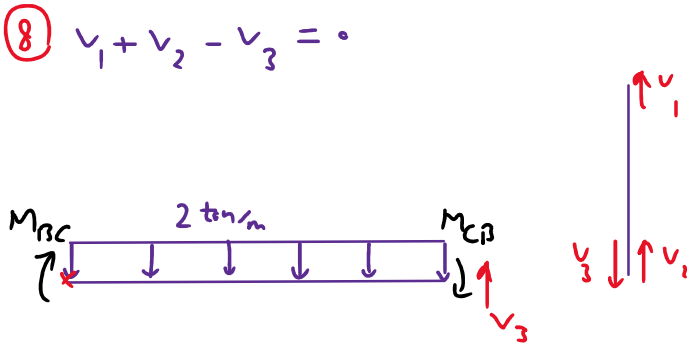
$$V_4 = \frac{M_{GD} + M_{DG}}{3}$$

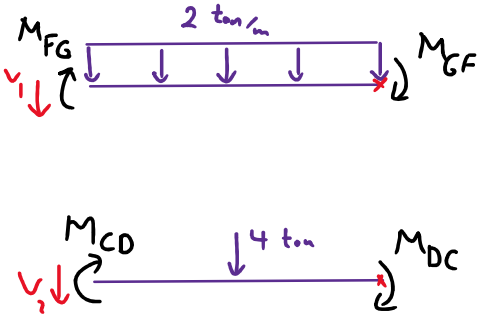
⑦ $V_1 + V_2 - V_3 - V_4 + 3 = 0$

$$V_1 = \frac{M_{AB} + M_{BA}}{4}$$

$$V_2 = \frac{M_{DE} + M_{ED}}{4}$$

⑧ $V_1 + V_2 - V_3 = 0$





$$V_1 = \frac{1}{6}\left[M_{FG} + M_{GF} - 2(6)(3)\right] \quad (36)$$

$$V_2 = \frac{1}{6}\left[M_{CD} + M_{DC} - 4(3)\right] \quad (12)$$

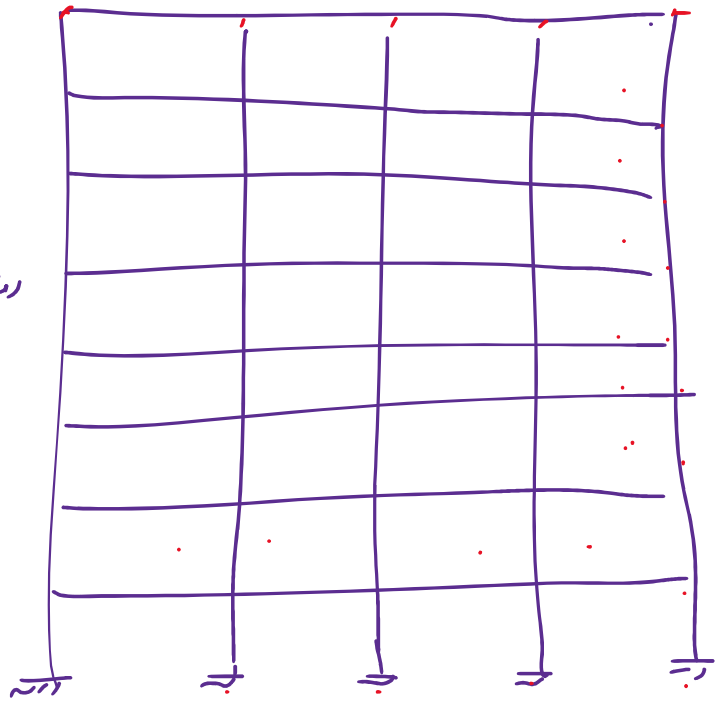
$$V_2 = \frac{1}{6}\left[M_{CD} + M_{DC} - \overset{12}{4}(3)\right]$$

$$V_3 = \frac{1}{8}\left[M_{CB} + M_{BC} + 2\overset{64}{(8)}(4)\right]$$

مقایسه‌ی

تعداد مجهولات و معادلات

به روش شیب و دیر



$\frac{\partial U}{\partial M_1} = 0$

$\int \frac{M}{EI}\frac{\partial M}{\partial M_1} dx = 0$

روش سازگاری ← تغییر شکل ها و روش کاستیلیانو

$(28+4) \times 3 = \boxed{96}$

(8×5) 0

8 5

روش شیب - افت

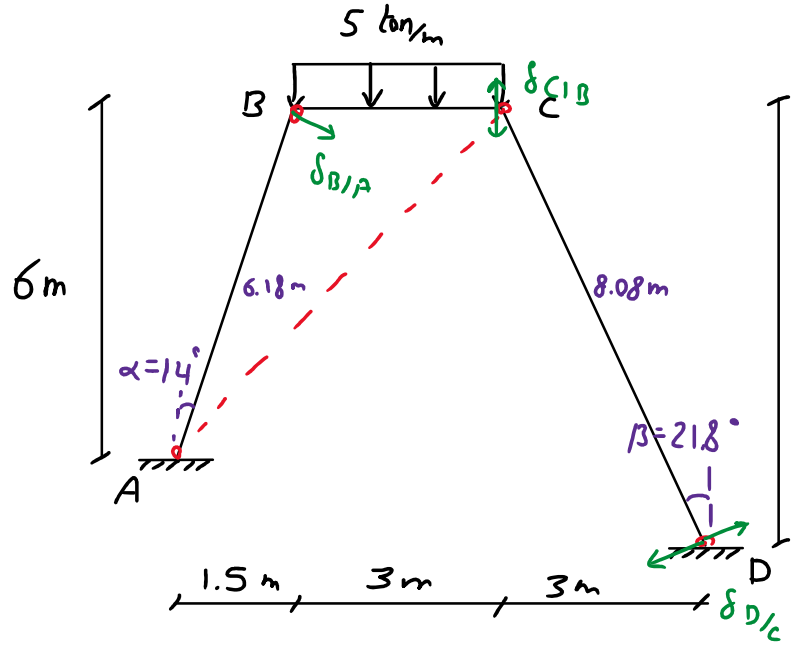
$\boxed{48}$

Video 8

Slope-Deflection 8

Thursday, March 14, 2024 14:17

مثال: مطلوب است تشکیل دستگاه معادلات جهت تحلیل سازه به روش شیب - افت.



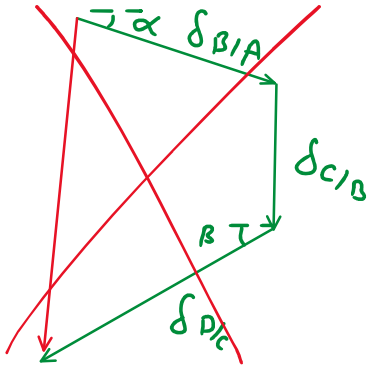
مجهول: θ_B , θ_C , Δ

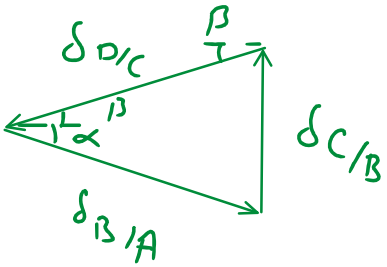
معادله: $\sum M_B = 0$

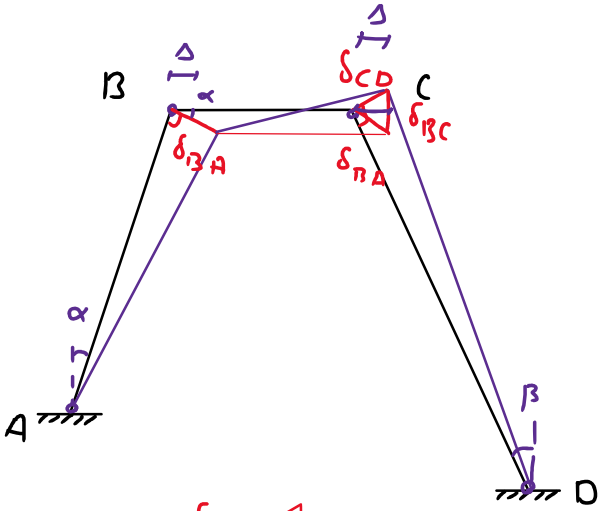
$\sum M_C = 0$

معادله برش روستیر BC

$$\delta_D = \delta_A + \delta_{B/A} + \delta_{C/B} + \delta_{D/C}$$



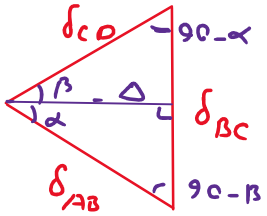




$$\delta_{CD} = \frac{\Delta}{\cos\beta} = 1.08\,\Delta$$

$$\delta_{AB} = \frac{\Delta}{\cos\alpha} = 1.03\,\Delta$$

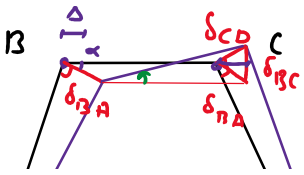
$$\delta_{BC} = \Delta(\tan\alpha + \tan\beta) = 0.65\,\Delta$$



$$\frac{\delta_{BC}}{\sin(\alpha+\beta)} = \frac{\delta_{CD}}{\sin(90-\beta)} = \frac{\delta_{AB}}{\sin(90-\alpha)}$$

$$M_{AB} = \frac{2EI}{6.18}\left(\theta_B - 3\,\frac{1.03\,\Delta}{6.18}\right)$$

$$M_{BA} = \frac{2EI}{6.18}\left(2\theta_B - 3\,\frac{1.03\,\Delta}{6.18}\right)$$



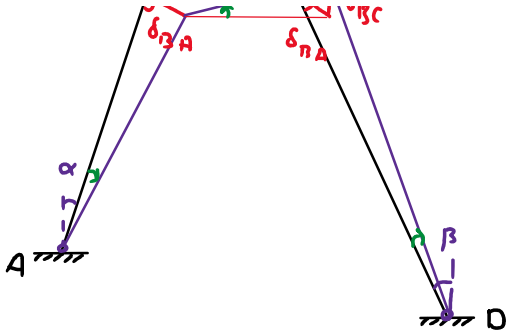
$$M_{BA} = \frac{2EI}{6.18}\left(2\theta_B - 3\,\frac{1.03\Delta}{6.18}\right)$$

$$M_{BC} = \frac{2EI}{3}\left(2\theta_B + \theta_C + 3\,\frac{0.65\Delta}{3}\right) - \frac{5\times 3^2}{12}$$

$$M_{CB} = \frac{2EI}{3}\left(2\theta_C + \theta_B + 3\,\frac{0.65\Delta}{3}\right) + \frac{5\times 3^2}{12}$$

$$M_{CD} = \frac{2EI}{8.08}\left(2\theta_C - 3\,\frac{1.08\Delta}{8.08}\right)$$

$$M_{DC} = \frac{2EI}{8.08}\left(\theta_C - 3\,\frac{1.08\Delta}{8.08}\right)$$



---

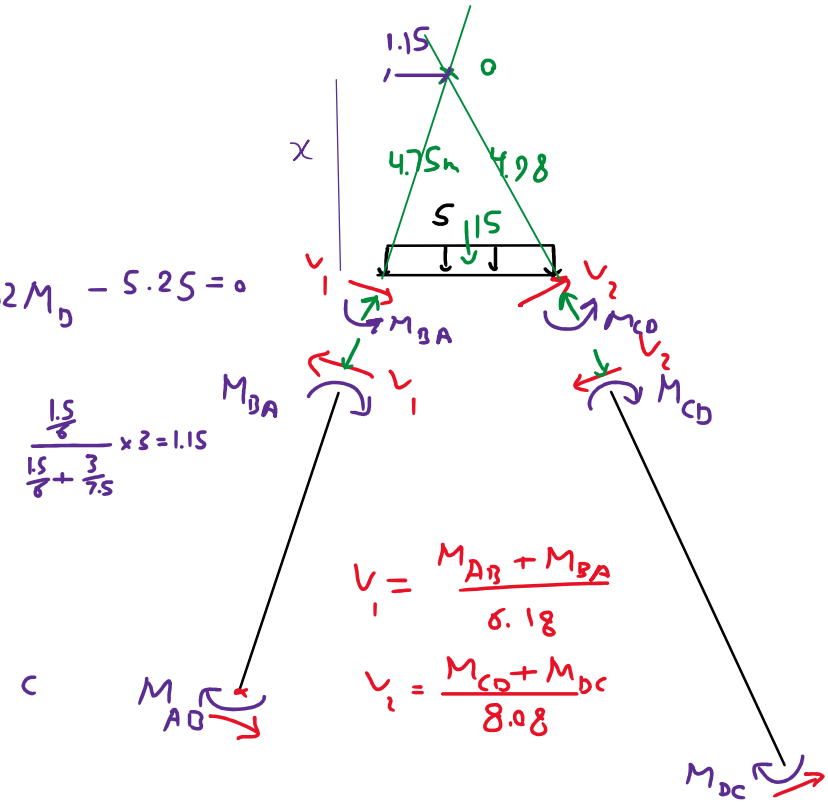
① $M_{BA} + M_{BC} = 0$

② $M_{CD} + M_{CB} = 0$

③ $0.77M_{AB} + 1.77M_{BA} + 1.62M_{CD} + 0.62M_D - 5.25 = 0$

$+\circlearrowleft \Sigma M_o = 0$

$M_{BA} + M_{CD} + 4.75V_1 + 4.98V_2 - 15\times 0.35 = 0$



Video 9

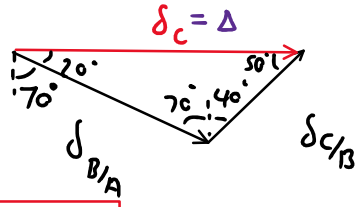
Slope-Deflection 9

Thursday, March 14, 2024 14:18

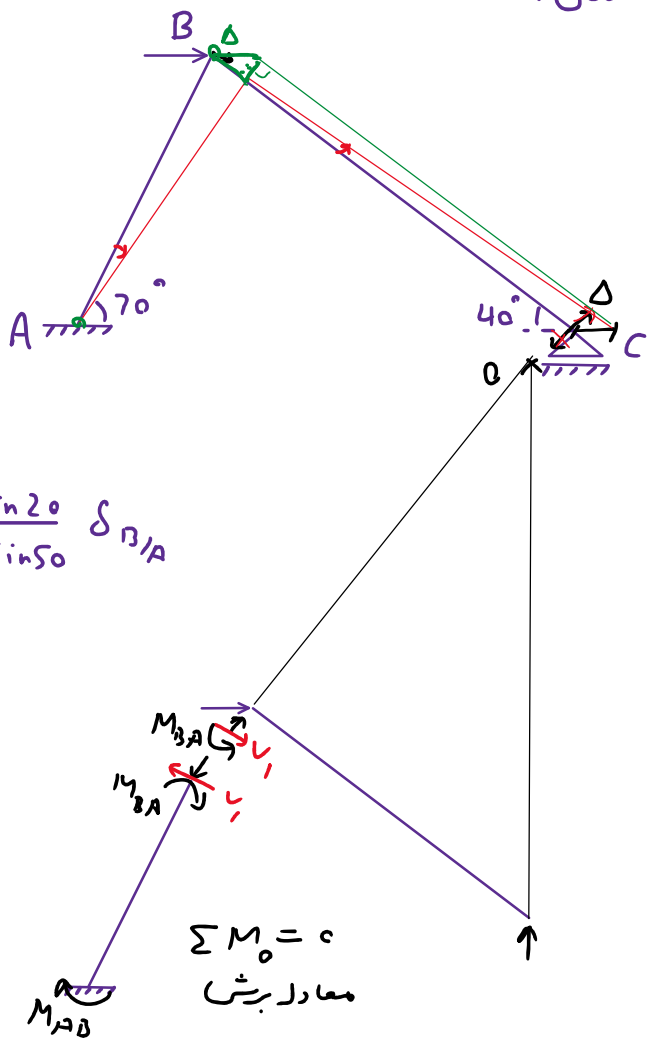
نوشتن درجات آزادی وابسته

مثال :

$$\vec{\delta_C} = \vec{\delta_{B/A}} + \vec{\delta_{C/B}}$$



$$\frac{\delta_{B/A}}{\sin 50} = \frac{\delta_{C/B}}{\sin 20}$$



$$\uparrow y: -\delta_{B/A}\sin 20 + \delta_{C/B}\sin 50 = 0 \rightarrow \delta_{C/B} = \frac{\sin 20}{\sin 50}\delta_{B/A}$$

$$\rightarrow x: \delta_{B/A}\cos 20 + \delta_{C/B}\cos 50 = \Delta$$

$$\delta_{B/A}\cos 20 + \frac{\sin 20}{\sin 50}\cos 50\,\delta_{B/A} = \Delta$$

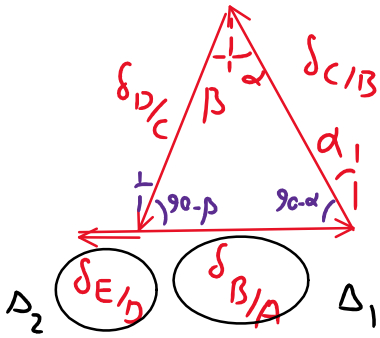
$$\boxed{\delta_{B/A} = 1.12\Delta} \qquad \boxed{\delta_{C/B} = 0.5\Delta}$$

$$M_{AB} = \frac{2EI}{L}\left(-3\,\frac{1.12\Delta}{L}\right)$$

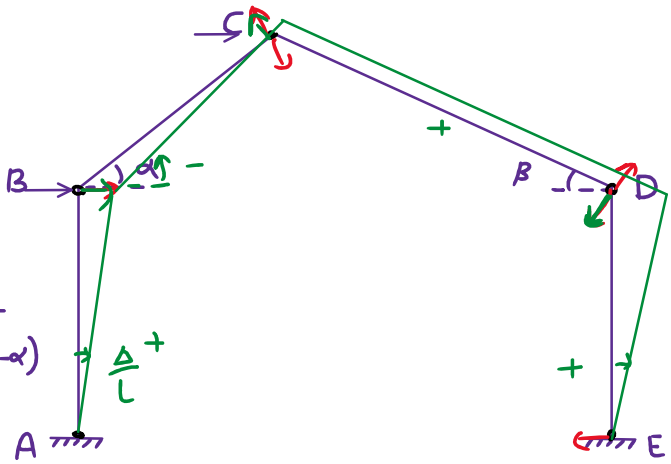
$$M_{BC} = \frac{2EI}{L}\left(+3\,\frac{0.5\Delta}{L}\right)$$

مثال :

$$\delta_E = \delta_A + \delta_{B/A} + \delta_{C/B} + \delta_{D/C} + \delta_{E/D}$$



$$\frac{\Delta_1 - \Delta_2}{\sin(\alpha+\beta)} = \frac{\delta_{CB}}{\sin(90-\beta)} = \frac{\delta_{DC}}{\sin(90-\alpha)}$$



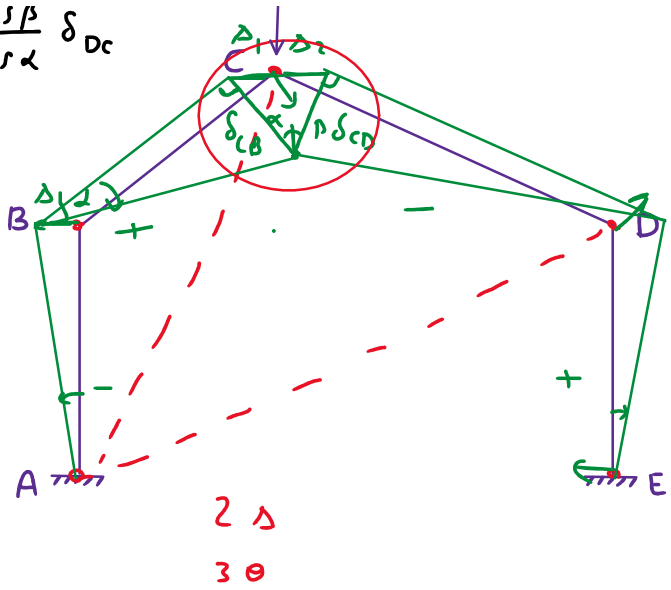
$$y: \delta_{CB}\cos\alpha - \delta_{DC}\cos\beta = 0 \rightarrow \delta_{CB} = \frac{\cos\beta}{\cos\alpha}\delta_{DC}$$

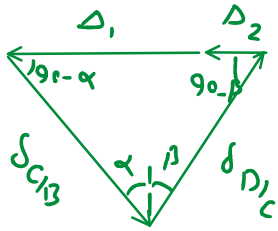
$$y:\ \delta_{CB}\cos\alpha - \delta_{DC}\cos\beta = 0 \rightarrow \delta_{CB} = \frac{\cos\beta}{\cos\alpha}\delta_{DC}$$

$$x:\ \Delta_1 - \delta_{CB}\sin\alpha - \delta_{DC}\sin\beta - \Delta_2 = 0$$

$$\left(-\frac{\cos\beta}{\cos\alpha}\sin\alpha - \sin\beta\right)\delta_{DC} = \Delta_2 - \Delta_1$$

$$\boxed{\delta_{DC} = \frac{\Delta_1 - \Delta_2}{\sin\beta + \frac{\cos\beta}{\cos\alpha}\cos\alpha}}$$

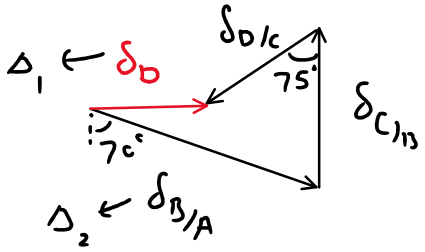


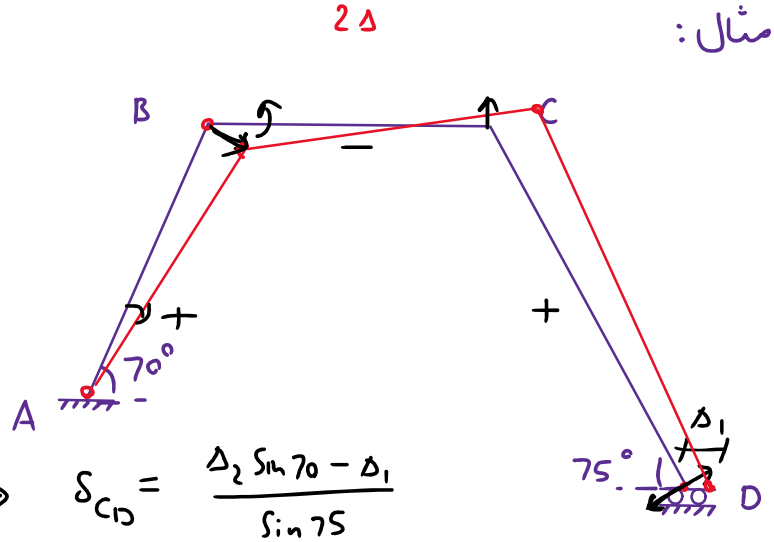


$$\frac{\Delta_1 + \Delta_2}{\sin(\alpha+\beta)} = \frac{\delta_{C/B}}{\sin(90-\beta)} = \frac{\delta_{D/C}}{\sin(90-\alpha)}$$

مثال:

$$\delta_D = \delta_A + \delta_{B/A} + \delta_{C/B} + \delta_{D/C}$$



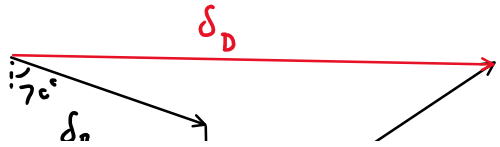


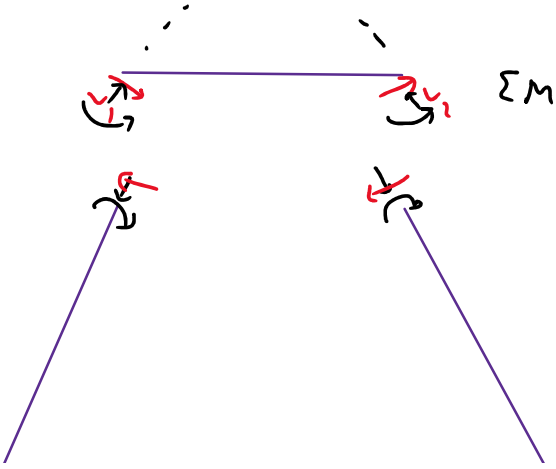
$$\rightarrow x:\ \Delta_2\sin 70 + 0 - \delta_{CD}\sin 75 = \Delta_1 \rightarrow \delta_{CD} = \frac{\Delta_2\sin 70 - \Delta_1}{\sin 75}$$

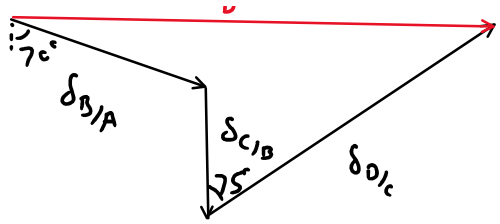
$$\uparrow y:\ -\Delta_2\cos 70 + \delta_{CB} - \delta_{CD}\cos 75 = 0 \rightarrow \delta_{CB} = \Delta_2\cos 70 + (0.97\Delta_2 - 1.04\Delta_1)\cos 75$$

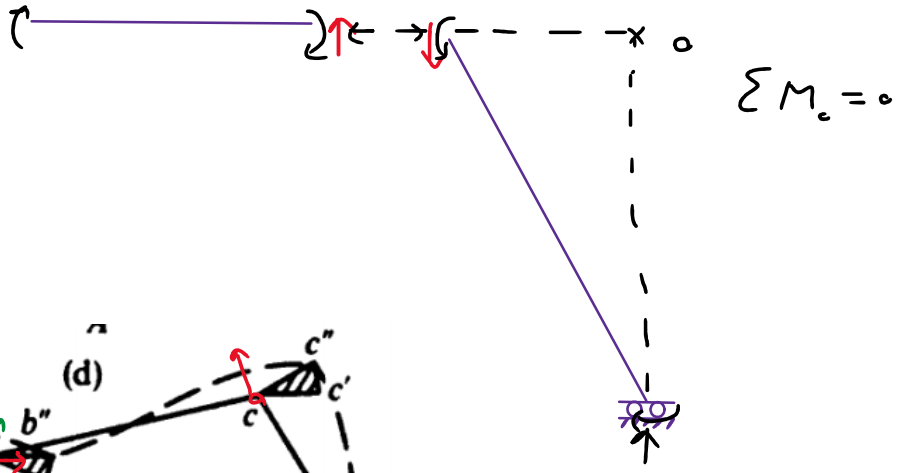
$$\boxed{\delta_{CD} = 0.97\Delta_2 - 1.04\Delta_1}$$

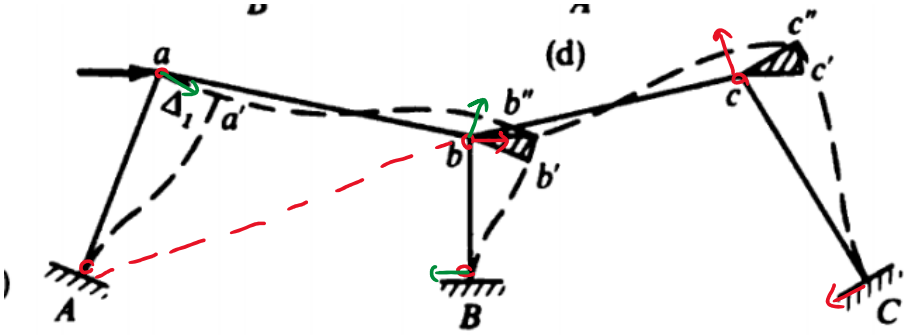
$$\boxed{\delta_{CB} = 0.59\Delta_2 - 0.27\Delta_1}$$





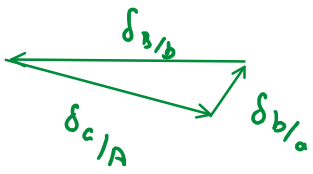




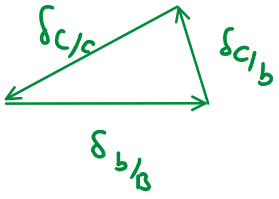


1 Δ

$$\delta_B = \delta_{C/A} + \delta_{b/a} + \delta_{B/b}$$



$$\delta_C = \delta_{b/A} + \delta_{C/b} + \delta_{C/c}$$

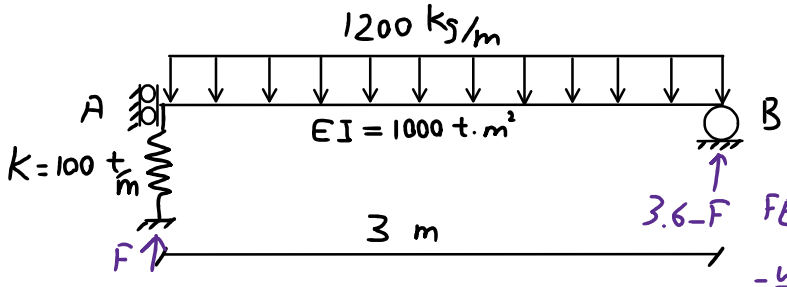


Video 10

Slope-Deflection 10

Thursday, March 14, 2024 14:18

مثال : نیروی فنر را به دست آورید .



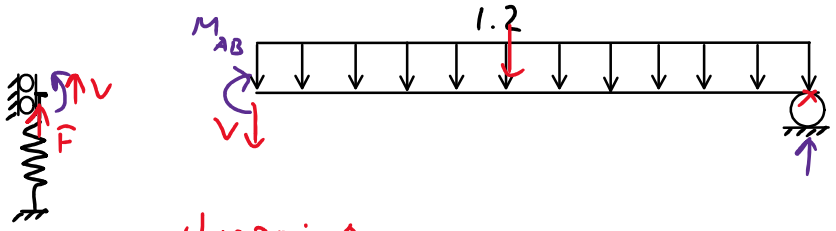
$FEM_{AB} - \frac{1}{2} FEM_{BA} = -\frac{wL^2}{12} - \frac{1}{2}\frac{wL^2}{12} = -\frac{3}{2}\frac{wL^2}{12}$

$$M_{AB} = \frac{3EI}{3}\left(\frac{\Delta}{3}\right) - \frac{1.2\times 3^2}{8} = \frac{1000}{3}\Delta - 1.35$$

$$(3.6 - F)\times 3 - 3.6\times\frac{3}{2} = \frac{1000}{3}\times\frac{F}{100} - 1.35$$

$$6.75 = \frac{19}{3}F \rightarrow \boxed{F = 1.066 \text{ ton}}$$

روش شیب-انحنا



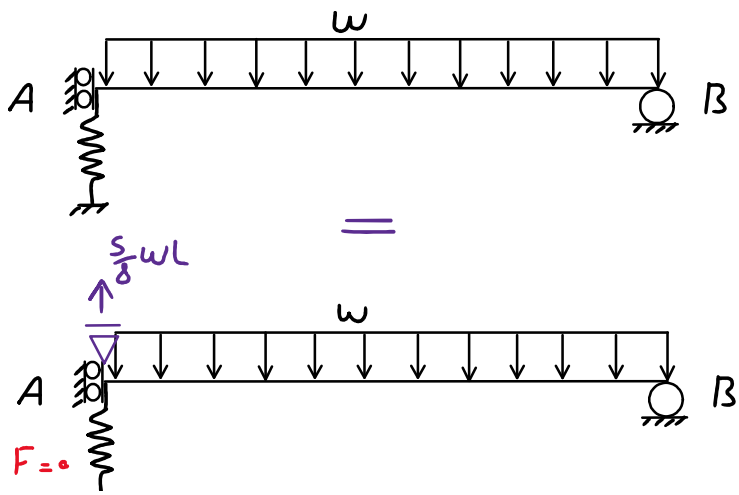
مجهول : Δ

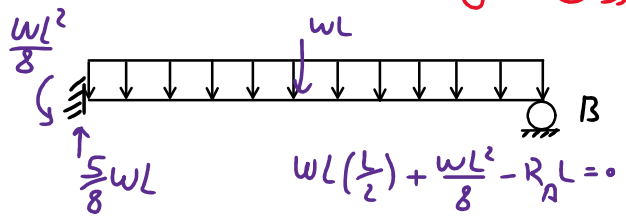
معادله : $V + F = 0$

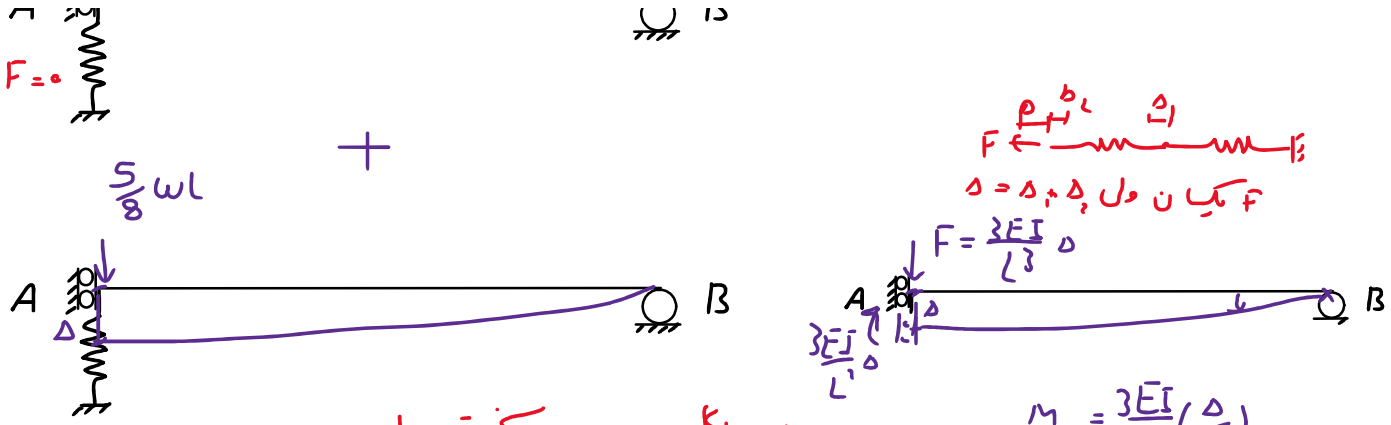
$$V = \frac{1}{3}\left(M_{AB} - 3.6\times\frac{3}{2}\right) = \frac{1}{3}\left(\frac{1000}{3}\Delta - 1.35 - 5.4\right) = \frac{1000}{9}\Delta - 2.25$$

$$V + F = \frac{1000}{9}\Delta - 2.25 + 100\Delta = 0 \rightarrow \Delta = 0.01066 \text{ m} \rightarrow \boxed{F = 1.066 \text{ ton}}$$

روش کنتی







سختی سازی

$\Delta = \frac{F}{\Sigma k}$

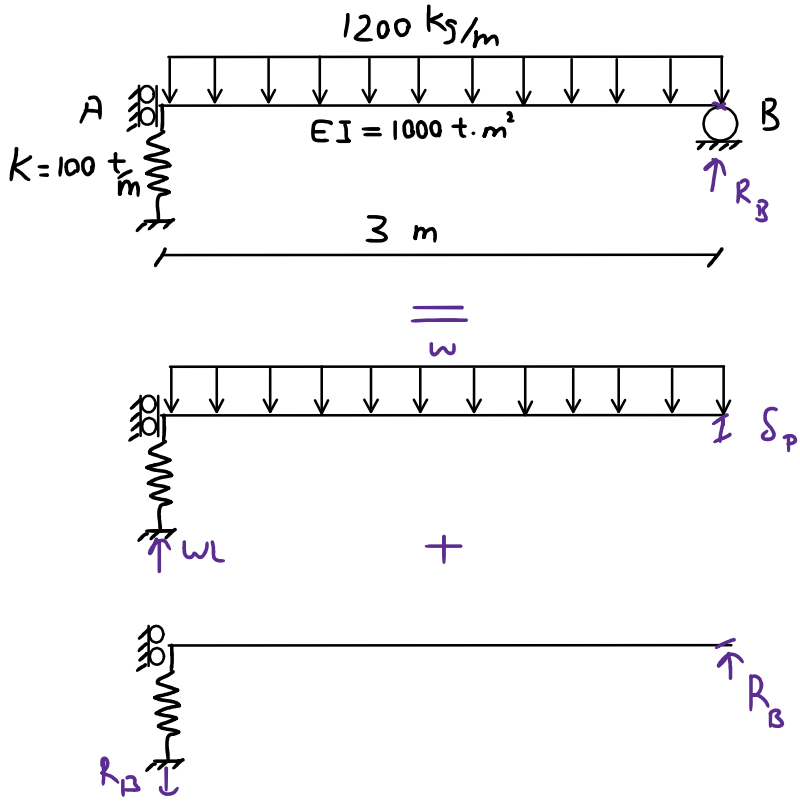
$F = F_1 + F_2$ و Δ یکسان

$F = k_1\Delta + k_2\Delta \rightarrow \Delta = \frac{F}{\Sigma k} \rightarrow F_1 = \frac{k_1}{\Sigma k} F$

$k = \frac{3EI}{L^3} = \frac{3\times 1000}{3^3} = \frac{1000}{9}$

$$F = \frac{k_s}{k_1 + k_b} \times \frac{5}{8} wL = \frac{100}{100 + 111.11} \times \frac{5}{8} \times 1.2 \times 3 = 1.056 \text{ ton}$$

روش سازگاری تغییر شکل‌ها



مجهول: R_B

معادله: $\Delta_B = 0$

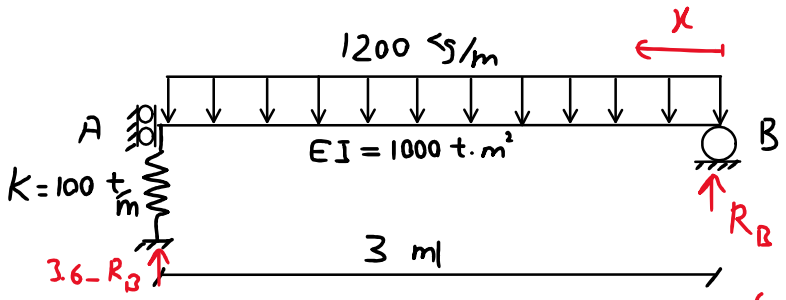
$\delta_p + R_B \delta_i = 0$

$$\frac{wL}{k} + \frac{wL^4}{8EI} = \frac{R_B}{k} + \frac{R_B L^3}{3EI}$$

$$\frac{1.2\times 3}{100} + \frac{1.2\times 3^4}{8\times 1000} = R_B\left(\frac{1}{100} + \frac{3^3}{3\times 1000}\right) \rightarrow R_B = 2.534$$

$$F = 1.066 \text{ tm}$$

روش کاستیلیانو



$$U = \frac{1}{2}\int \frac{M^2}{EI}dx + \frac{1}{2}\frac{F^2}{k}$$

$$\Delta_B = \frac{\partial U}{\partial R_B} = 0$$

$$\begin{cases} M = R_B x - \frac{1.2x^2}{2} & \frac{\partial M}{\partial R_B} = x \\ F = 3.6 - R_B & \frac{\partial F}{\partial R_B} = -1 \end{cases}$$

$$\int \frac{M}{EI}\left(\frac{\partial M}{\partial R_B}\right)dx + \frac{F}{k}\left(\frac{\partial F}{\partial R_B}\right) = 0$$

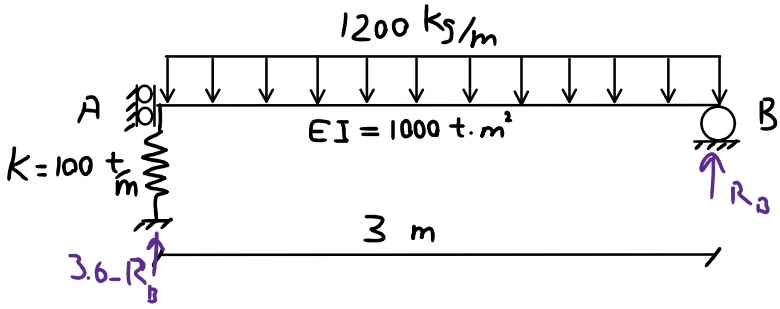
$$\frac{1}{EI}\int_0^3 (R_B x - 0.6x^2)x\,dx + \frac{1}{k}(3.6 - R_B)(-1) = 0$$

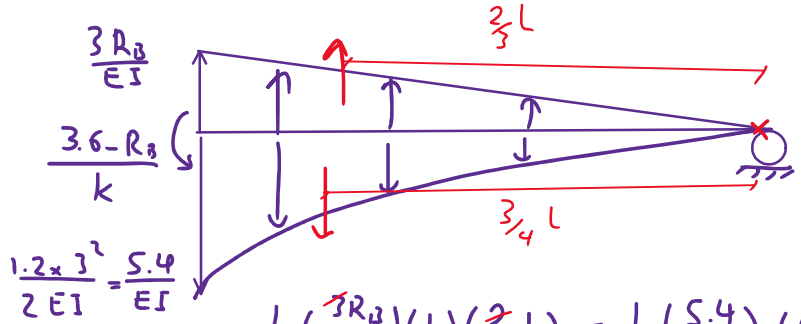
$$\frac{1}{1000}\left(R_B\frac{x^3}{3} - \frac{0.6}{4}x^4\right)\Big|_0^3 + \frac{1}{100}(R_B - 3.6) = 0$$

$$\frac{1}{1000}(9R_B - 12.15) + \frac{1}{100}(R_B - 3.6) = 0$$

$$9R_B - 12.15 + 10R_B - 36 = 0 \longrightarrow R_B = 2.534 \longrightarrow F = 1.066 \text{ tm}$$

روش تیر مزدوج





$$+\circlearrowleft \sum M_B = 0$$

$$\frac{1}{2}\left(\frac{3R_B}{EI}\right)(L)\left(\frac{2}{3}L\right) - \frac{1}{2}\left(\frac{5.4}{EI}\right)(L)\left(\frac{3}{4}L\right) - \frac{3.6 - R_B}{\quad} = 0$$

$\frac{1.2 \times 3^3}{2EI} = \frac{5.4}{EI}$ ✓

$$\frac{1}{2}\left(\frac{3R_B}{EI}\right)(L)\left(\frac{2}{3}L\right) - \frac{1}{3}\left(\frac{5.4}{EJ}\right)(L)\left(\frac{3}{4}L\right) - \frac{3.6 - R_B}{k} = 0$$

$$\frac{1}{1000}\left(3^2 R_B - 12.15\right) - \frac{1}{100}\left(3.6 - R_B\right) = 0 \rightarrow R_B = 2.534$$

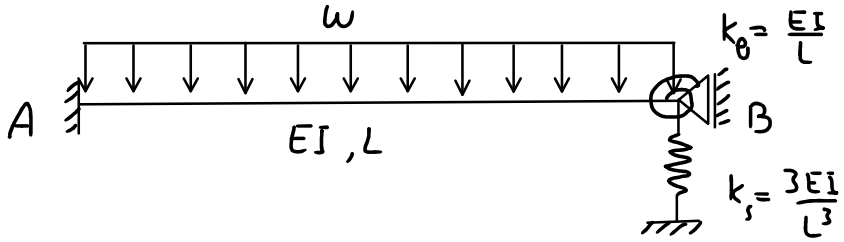
$F = 1.066$ ton

Video 11

Slope-Deflection 11

Wednesday, March 27, 2024 16:11

مثال : نیروهای فنرها را به دست آورید .



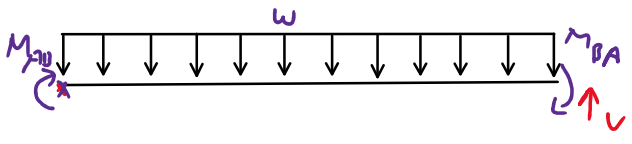
① روش شیب - افت :

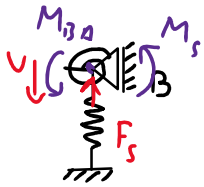
مجهولات : θ_B , Δ

معادلات : $\sum M_B = 0$ ، معادله برش

$$M_{AB} = \frac{2EI}{L}\left(\theta_B - 3\frac{\Delta}{L}\right) - \frac{wL^2}{12}$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B - 3\frac{\Delta}{L}\right) + \frac{wL^2}{12}$$





$$V = \frac{1}{L}(M_{AB} + M_{BA}) + \frac{wL}{2} = \frac{6EI}{L^2}\theta_B - 12\frac{EI}{L^3}\Delta + \frac{wL}{2}$$

① $M_{BA} + M_s = 0 \rightarrow$ ($M_s = k_\theta \theta_B$) $\quad \frac{5EI}{L}\theta_B - \frac{6EI}{L^2}\Delta + \frac{wL^2}{12} = 0 \quad \times \frac{L}{EI}$

② $V - F_s = 0 \rightarrow$ ($F_s = k_s \Delta$) $\quad \frac{6EI}{L^2}\theta_B - 15\frac{EI}{L^3}\Delta + \frac{wL}{2} = 0 \quad \times \frac{L^2}{EI}$

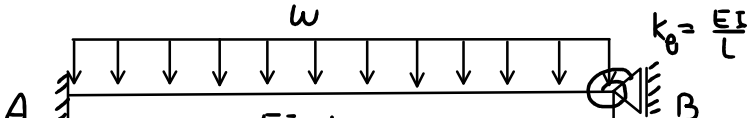
$$\begin{cases} 5\theta_B - 6\frac{\Delta}{L} = -\frac{wL^3}{12EI} & \times 6 \\ 6\theta_B - 15\frac{\Delta}{L} = -\frac{wL^3}{2EI} & \times -5 \end{cases}$$

$$(75 - 36)\frac{\Delta}{L} = \left(\frac{5}{2} - \frac{1}{2}\right)\frac{wL^3}{EI} \rightarrow \boxed{\frac{\Delta}{L} = \frac{2}{39}\frac{wL^3}{EI}}$$

$$6\theta_B = \frac{5}{15} \times \frac{2}{39}\frac{wL^3}{EI} - \frac{1}{2}\frac{wL^3}{EI} = \frac{20-13}{26}\frac{wL^3}{EI} \rightarrow \boxed{\theta_B = \frac{7}{156}\frac{wL^3}{EI}}$$

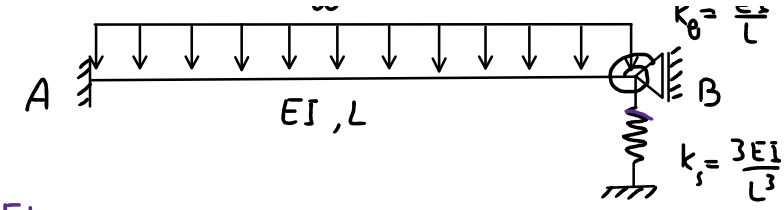
$$M_s = k_\theta \theta_B = \frac{EI}{L} \times \frac{7}{156}\frac{wL^3}{EI} \rightarrow \boxed{M_s = \frac{7}{156}wL^2}$$

$$F_s = k_s \Delta = \frac{3EI}{L^3} \times \frac{2}{39}\frac{wL^4}{EI} \rightarrow \boxed{F_s = \frac{2}{13}wL}$$



② روش تیر مزدوج

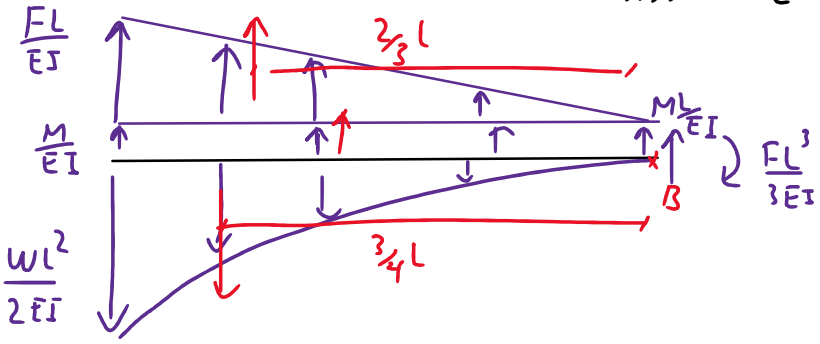
$\theta = \frac{M}{k_\theta} = \frac{Ml}{EI} \rightarrow V$

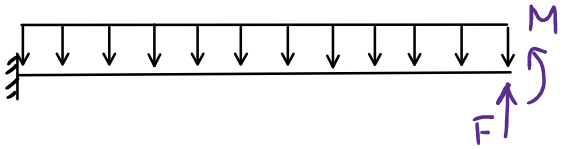


۱) روش تیر مزدوج

$\theta = \frac{M}{k_\theta} = \frac{ML}{EI}$ → V

$\Delta = \frac{F}{k_s} = \frac{FL^3}{3EI}$ → M





$$+\uparrow \sum F_y = 0 \quad \frac{1}{2}\left(\frac{FL}{EI}\right)(L) + \left(\frac{M}{EI}\right)(L) - \frac{1}{3}\left(\frac{wL^2}{2EI}\right)(L) + \frac{ML}{EI} = 0 \quad \times \frac{EI}{L}$$

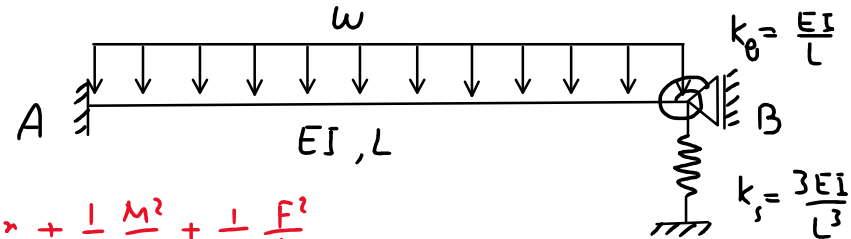
$$+\circlearrowright \sum M_B = 0 \quad \frac{FL^2}{2EI}\left(\frac{2}{3}L\right) + \frac{ML}{EI}\left(\frac{L}{2}\right) - \frac{wL^4}{8EI}\left(\frac{3}{4}L\right) + \frac{FL^3}{3EI} = 0 \quad \times \frac{EI}{L^2}$$

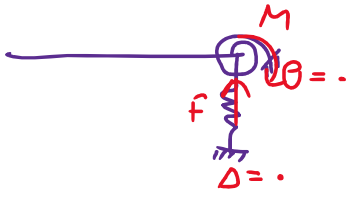
$$-4 \begin{cases} 2M + \frac{1}{2}FL = \frac{1}{6}wL^2 \\ \frac{M}{2} + \frac{2}{3}FL = \frac{1}{8}wL^2 \end{cases}$$

$$\left(\frac{1}{2} - \frac{2}{3}\right)FL = \left(\frac{1}{6} - \frac{1}{2}\right)wL^2 \rightarrow \left(\frac{3-16}{12}\right)FL = -\frac{1}{3}wL^2 \rightarrow \boxed{F = \frac{2}{13}wL}$$

$$2M = -\frac{1}{2} \times \frac{2}{13}wL^2 + \frac{1}{6}wL^2 = \frac{-6+13}{78}wL^2 = \frac{7}{78}wL^2 \rightarrow \boxed{M = \frac{7}{156}wL^2}$$

۲) روش کاستیلیانو



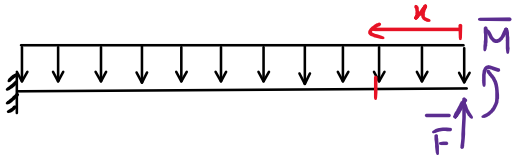


$$U = \frac{1}{2}\int \frac{m^2}{EI}dx + \frac{1}{2}\frac{M^2}{k_\theta} + \frac{1}{2}\frac{F^2}{k_s}$$

$$\frac{\partial U}{\partial \bar{M}} = 0 \quad , \quad \frac{\partial U}{\partial \bar{F}} = 0$$

$$m = \bar{M} + \bar{F}x - \frac{wx^2}{2} \quad ; \quad \frac{\partial m}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial m}{\partial \bar{F}} = x$$

$$M = \bar{M} \quad ; \quad \frac{\partial M}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial M}{\partial \bar{F}} = 0$$



$$M = \bar{M} \quad ; \quad \frac{\partial M}{\partial \bar{M}} = 1 \quad ; \quad \frac{\partial M}{\partial \bar{F}} = 0$$

$$F = \bar{F} \quad ; \quad \frac{\partial F}{\partial \bar{M}} = 0 \quad ; \quad \frac{\partial F}{\partial \bar{F}} = 1$$

$$\frac{\partial U}{\partial \bar{M}} = \int \frac{M}{EI}\frac{\partial M}{\partial \bar{M}}\,dx + \frac{M}{k_0}\frac{\partial M}{\partial \bar{M}} + \frac{F}{k_1}\frac{\partial F}{\partial \bar{M}} = 0$$

$$\frac{1}{EI}\int_0^L \left(\bar{M} + \bar{F}x - \frac{wx^2}{2}\right)(1)\,dx + \left(\frac{\bar{M}L}{EI}\right)(1) + 0 = 0$$

$$\frac{1}{EI}\left(\bar{M}x + \bar{F}\frac{x^2}{2} - \frac{wx^3}{6}\right)\Big|_0^L + \frac{\bar{M}L}{EI} = 0 \quad \rightarrow ① \quad 2\frac{\bar{M}L}{EI} + \frac{1}{2}\frac{\bar{F}L^2}{EI} - \frac{wL^3}{6EI} = 0 \quad \times \frac{EI}{L}$$

$$\frac{\partial U}{\partial \bar{F}} = \int \frac{M}{EI}\frac{\partial M}{\partial \bar{F}}\,dx + \frac{M}{k_0}\frac{\partial M}{\partial \bar{F}} + \frac{F}{k_1}\frac{\partial F}{\partial \bar{F}} = 0$$

$$\frac{1}{EI}\int_0^L \left(\bar{M} + \bar{F}x - \frac{wx^2}{2}\right)(x)\,dx + 0 + \left(\frac{\bar{F}L^3}{3EI}\right)(1) = 0$$

$$\frac{1}{EI}\left(\bar{M}\frac{x^2}{2} + \bar{F}\frac{x^3}{3} - \frac{wx^4}{8EI}\right)\Big|_0^L + \frac{\bar{F}L^3}{3EI} = 0 \quad \rightarrow ② \quad \frac{1}{2}\frac{\bar{M}L^2}{EI} + \frac{2}{3}\frac{\bar{F}L^3}{EI} - \frac{wL^4}{8EI} = 0 \quad \times \frac{EI}{L^3}$$

$$\begin{cases} ① & 2\bar{M} + \frac{1}{2}\bar{F}L = \frac{1}{6}wL^2 \\ ② & \frac{1}{2}\bar{M} + \frac{2}{3}\bar{F}L = \frac{1}{8}wL^2 \end{cases}$$

$$\bar{F} = \frac{2}{13}wL$$

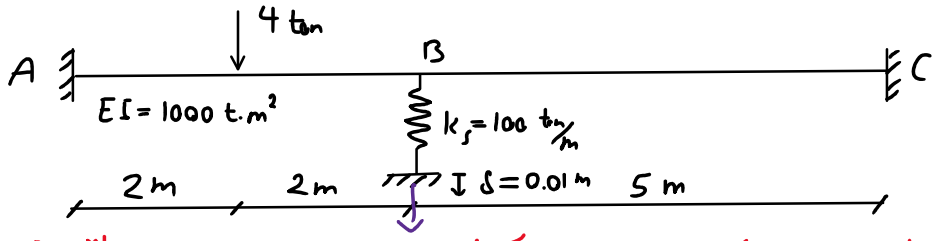
$$\bar{M} = \frac{7}{156}wL^2$$

Video 12

Slope-Deflection 12

Wednesday, March 27, 2024 16:11

مثال: تیر شکل زیر را تحلیل کنید.



۲ مجهول تغییرمکان Δ , θ_B : گتی ۳ مجهول نیروی: نرن

روش شیب - افت (1)

$$M_{AB} = \frac{2\times1000}{4}\left(\theta_B - \frac{3\Delta}{4}\right) - \frac{4\times4}{8} = 500\theta_B - 375\Delta - 2$$

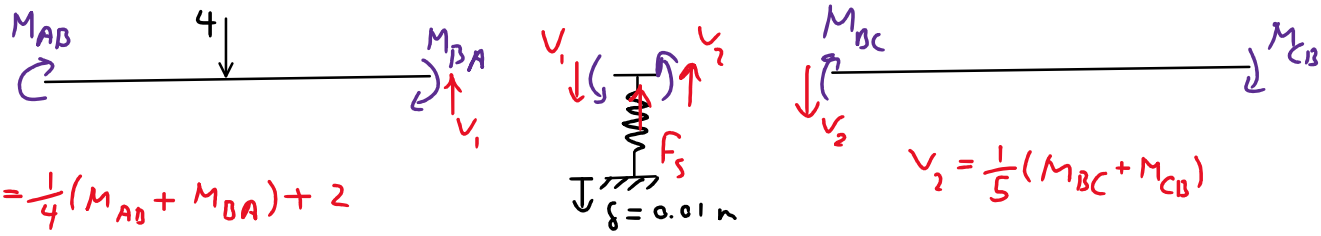
$$M_{BA} = 500\left(2\theta_B - 3\frac{\Delta}{4}\right) + 2 = 1000\theta_B - 375\Delta + 2$$

$$M_{BC} = \frac{2\times1000}{5}\left(2\theta_B + 3\frac{\Delta}{5}\right) = 800\theta_B + 240\Delta$$

$$M_{CB} = 400\left(\theta_B + 3\frac{\Delta}{5}\right) = 400\theta_B + 240\Delta$$

(1) $M_{BA} + M_{BC} = 0$

$$1800\theta_B - 135\Delta + 2 = 0$$



$$V_1 = \frac{1}{4}(M_{AB} + M_{BA}) + 2$$

$$V_2 = \frac{1}{5}(M_{BC} + M_{CB})$$

(2) $V_2 - V_1 + F_s = 0$

$$\frac{1}{5}(1200\theta_B + 480\Delta) - \frac{1}{4}(1500\theta_B - 750\Delta) - 2 + k_s(\Delta - 0.01) = 0$$

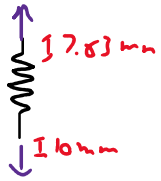
$k_s = 100$

$$-135\theta_B + 383.5\Delta = 3$$

$$\begin{cases} 1800\theta_B - 135\Delta = -2 \\ -135\theta_B + 383.5\Delta = 3 \end{cases} \quad \frac{40}{3}$$

$$4978.3\Delta = 38 \longrightarrow \Delta = 7.63\times10^{-3}\ m = 7.63\ mm$$

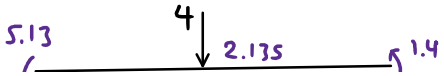
$$\theta_B = -5.47\times10^{-4}\ rad$$



$$F_s = 100(7.63\times10^{-3} - 0.01) = -0.237\ ton$$

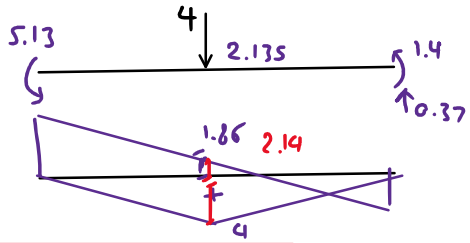
$F_s = 0.237$ ton

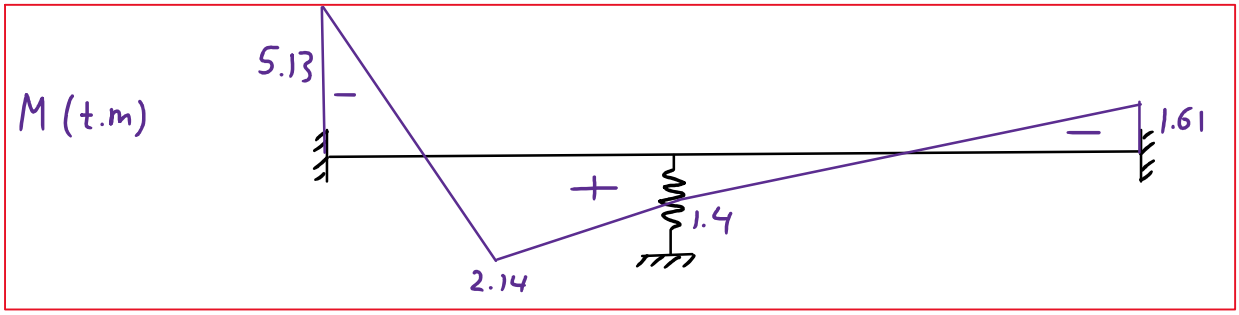
$M_{AB} = -5.13$ t.m $\quad M_{BC} = 1.4$ t.m



$M_{AB} = -5.13$ t.m $M_{BC} = 1.4$ t.m

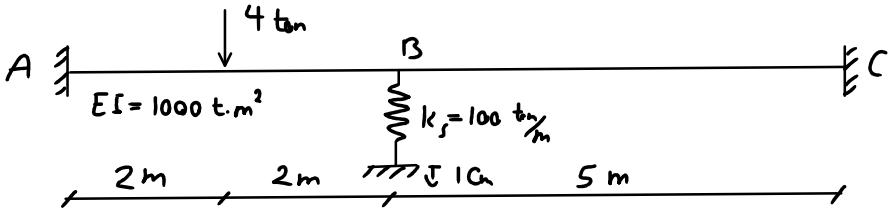
$M_{BA} = -1.4$ t.m $M_{CB} = 1.61$ t.m

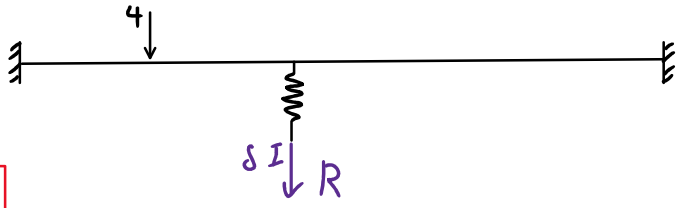




(۲)

دوتی نیردی

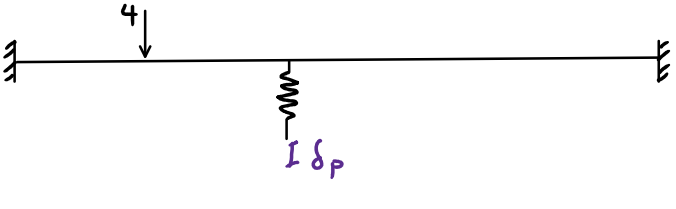




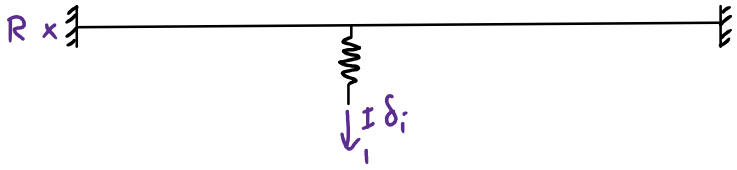
$\delta = 0.01$

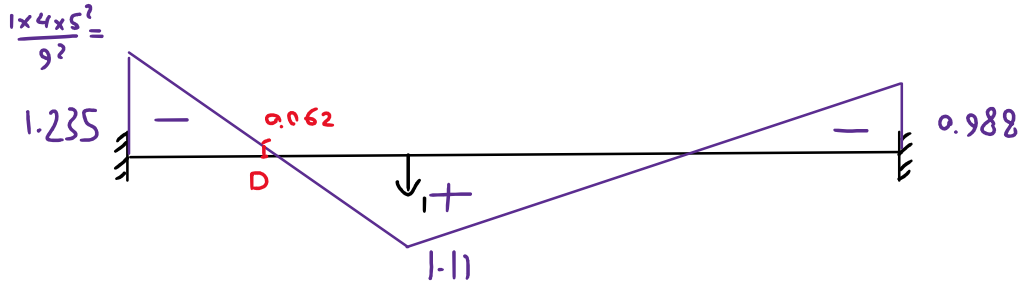
$\delta_p + R\,\delta_i = 0.01$

=

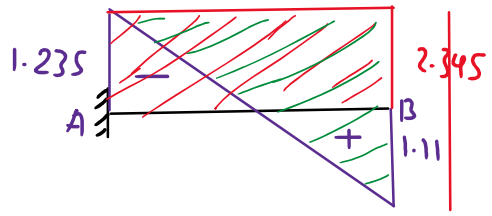


+





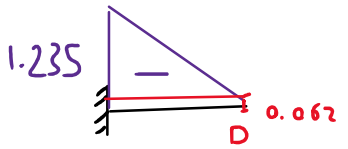
$$-\left(\frac{1.235+0.988}{2}\right)+\frac{1\times4\times5}{9}=1.11$$



$\delta_{B/A}=S\bar{X}$ $\quad -\frac{ML^2}{2}$ $\quad \frac{ML^2}{6}$

$$\delta_{B/A}=-1.235\times\frac{4^2}{2}+2.345\times\frac{4^2}{6}=\frac{-3.63}{EI}=-3.63\times10^{-3}\ m$$

$$\delta_i=\delta_b+\delta_s=-3.63\times10^{-3}-\frac{1}{100}=-13.63\times10^{-3}\ m$$

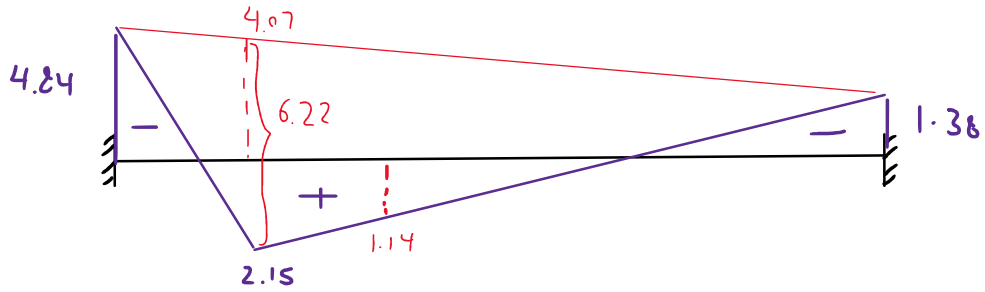


قانون ماکسول

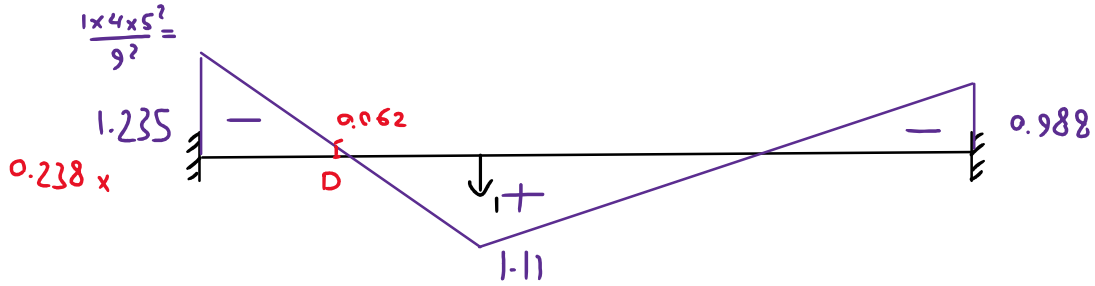
$$\delta_{D/A}=-0.062\times\frac{2^2}{2}-1.172\times\frac{2^2}{3}=\frac{1.687}{EI}\longrightarrow\delta_p=4\times\frac{1.687}{EI}=\frac{6.75}{EI}=6.75\times10^{-3}\ m$$

$\delta_p+R\,\delta_i=0.01$ معادله سازگاری

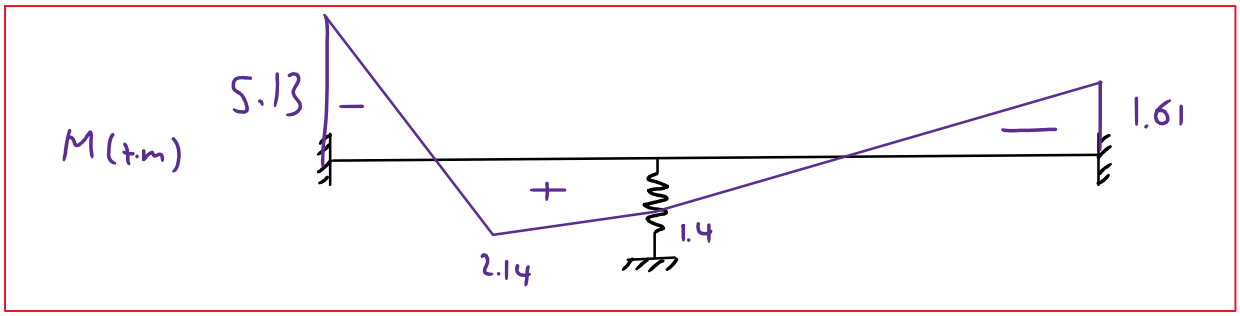
$$6.75\times10^{-3}+R\,(13.63\times10^{-3})=0.01\longrightarrow R=0.238\ ton$$



+



=


M (t.m)
5.13
1.61
2.14
1.4

Video 13

Moment Distribution 1

Wednesday, March 27, 2024 16:11

فصل دوم :

روش توزیع لنگر (روش کراس)

روش توزیع لنگر، در سال 1930 توسط پروفسور هاردی کراس (Hardy Cross) طی مقاله‌ای انتشار یافت. این روش از نظر مفهومی مشابه روش شیب-افت است با این تفاوت که معادلات تعادل لنگر گره‌ها طی یک روش عددی با ماهیت محاسباتی دوره‌ای ارضا می‌شود.

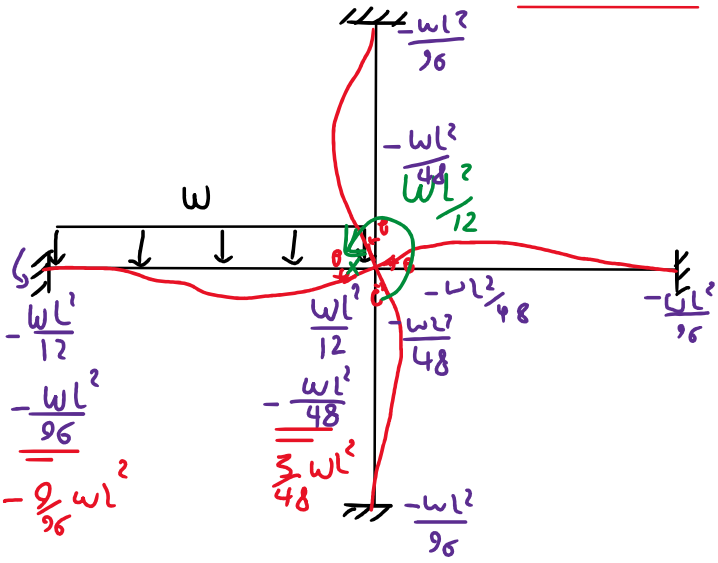
به طور کلی همه روش‌های قبلی تحلیل سازه‌های نامعین اعم از روش‌های نیرویی و روش شیب-افت نیازمند حل همزمان n معادله n مجهولی هستند؛ در حالی که روش توزیع لنگر طی چند سیکل محاسباتی، به دقت مورد نظر می‌رسد.

البته اگر سازه دارای درجه آزادی تغییرمکانی باشد، نیاز به حل معادلاتی به تعداد این درجات آزادی است.

توضیح ابتدایی

۱ θ درجه آزادی ندارد.

$$M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B) + FEM_{AB}$$

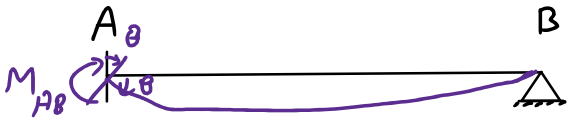


تعاریف پایه

① سختی دورانی تیر: لنگر لازم برای دوران واحد $k = \frac{M}{\theta}$

$$M_{AB} = \frac{2EI}{L}(2\theta) = \frac{4EI}{L}\theta$$

$$k = \frac{4EI}{L}$$ سختی تیر

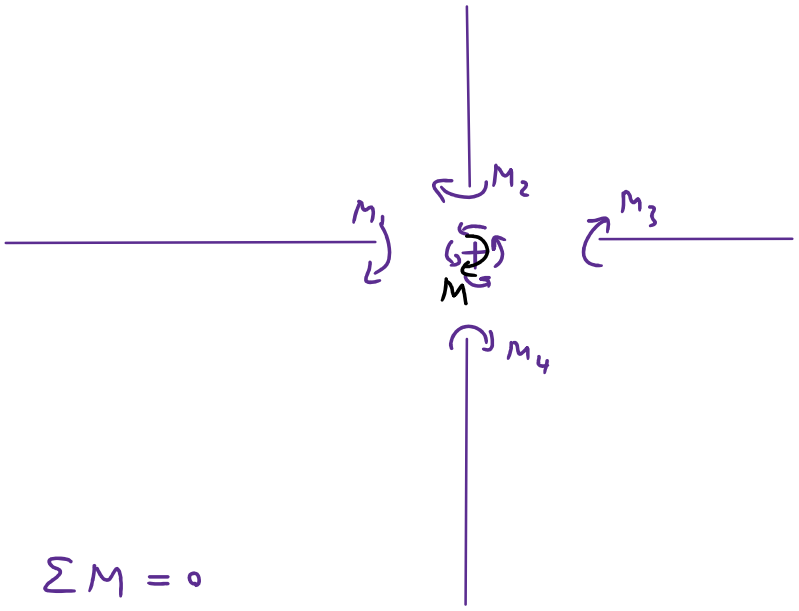


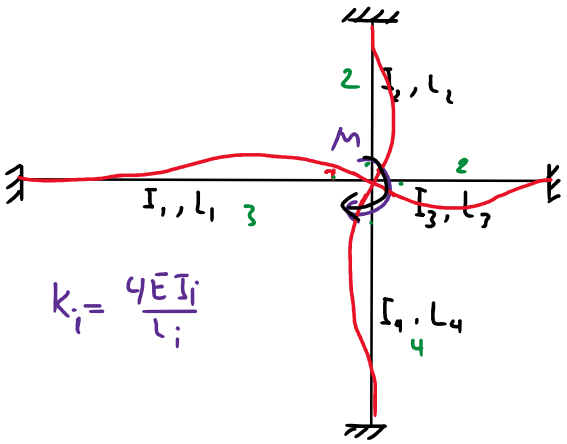
$$M_{AB} = \frac{3EI}{L}(\theta)$$

$$k = \frac{3EI}{L}$$ سختی اصلاح شده

② ضریب توزیع : اگر لنگر M روی گره نشان داده شده وارد شود، سهم هریک از اعضا در جذب لنگر، همان ضریب توزیع است.

ضریب توزیع است.





$k_i = \frac{4EI_i}{L_i}$

$\Sigma M = 0$

$M_1 + M_2 + M_3 + M_4 = M$

$k_1\theta + k_2\theta + k_3\theta + k_4\theta = M$

$\theta = \frac{M}{\Sigma k_i}$

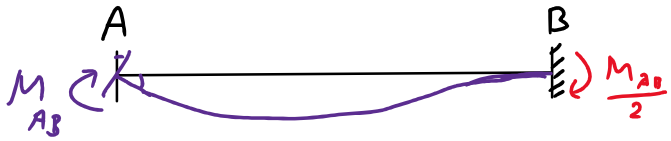
$DF_i = \frac{k_i}{\Sigma k_i}$

DF

$M_1 = k_1\theta = \frac{k_1}{\Sigma k_i} M$

$M_2 = k_2\theta = \frac{k_2}{\Sigma k_i} M$

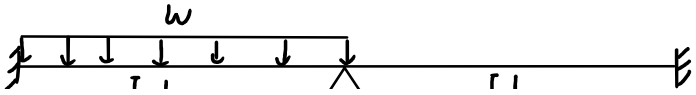
③ ضریب انتقال: در صورتی که به یک سر تیر لنگری وارد شود، سر دیگر تیر هم لنگری جذب می‌کند. نسبت این لنگر به لنگر وارده را ضریب انتقال گویند.

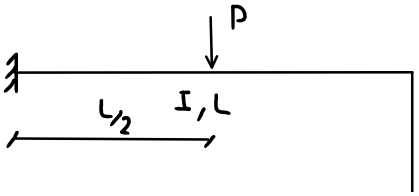


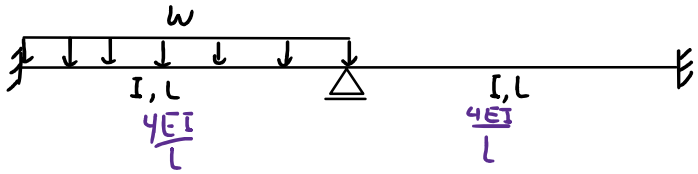
$$\begin{cases} M_{AB} = \frac{4EI}{L}\theta_A \\ M_{BA} = \frac{2EI}{L}\theta_A \end{cases} \rightarrow C = \frac{M_{BA}}{M_{AB}} = 0.5$$

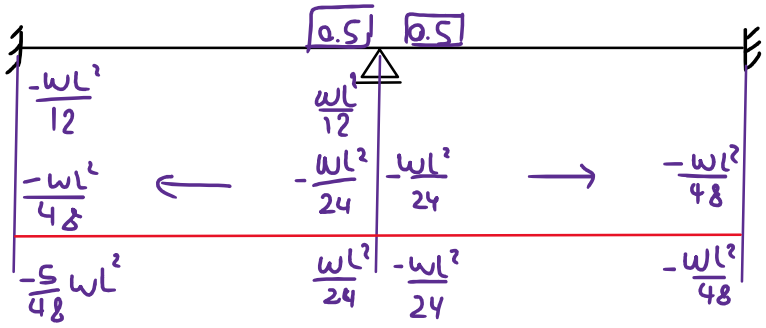
مثال: در تیر دوقاب شکل زیر، لنگرهای انتهایی اعضا را به دست آورید.

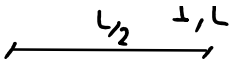
Δ ندارد
1 θ

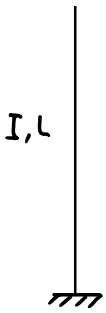


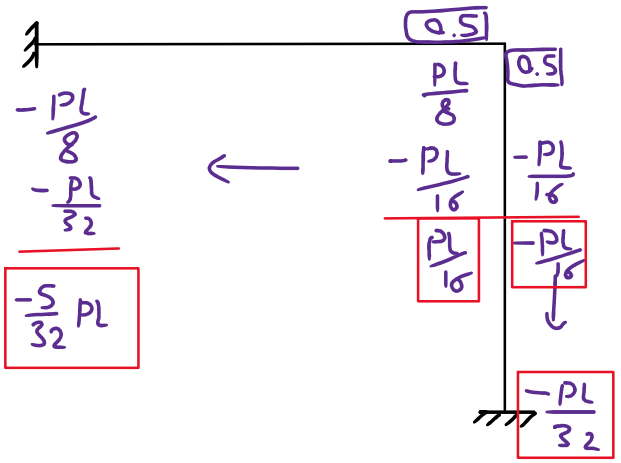



w
I, L
I, L
4EI/L
4EI/L


0.5
0.5
-wL²/12
wL²/12
-wL²/48
-wL²/24
-wL²/24
-wL²/48
-5/48 wL²
wL²/24
-wL²/24
-wL²/48


L/2
I, L


I, L


0.5
0.5
PL/8
-PL/8
-PL/32
-PL/16
-PL/16
PL/16
-PL/16
-5/32 PL
-PL/32

Video 14

Moment Distribution 2

Wednesday, March 27, 2024 16:11

توضیح روش توزیع لنگر

* در متن زیر منظور از گره‌ها، گره‌هایی است که درجه آزادی دورانی (θ) دارند.

1. *محاسبه لنگرهای گیرداری:* ابتدا با یک سری تکیه گاه مجازی جلوی دوران کلیه گره‌ها را می‌گیریم، سپس بار را وارد می‌کنیم. در این صورت، لنگرهای انتهایی هر عضو همان لنگرهای گیرداری FEM است.
2. *یک سیکل محاسبات:* با شروع از یکی از گره‌ها، عملیات برداشتن تکیه گاه مجازی (آزاد کردن گره یا به عبارت دیگر متوازن کردن لنگر گره) به صورت زیر انجام می‌شود:

   2-1. برداشتن تکیه گاه مجازی: لنگر نامتوازن هر گره (مجموع لنگرهای ایجاد شده در تکیه گاه مجازی) را محاسبه نموده و و یک لنگر متعادل کننده در خلاف جهت آن وارد می‌شود.

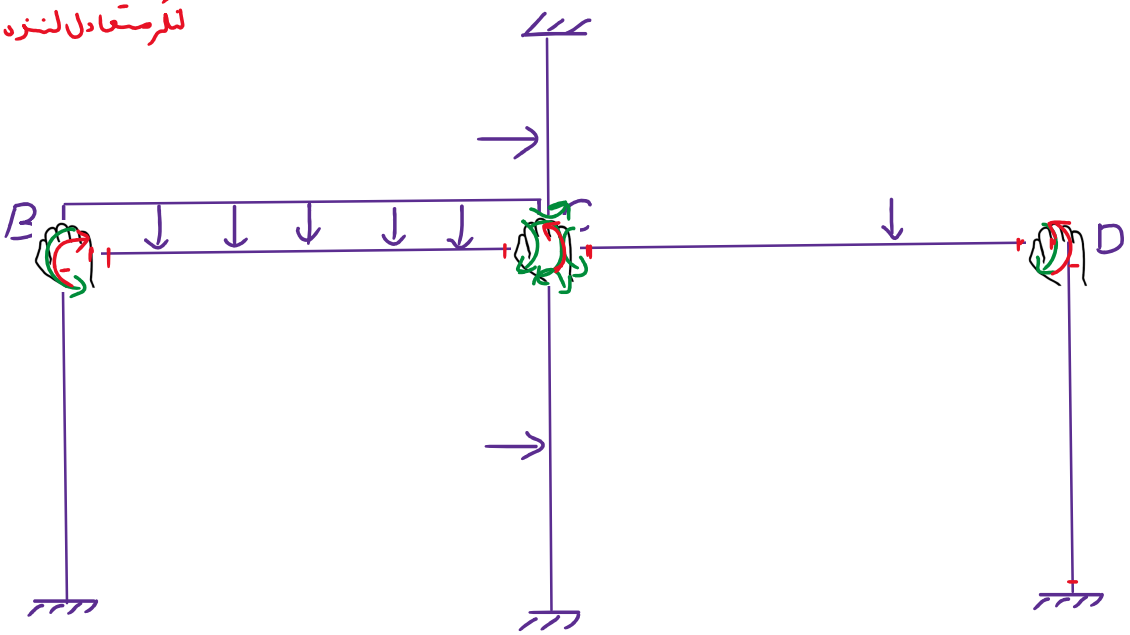
   2-2. توزیع لنگر: لنگر متعادل کننده با استفاده از ضریب توزیع بین اعضای متصل به گره توزیع می‌شود.

   2-3. انتقال لنگر: لنگر گام قبل به سر دیگر عضو انتقال داده می‌شود.

   پس از 3 مرحله فوق، دوباره گره بسته می‌شود. با انجام این عملیات برای تک تک گره‌های سازه، سیکل اول محاسبات خاتمه می‌یابد.

3. *تکرار سیکل محاسباتی:* پس از انجام هر سیکل، به دلیل لنگرهای انتقال یافته از سیکل قبل، باز هم لنگر نامتوازن در گره وجود دارد. بنابراین سیکل‌های محاسباتی تا جایی که لنگرهای نامتوازن قابل صرف نظر باشد، ادامه می‌یابد.
4. *محاسبه لنگرهای انتهایی اعضا:* لنگر انتهایی اعضا با جمع کردن لنگر گیرداری FEM، لنگر توزیع شده و لنگر انتقال یافته در سیکل‌های محاسباتی به دست می‌آید.

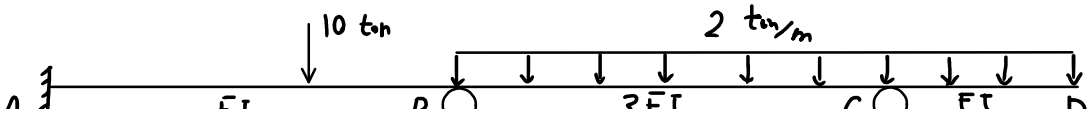
لنگر نامتوازن
لنگر متعادل کننده

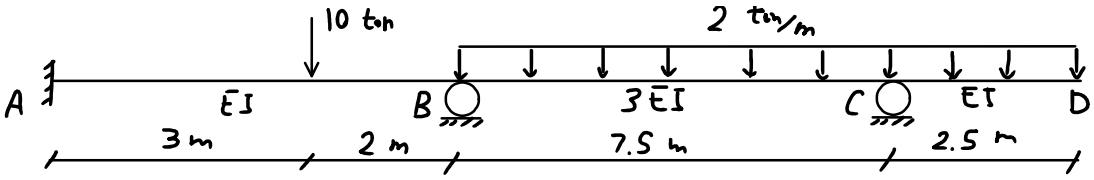


مثال: تیر سراسری شکل زیر را دوبار به روش توزیع لنگر تحلیل کنید.

(الف) سختی معمولی برای تیر BC ← دیگر از این روش استفاده نمی‌کنم

(ب) سختی اصلاح شده تیر BC





(الف) کنش معمولی

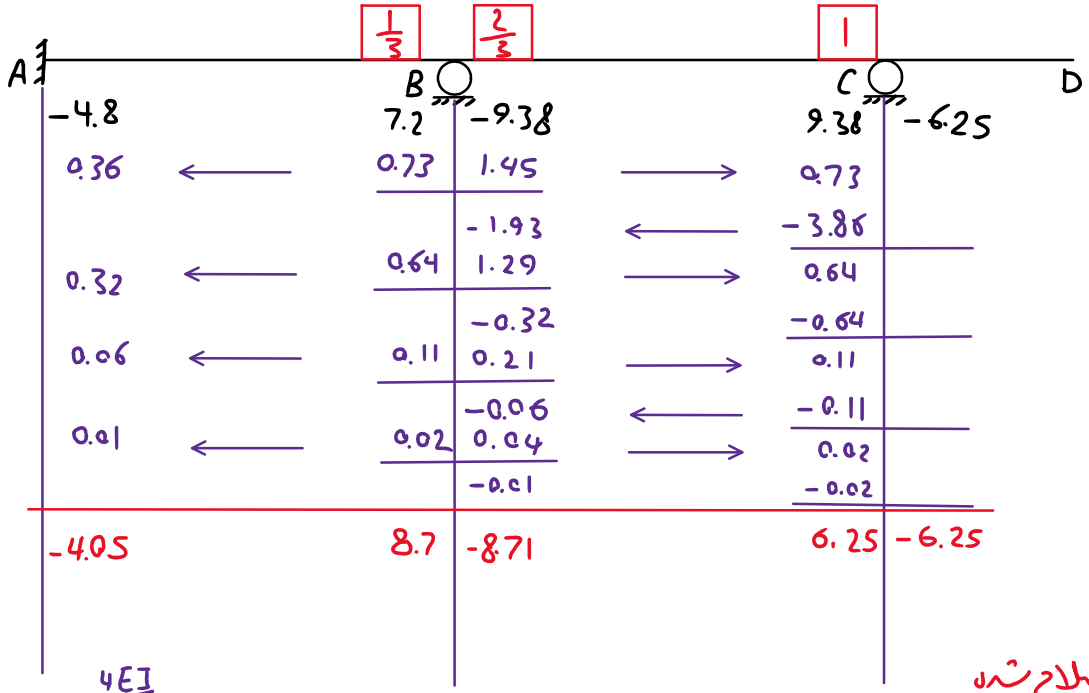
$$FEM_{AB} = -\frac{10 \times 3 \times 2^2}{5^2} = -4.8 \ t.m$$

$$FEM_{BA} = \frac{10 \times 2 \times 3^2}{5^2} = 7.2$$

$$FEM_{BC} = -FEM_{CB} = \frac{-2 \times 7.5^2}{12} = -9.38$$

$$FEM_{CD} = -\frac{2 \times 2.5^2}{2} = -6.25$$

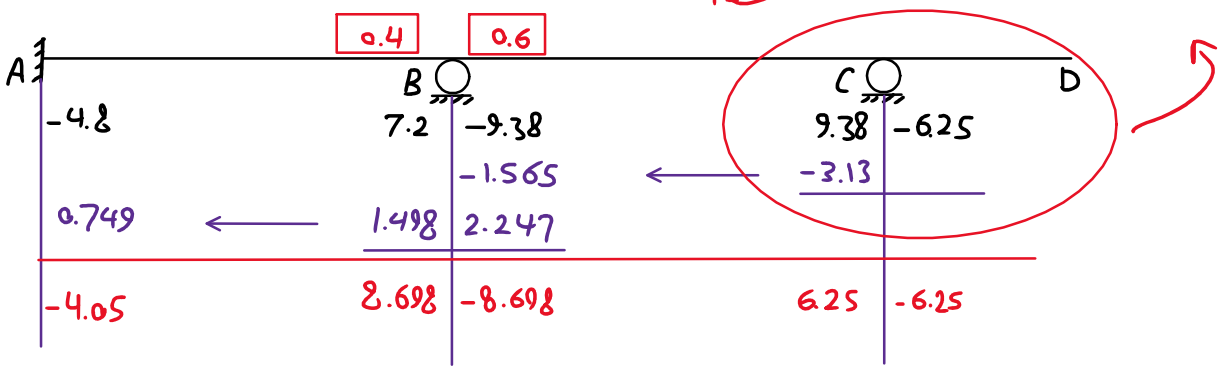
$$\begin{cases} DF_{BA} = \dfrac{\frac{4EI}{5}}{\frac{4EI}{5} + \frac{4(2EI)}{7.5}} = \frac{1}{3} \\ DF_{BC} = 1 - \frac{1}{3} = \frac{2}{3} \end{cases}$$



(ب) کنش اصلاح شده

$$\begin{cases} DF_{BA} = \dfrac{\frac{4EI}{5}}{\frac{4EI}{5} + \frac{3(3EI)}{7.5}} = 0.4 \\ DF_{BC} = 0.6 \end{cases}$$

بعد از انتقال لنگر، دیگر به منفصل کار نداریم.



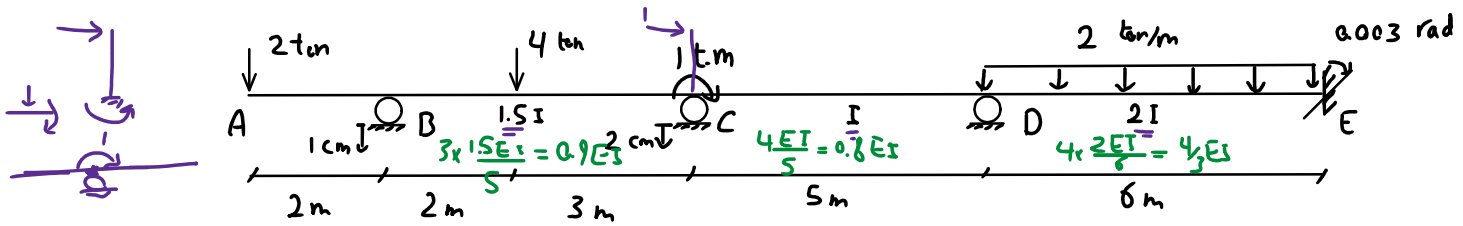
Video 15

Moment Distribution 3

Wednesday, March 27, 2024 16:11

مثال: تیر شکل زیر را تحت بارهای وارده و نشست تکیه گاه ها تحلیل کنید.

$E = 2\times10^6\ kg/cm^2$, $I = 3000\ cm^4$



$M_{BA} = 4$ t.m

$FEM_{BC} = -\frac{4\times2\times3^2}{5^2} = -2.88$

$FEM_{DE} = -FEM_{ED} = -\frac{2\times6^2}{12} = -6$

$FEM_{CB} = \frac{4\times3\times2^2}{5^2} = 1.92$

FEM ناشی از بارخارجی:

FEM ناشی از نشست:

$FEM_{BC} = FEM_{CB} = -\frac{6EI}{L^2}\Delta = -6\frac{1.5\times600}{5^2}\times0.01 = -2.16$ t.m

$FEM_{CD} = FEM_{DC} = 6\times\frac{600}{5^2}\times0.02 = 2.88$ t.m

$FEM_{DE} = \frac{2EI}{L}\theta = 2\times\frac{2\times600}{6}\times0.003 = 1.2$ t.m

$FEM_{ED} = \frac{4EI}{L}\theta = 2.4$ t.m

$EJ = 2\times10^6\times3000 = 6\times10^9\ kg.cm^2 = 600\ t.m^2$

$M_{AB} = M_{BA} = \frac{2EI}{L}\left(-\frac{3\Delta}{L}\right) = -\frac{6EI}{L^2}\Delta$

$\frac{2EI}{L}\theta$ $\quad$ $\frac{4EI}{L}\theta$

FEM کل ناشی از بار و نشست:

$FEM_{BA} = 4$	$FEM_{CD} = FEM_{DC} = 2.88$
$FEM_{BC} = -2.88 - 2.16 = -5.04$	$FEM_{DE} = -6 + 1.2 = -4.8$
$FEM_{CB} = 1.92 - 2.16 = -0.24$	$FEM_{ED} = 6 + 2.4 = 8.4$

ضریب توزیع:

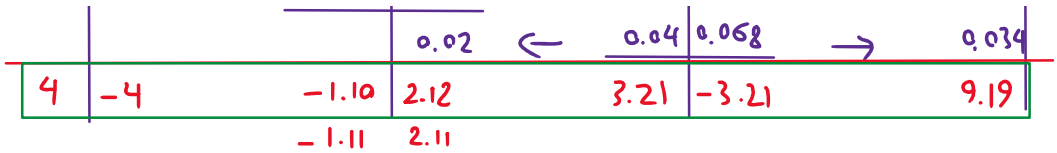
$DF_{CB} = \frac{0.9}{0.9+0.8} = 0.529$

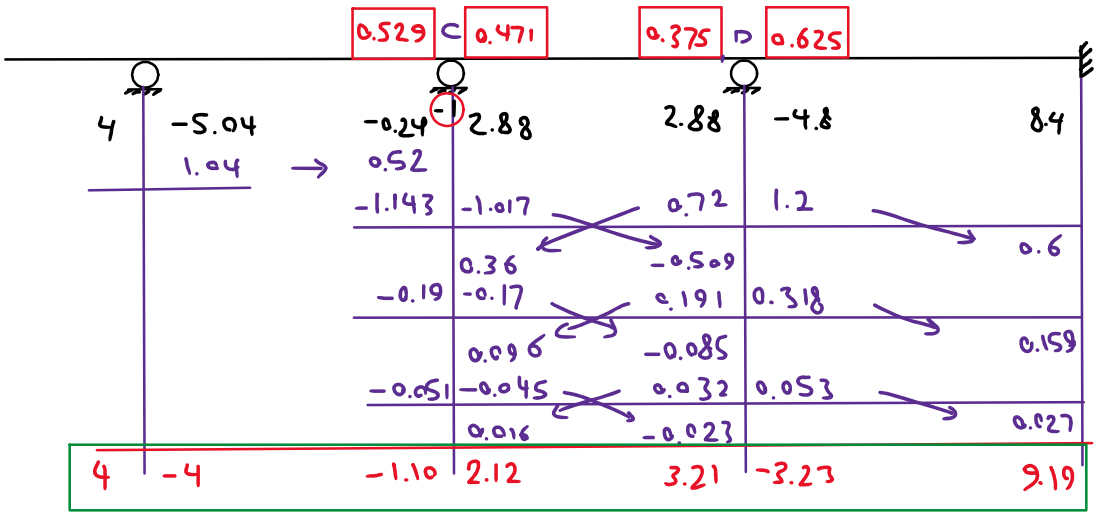
$DF_{CD} = 1 - DF_{CB} = 0.471$

$DF_{DC} = \frac{0.8}{0.8+\frac{4}{3}} = 0.375$

$DF_{DE} = 0.625$

B	0.529 C	0.471	0.375 D	0.625	E
4	-5.04	-0.24	2.88	-4.8	8.4
	1.04 →	0.52			
		-1.143	-1.017 →	-0.509	
			0.456 ←	0.911 1.518 →	0.759
		-0.241	-0.215 →	-0.108	
			0.02 ←	0.04 0.068 →	0.034
4	-4	-1.10	2.12	3.21	-3.21	9.19


0.02
0.04
0.068
0.034
4
-4
-1.10
2.12
3.21
-3.21
9.19
-1.11
2.11

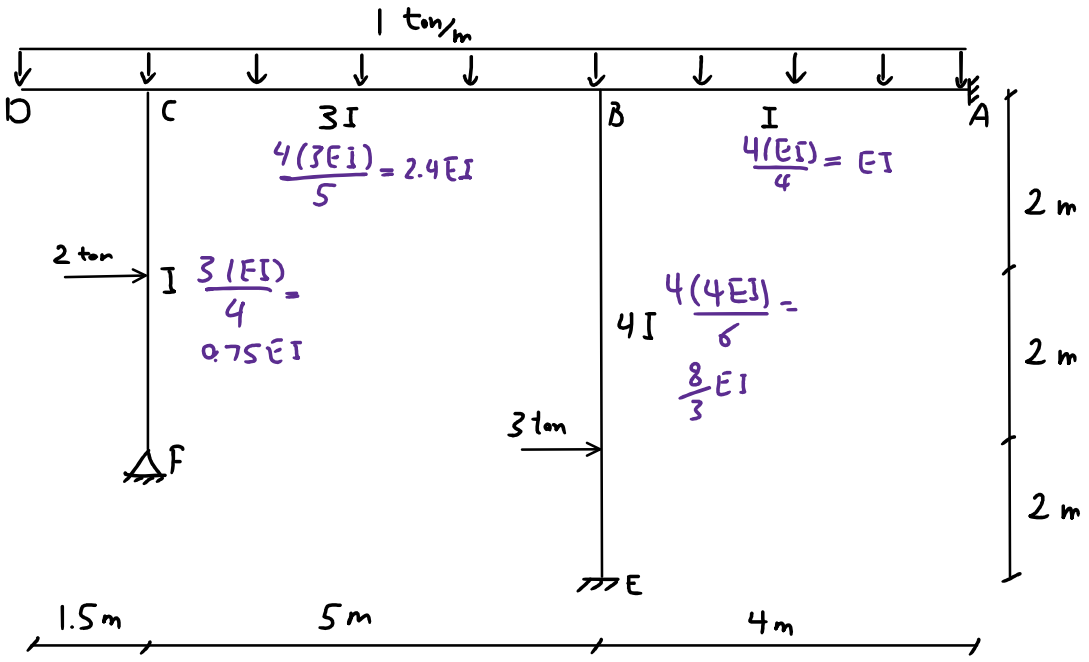

0.529
0.471
0.375
0.625
4
-5.04
-0.24
-1
2.88
2.88
-4.8
8.4
1.04
0.52
-1.143
-1.017
0.72
1.2
0.6
0.36
-0.509
-0.19
-0.17
0.191
0.318
0.159
0.096
-0.085
-0.051
-0.045
0.032
0.053
0.027
0.016
-0.023
4
-4
-1.10
2.12
3.21
-3.23
9.19

Video 16

Moment Distribution 4

Wednesday, March 27, 2024 16:11

مثال: قاب شکل زیر را تحلیل کنید.



درجه آزادی دوران ندارد

θ_B , θ_C

ضریب توزیع:

$DF_{CB} = \frac{2.4}{2.4+0.75} = 0.762$

$DF_{CF} = 0.238$

$DF_{BC} = \frac{2.4}{2.4+1+8/3} = 0.396$

$DF_{BE} = \frac{8/3}{2.4+1+8/3} = 0.44$

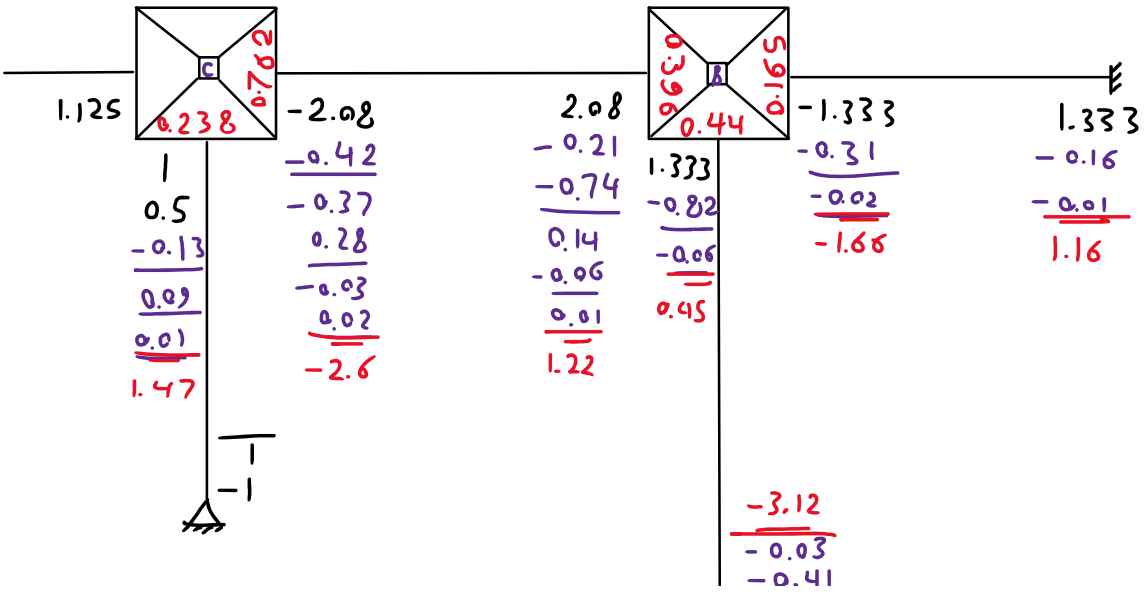
$DF_{BA} = 0.165$

FEM:

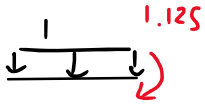
$\frac{1\times1.5^2}{2} = 1.125$ $\quad \frac{2\times4}{8} = 1$

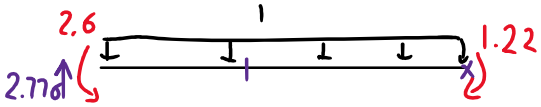
$\frac{1\times5^2}{12} = 2.08$ $\quad \frac{3\times2\times4^2}{6^2} = 2.667$

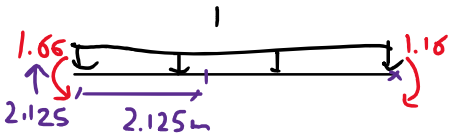
$\frac{1\times4^2}{12} = 1.333$ $\quad \frac{3\times4\times2^2}{6^2} = 1.333$

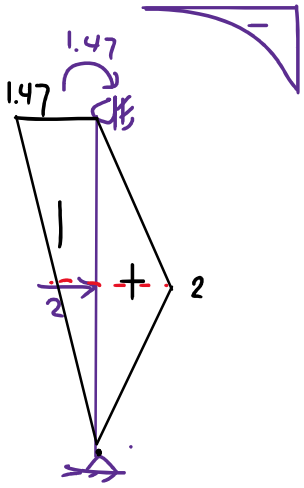


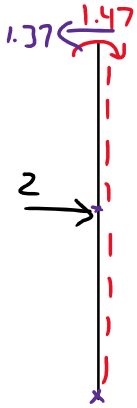
-3.12
-0.03
-0.41
-2.667

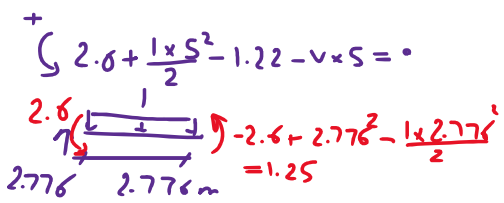

1
1.125

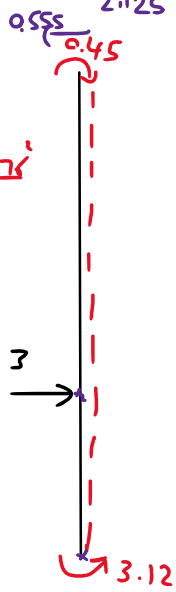

2.6
1
1.22
2.778

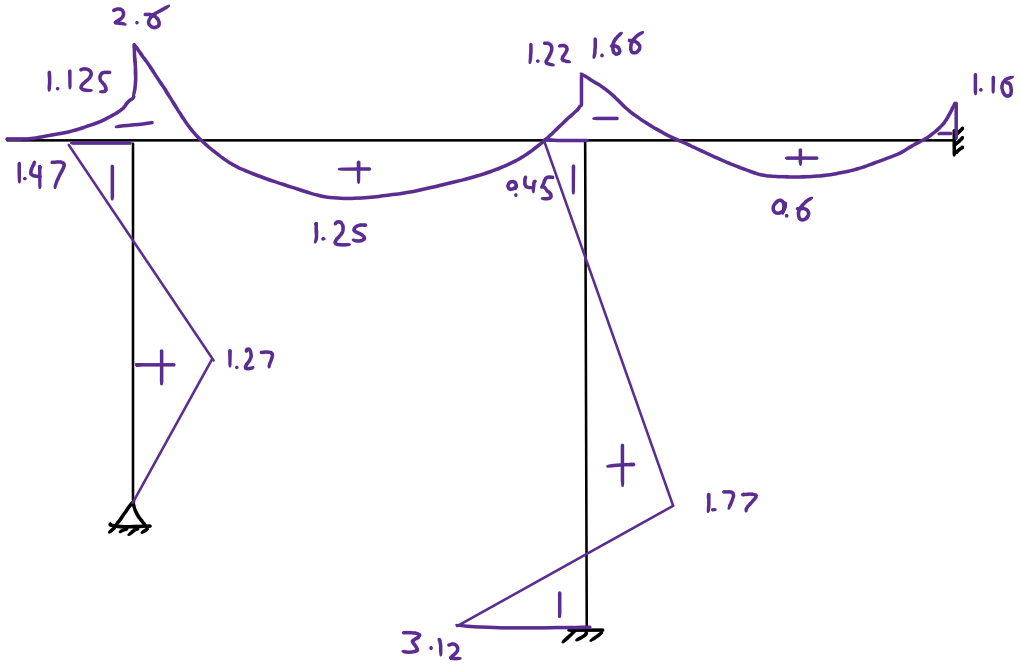

1.66
1
1.16
2.125
2.125m


1.47
1.47
1
2
2


1.37
1.47
2


+
2.6+1x5²/2-1.22-Vx5=0
2.6
1
-2.6+2.778²-1x2.778²/2
=1.25
2.778
2.778m


0.555
0.45
3
3.12


2.6
1.125
1.22
1.66
1.16
1.47
1
0.45
1
1.25
0.6
1.27
1.77
1
3.12

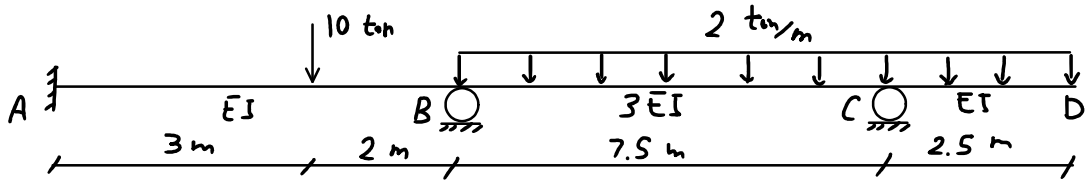
Video 17

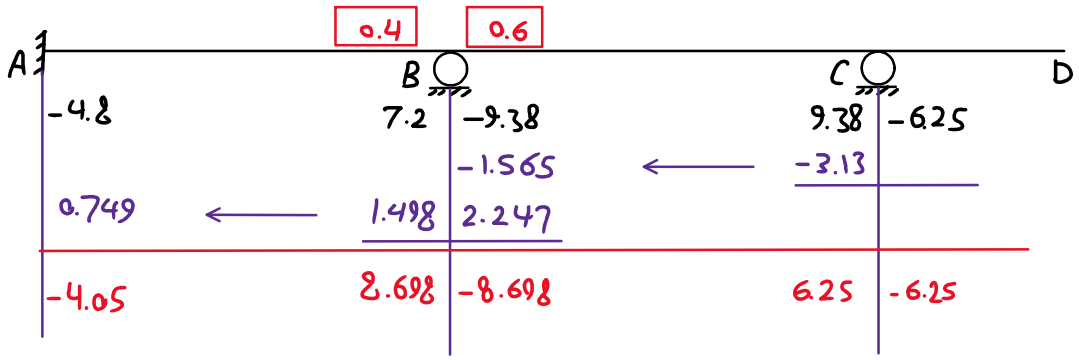
Moment Distribution 5

Wednesday, March 27, 2024 16:11

محاسبه دوران θ

مثال: دوران θ_B و θ_C را بیدا کنید.



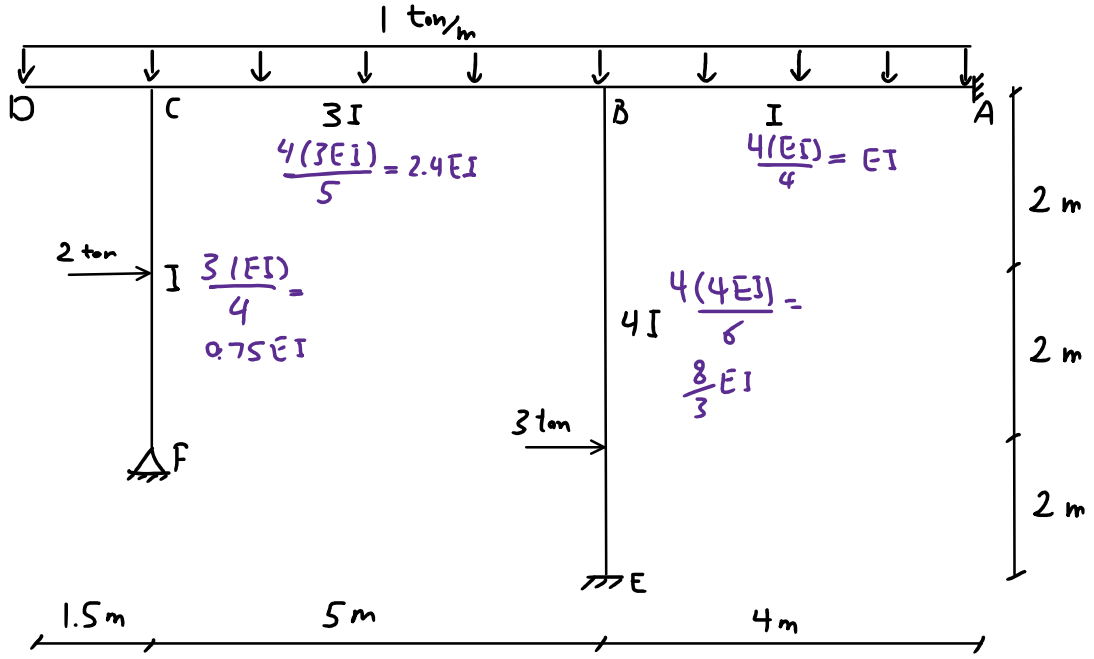


$$M_{BA} = \frac{2EI}{L}(2\theta_B) + FEM_{BA}$$

$$8.698 = \frac{4EI}{5}\theta_B + 7.2 \rightarrow \theta_B = \frac{1.87}{EI}$$

$$6.25 = M_{CB} = \frac{4(3EI)}{7.5}\theta_C + \frac{2(3EI)}{7.5}\left(\frac{1.87}{EI}\right) + 9.38 \rightarrow \theta_C = -\frac{2.89}{EI}$$

مثال: دوران θ_B و θ_C را بیدا کنید.



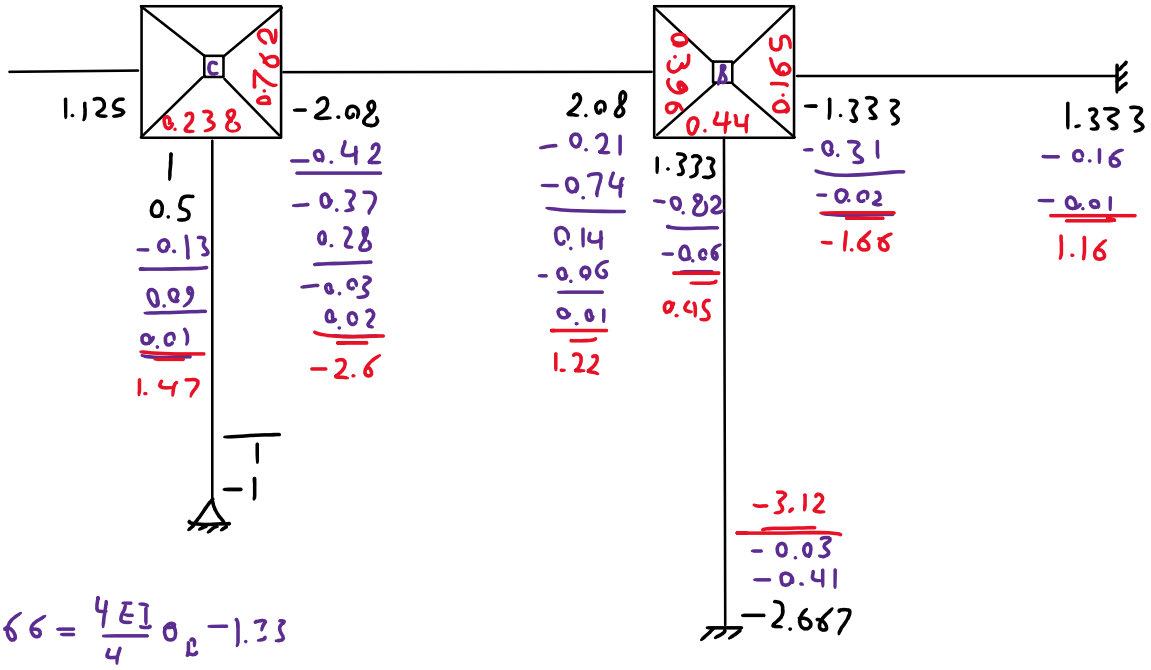
$$0.45 = M_{BE} = \frac{4(4EI)}{6}\theta_B + 1.333 \rightarrow \theta_B = -\frac{0.33}{EI}$$

$$M_{CE} = \frac{3EI}{\quad}\theta_C + (FEM_{\quad} - \tfrac{1}{2}FEM_{\quad}) \text{ is} \rightarrow \theta_C = -\frac{0.04}{\quad}$$

$$M_{CF} = \frac{3EI}{4}\theta_C + FEM_{CF} - \frac{1}{2}FEM_{FC} \quad (1.47,\ 1.5) \rightarrow \theta_C = -\frac{0.04}{EI}$$

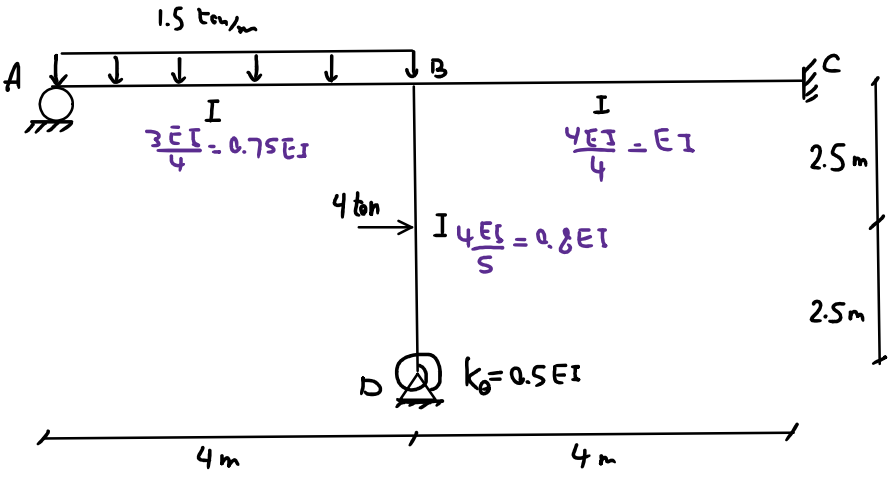
$$M_{FC} = \frac{4EI}{4}\theta_F + \frac{2EI}{4}\theta_C + FEM_{FC}$$

$$0 = EI\theta_F + \frac{1}{2}EI\left(\frac{-0.04}{EI}\right) - 1 \rightarrow \theta_F = \frac{1.02}{EI}$$



$$-1.66 = \frac{4EI}{4}\theta_B - 1.33$$

مثال: تاب شکل زیر را به روش توزیع لنگر تحلیل کنید.



DF

$\frac{0.75}{2.55} = 0.294$

$\frac{0.8}{2.55} = 0.314$

$\frac{1}{2.55} = 0.392$

$\frac{0.8}{1.3} = 0.615$

$\frac{0.5}{1.3} = 0.385$

درجه آزادی ندارد

θ_B ، θ_D

(۲) محاسبه دوران گره پس از محاسبه لنگرهای انتقالی

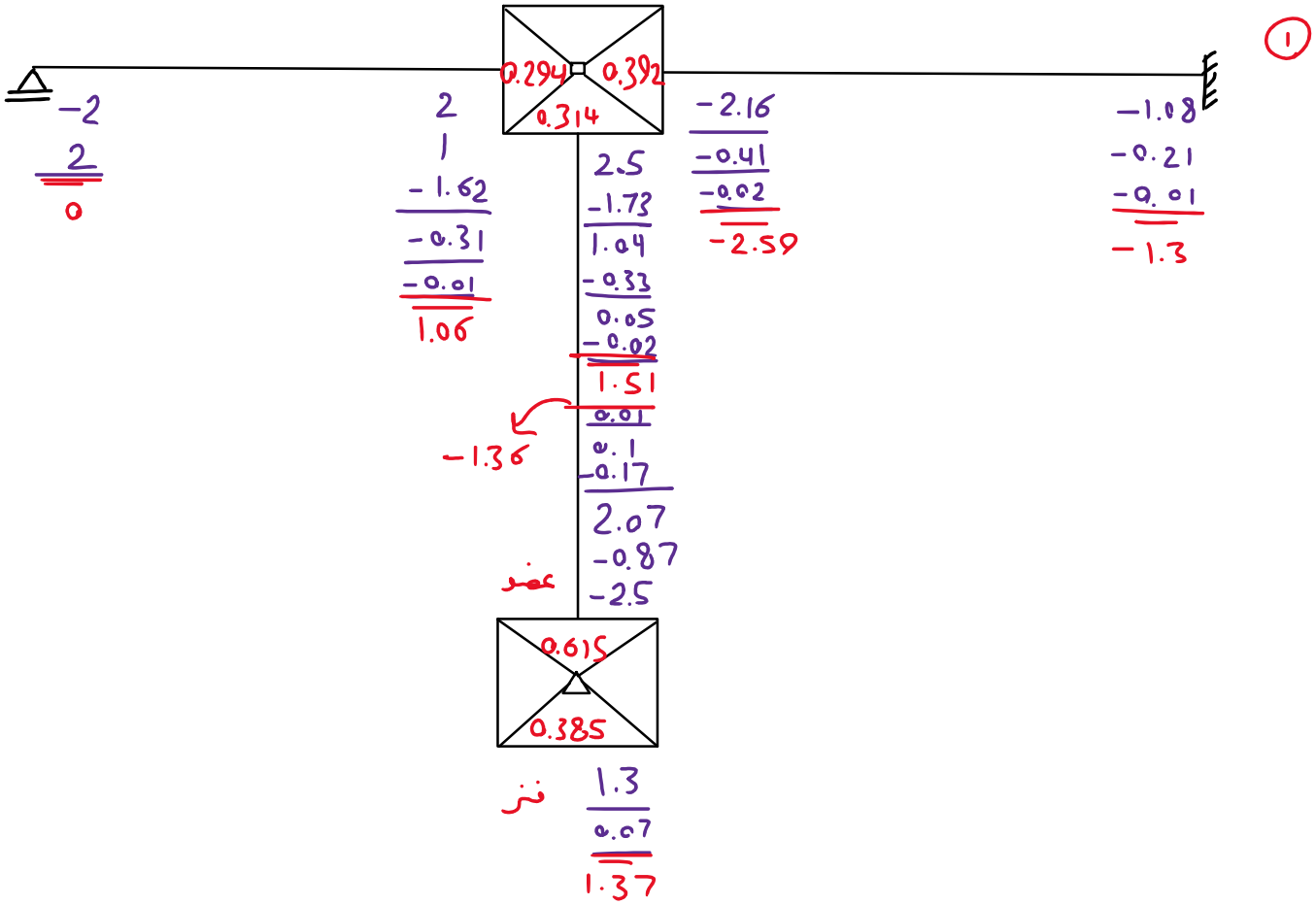
$$\theta_D = \frac{M_S}{k} = \frac{1.37}{0.5EI} = \frac{2.74}{EI}$$

محاسبه دوران گره بی از محاسبه لنگرهای انتهایی

$$\theta_D = \frac{M_S}{K_\theta} = \frac{1.37}{0.5EI} = \frac{2.74}{EI}$$

$$M_{BC} = \frac{4EI}{4}\theta_D \rightarrow \theta_B = \frac{-2.59}{EI}$$

$$M_{BA} = \frac{3EI}{4}\theta_B + 3 \rightarrow \theta_D = \frac{-2.59}{EI}$$

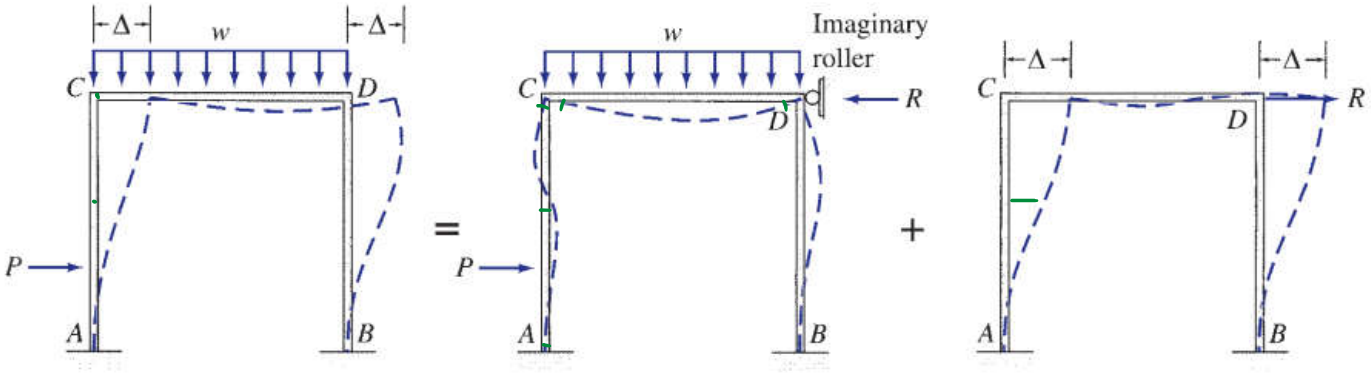


Video 18

Moment Distribution 6

Wednesday, March 27, 2024 16:11

تحلیل سازه دارای درجه آزادی Δ به روش توزیع لنگر



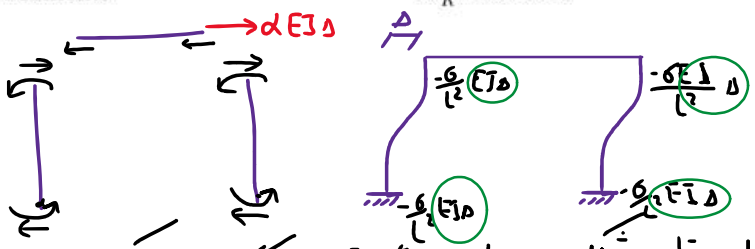
(a) Actual Frame — M Moments

(b) Frame with Sidesway Prevented — M_O Moments

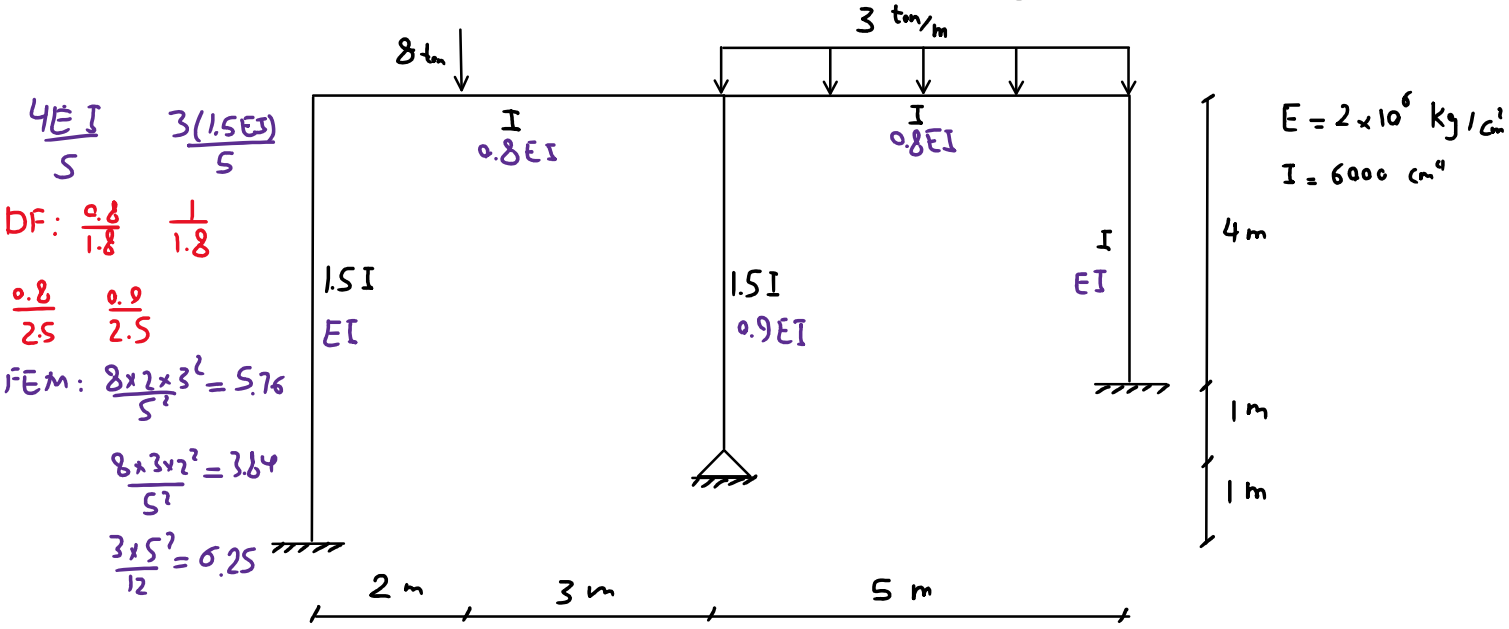
(c) Frame Subjected to R — M_R Moments

$\alpha EI\Delta = R \rightarrow EI\Delta = \frac{R}{\alpha}$

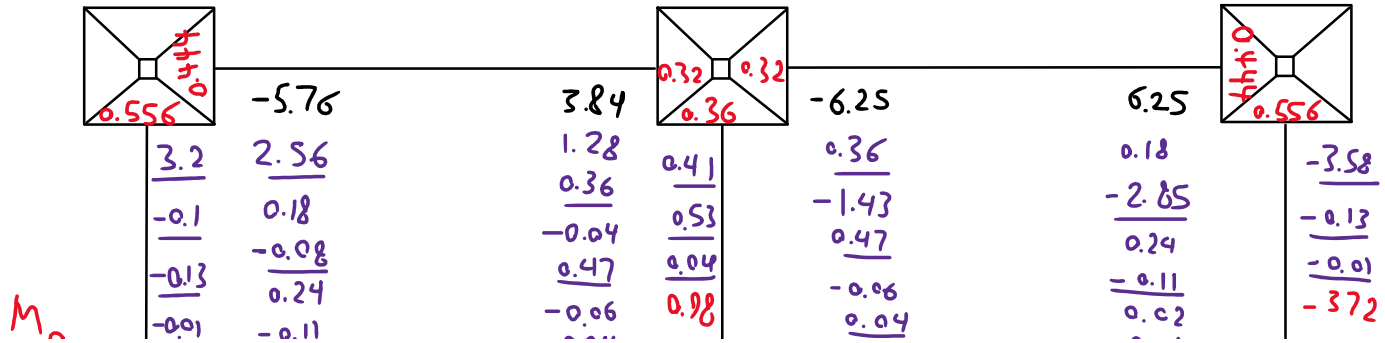
$M = M_o + M_R$

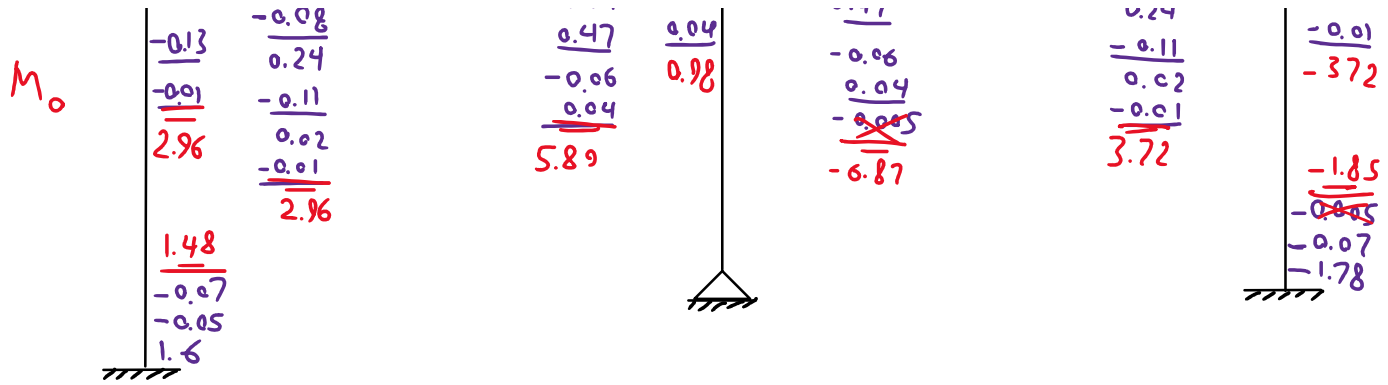


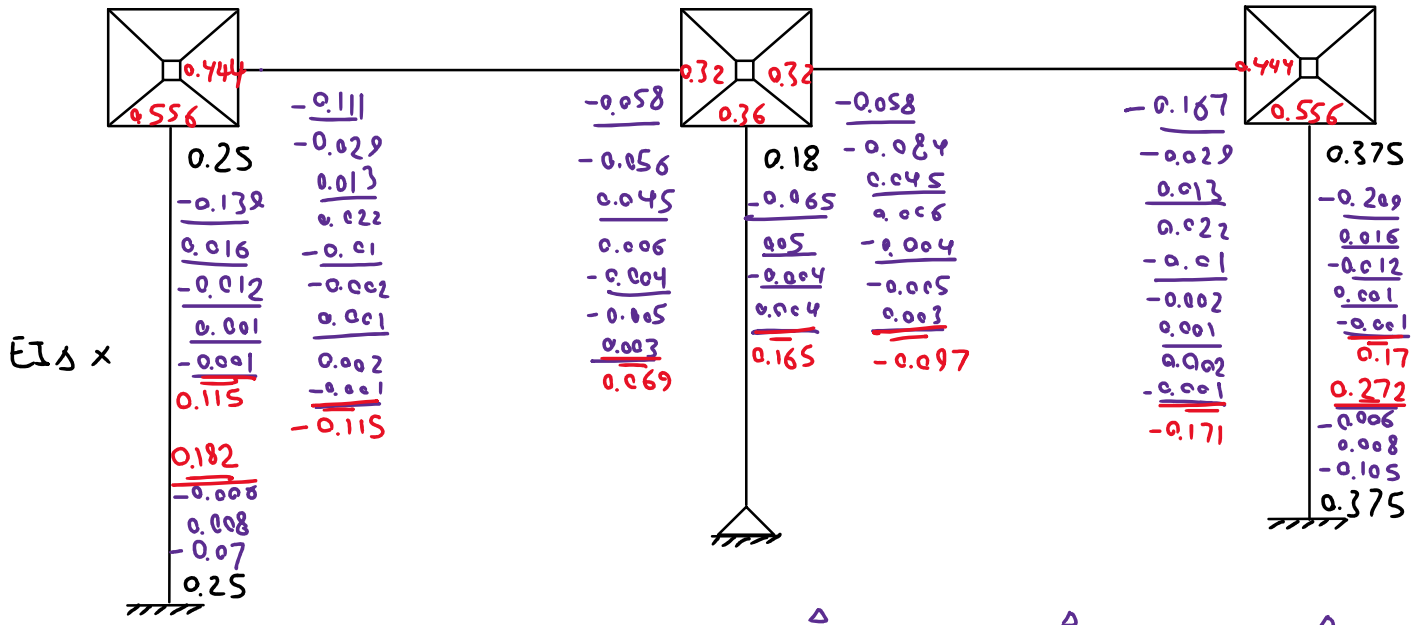
مثال: قاب شکل زیر را به روش توزیع لنگر تحلیل کنید.



تحلیل دراثر بارخارجی بدون Δ :



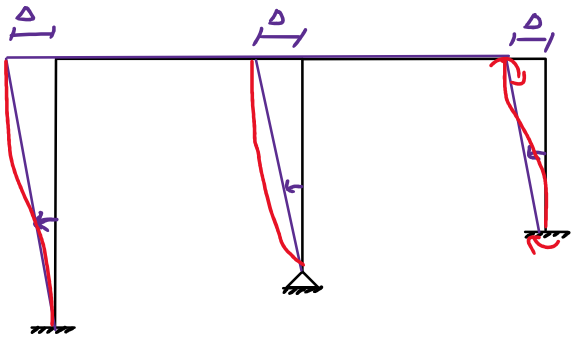


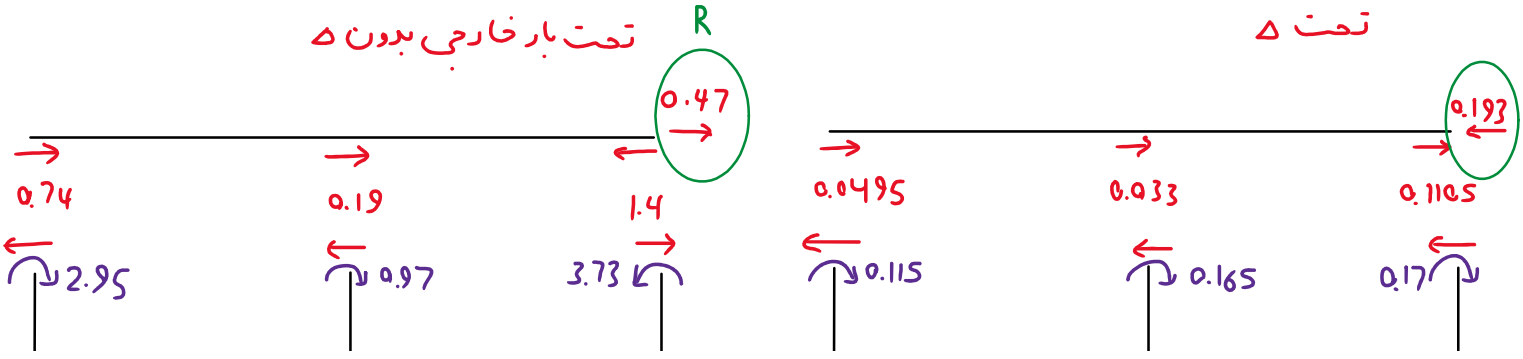


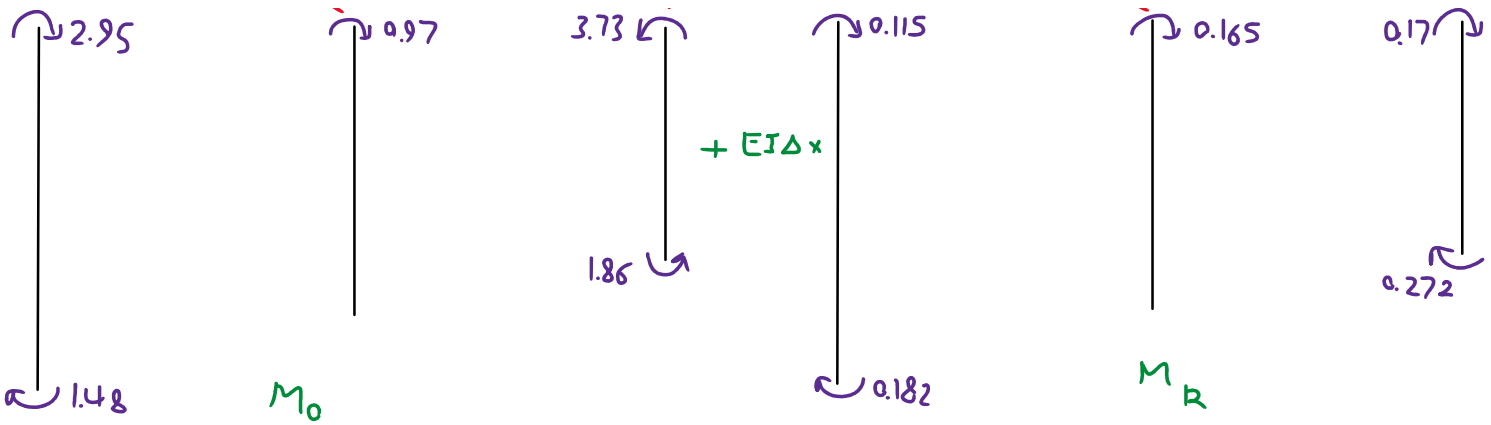
$$6\frac{(1.5EI)}{6^2}\Delta = 0.25EI\Delta$$

$$3\frac{(1.5EI)}{5^2}\Delta = 0.18EI\Delta$$

$$6\frac{(EI)}{4^2}\Delta = 0.375EI\Delta$$



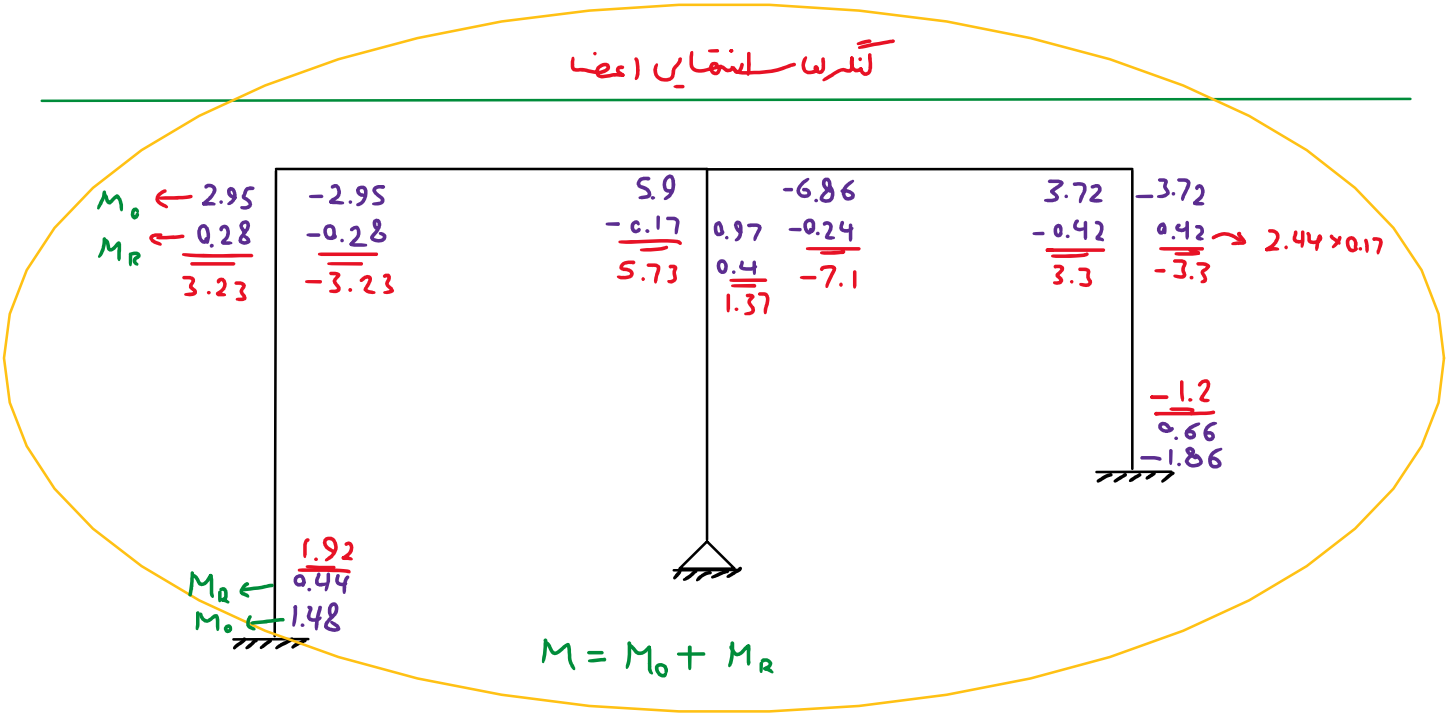




$0.193\ EI\Delta = 0.47 \longrightarrow \boxed{EI\Delta = 2.44}$

$EI = 2\times10^6\ kg/cm^2 \times 6000\ cm^4 = 12\times10^9\ kg.cm^2 = 1200\ t.m^2 \rightarrow \Delta = \frac{2.44}{1200} = 0.002\ m = 2\ mm$

لنگرهای انتهایی اعضا



$M = M_0 + M_R$

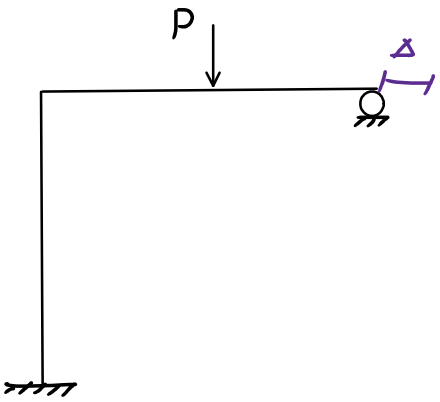
Video 19

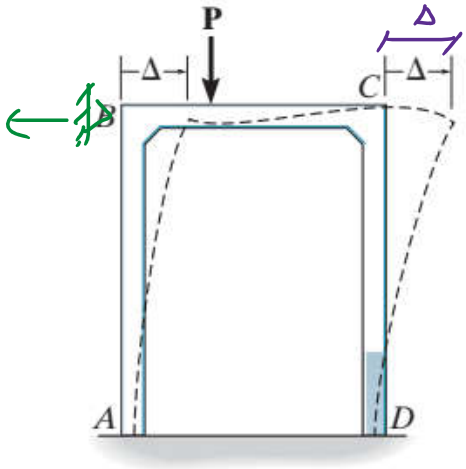
Moment Distribution 7

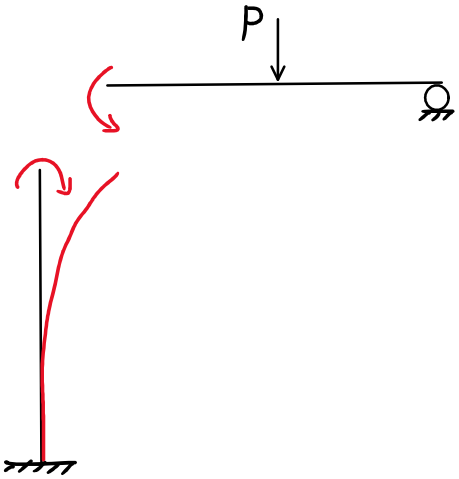
Wednesday, March 27, 2024 16:11

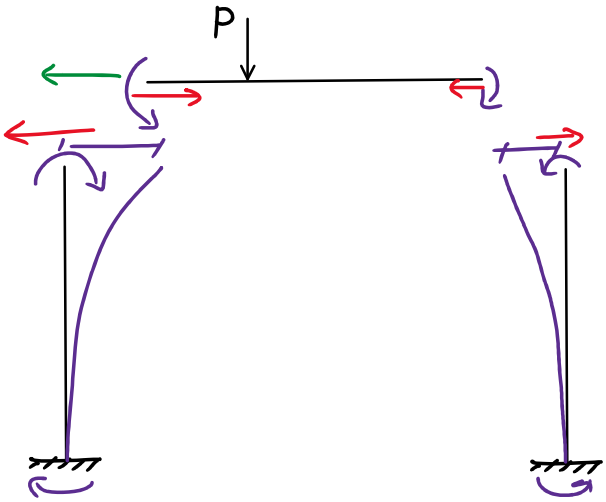
مباحث پیرامون درجه آزادی Δ

تشخیص جهت تقریبی Δ در قاب‌ها :





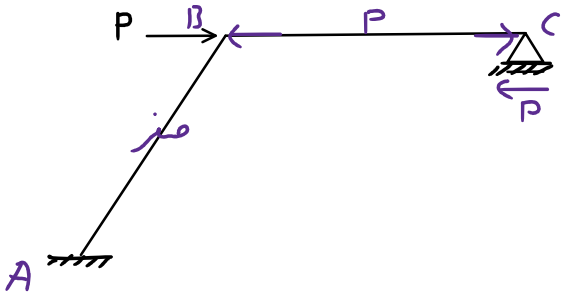




نکته مهم : اگر در سازه‌ای درجه آزادی Δ وجود نداشته باشد و یا بارگذاری به گونه‌ای باشد که $\Delta = 0$ باشد ؛

درصورتی که بارهای متمرکز روی گره‌ها وارد شود (در طول عضو بار وارد نشود)

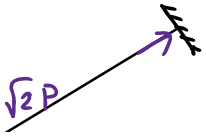
لنگر و برش در کلیه اعضا صفر است $M = 0$ و $V = 0$ و فقط نیروی محوری داریم. که رفتار خرپایی



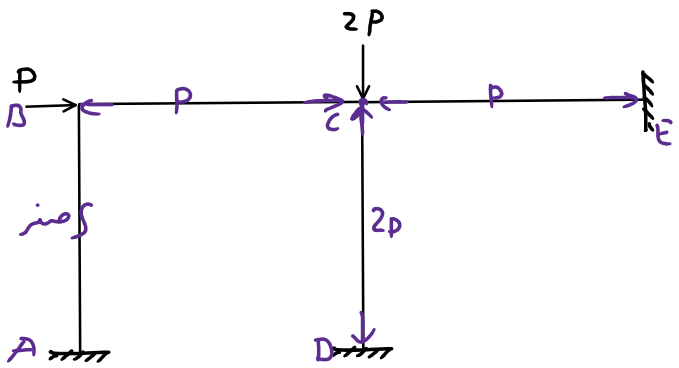
$$M_{BA} + M_{BC} = 0$$

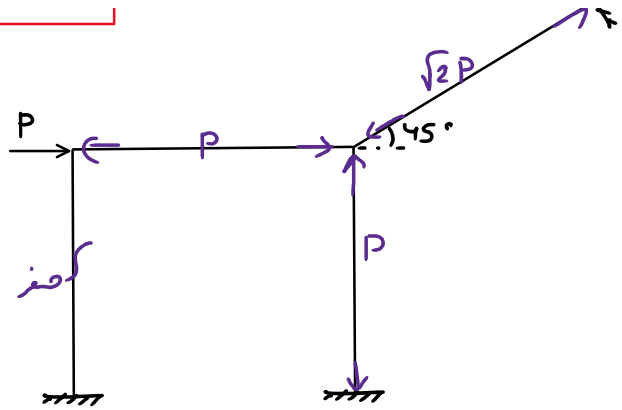
$$\frac{3EI}{L}(\theta_B) + \frac{2EI}{L}(2\theta_B) = 0 \rightarrow \boxed{\theta_B = 0}$$

$$\boxed{M = 0 \, , \, V = 0}$$



2 P





در سازه‌های برتر درجه آزادی دوران نداریم

$FEM = 0 \rightarrow \theta = 0 \rightarrow M = 0 \rightarrow V = 0$

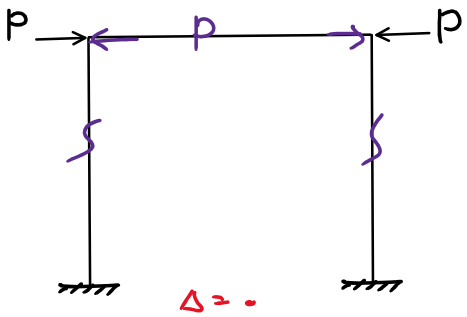
① بیان توزیع لنگر

② بیان شیب-افت

$$\sum M_B = 0 \rightarrow \frac{2EI}{L}(2\theta_B) + \frac{2EI}{L}(2\theta_B + \theta_C) = 0$$

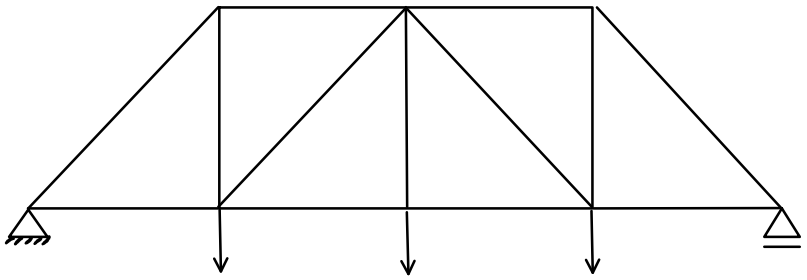
$$\sum M_C = 0 \rightarrow \frac{2EI}{L}(2\theta_C + \theta_B) + \frac{2EI}{L}(2\theta_C) + \frac{2EI}{L}(2\theta_C) = 0$$

$$\begin{cases} \alpha_1\theta_B + \beta_1\theta_C = 0 \\ \alpha_2\theta_B + \beta_2\theta_C = 0 \end{cases} \rightarrow \boxed{\begin{matrix}\theta_B = 0 \\ \theta_C = 0\end{matrix}} \rightarrow M_{ij} = 0 \rightarrow V = 0$$



$\Delta = 0$

$M = V = 0$

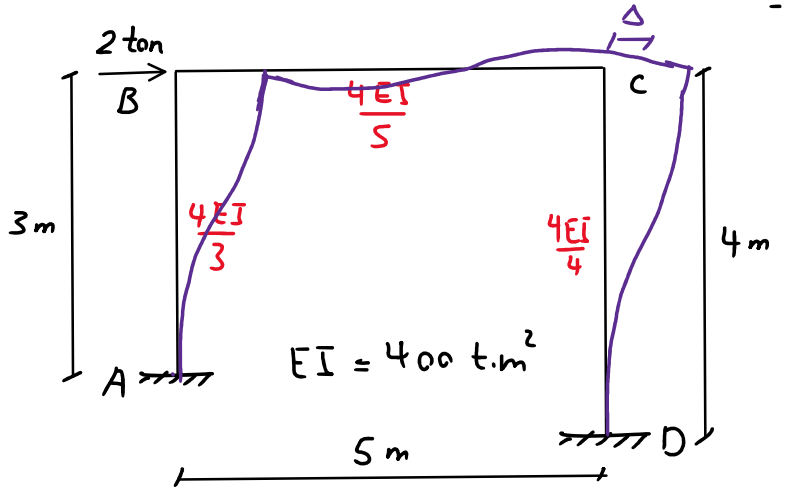


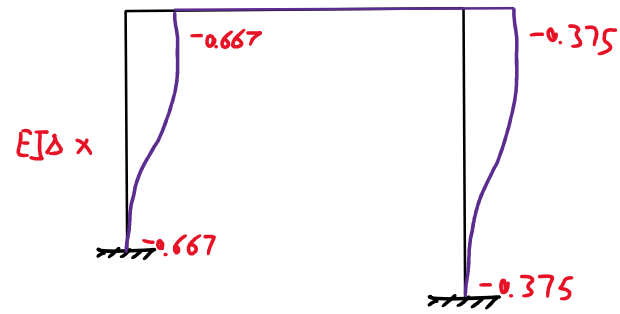
$M = 0 \leftarrow \theta = 0 \leftarrow FEM = 0 \leftarrow$ بارها در گره‌ها

درجه آزادی دوران نداریم ← بارها در گره‌ها

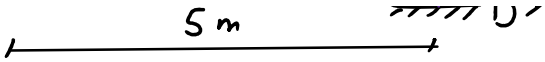
منظور تیرهای محوری

مثال: نیروهای داخلی و Δ و θ را به دست آورید.



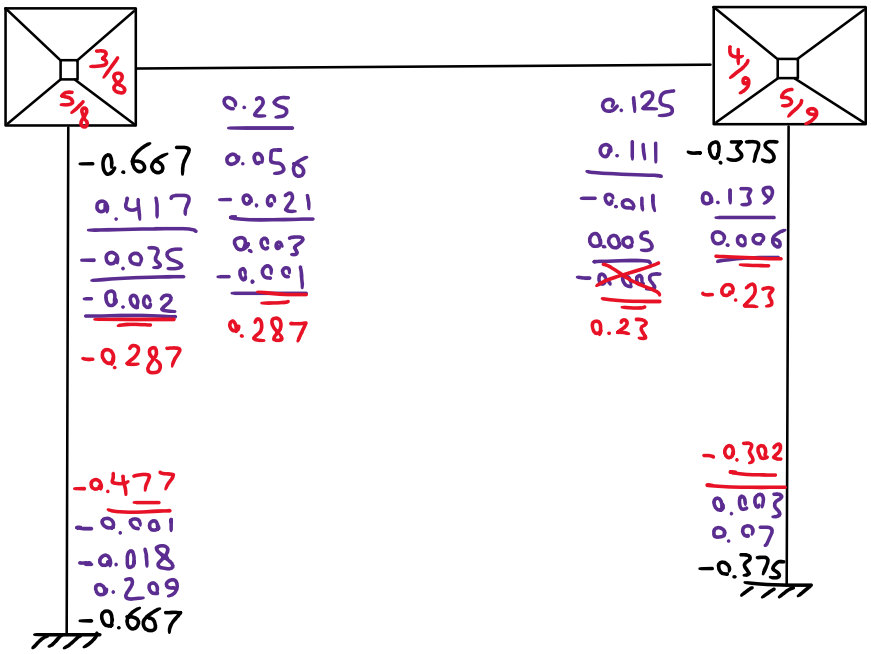


$FEM: -\frac{6EI}{3^2}\Delta = -0.667\,EI\Delta$

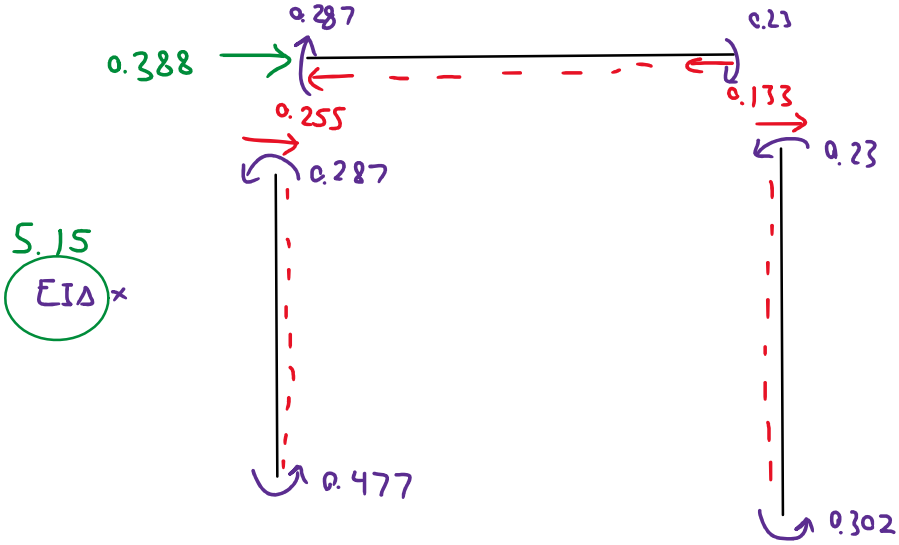


FEM : $-\frac{6EI}{3^2}\Delta = -0.667\,EI\Delta$

$-\frac{6EI}{4^2}\Delta = -0.375\,EI\Delta$



$EI\Delta \times$



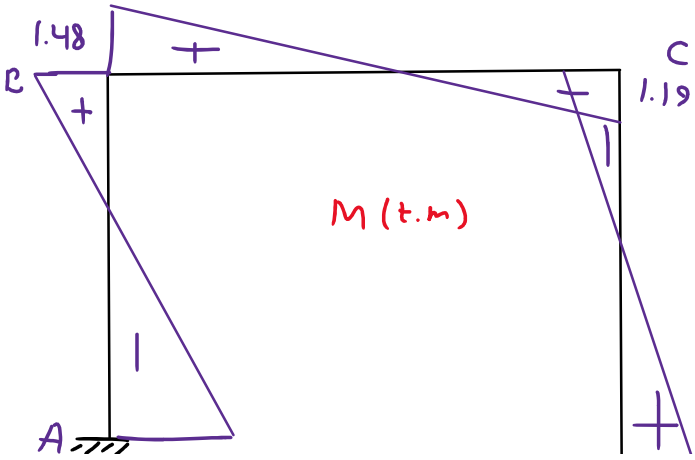
$0.388\,EI\Delta = 2$

$EI\Delta = 5.15$

$\Delta = 0.013\,m = 1.3\,cm$

5.15

$EI\Delta \times$

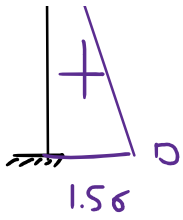


$M_{BA} = \frac{2EI}{L}(2\theta_B - \frac{3\Delta}{L})$

$-1.48 = \frac{4EI}{3}\theta_B - \frac{6}{3^2} \times 5.15 \rightarrow EI\theta_B = 1.47$

$\theta_B = 0.0037\,rad$

$M_{CD} = \frac{2EI}{L}(2\theta_C - \frac{3\Delta}{L})$



$$M_{CD} = \frac{2EI}{L}\left(2\theta_C - 3\frac{\Delta}{L}\right)$$

$$-1.19 = \frac{4}{4}EI\theta_C - \frac{6}{4^2} \times 5.15 \rightarrow EI\theta_C = 0.74$$

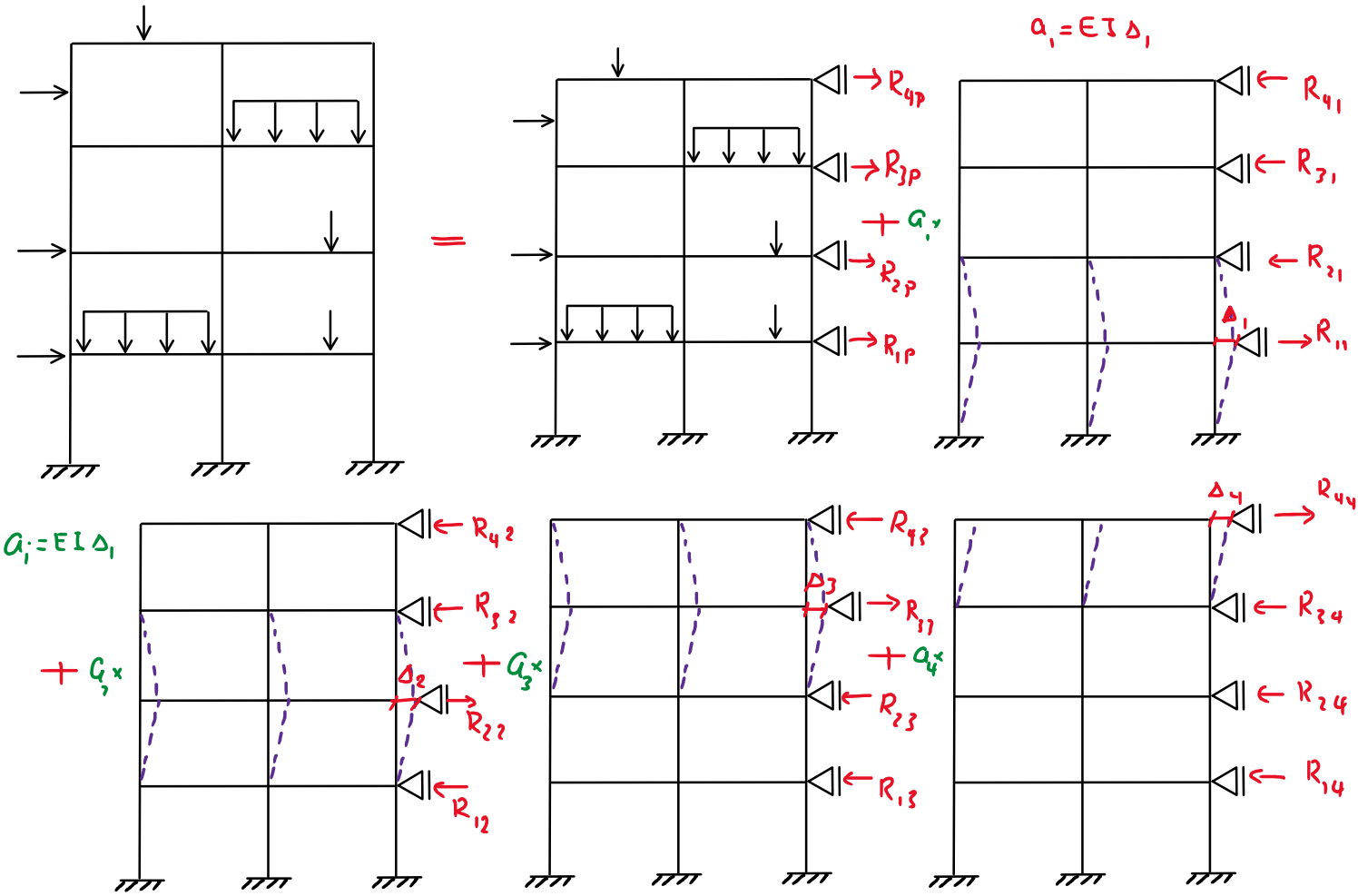
$$\theta_C = 0.0018 \text{ rad}$$

Video 20

Moment Distribution 8

Monday, April 29, 2024 22:53

تحلیل قاب با چند درجه آزادی Δ به روش توزیع لنگر

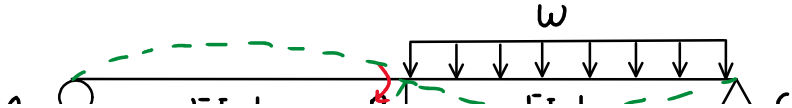


$$\begin{cases} R_{1P} + R_{11}a_1 + R_{12}a_2 + R_{13}a_3 + R_{14}a_4 = 0 \\ R_{2P} + R_{21}a_1 + R_{22}a_2 + R_{23}a_3 + R_{24}a_4 = 0 \\ R_{3P} + R_{31}a_1 + R_{32}a_2 + R_{33}a_3 + R_{34}a_4 = 0 \\ R_{4P} + R_{41}a_1 + R_{42}a_2 + R_{43}a_3 + R_{44}a_4 = 0 \end{cases} \rightarrow \begin{cases} a_1 \\ a_2 \\ a_3 \\ a_4 \end{cases}$$

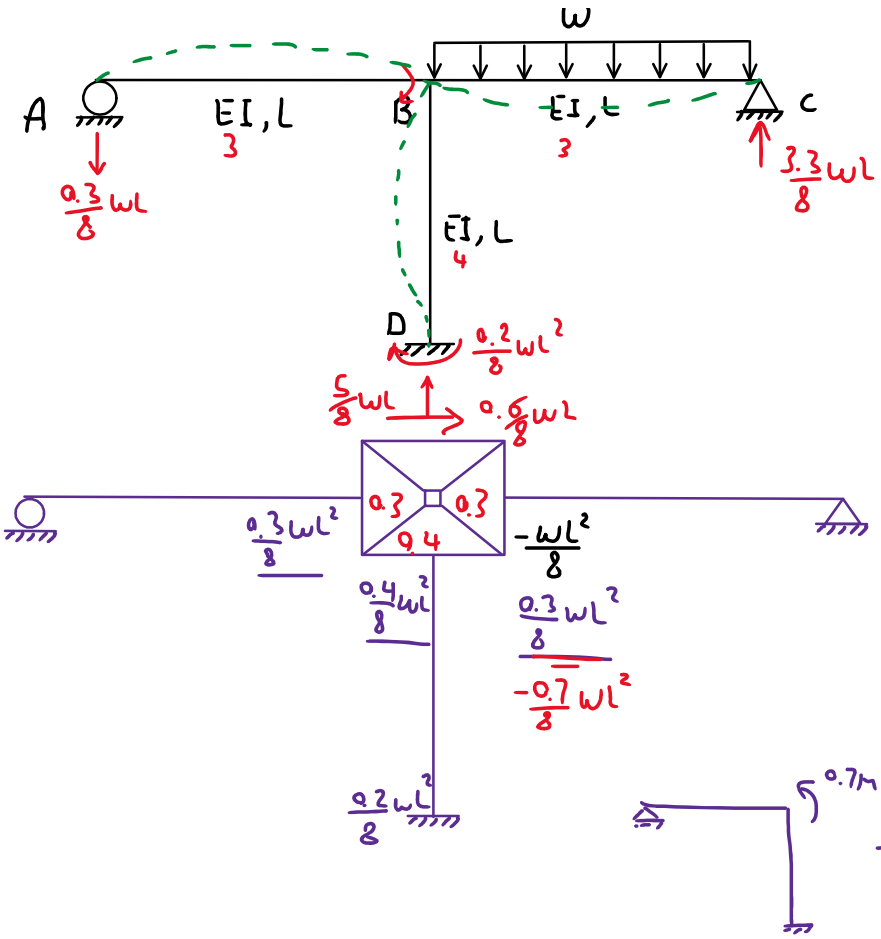
مجهولات Δ به دست آمده است.

$$M = M_P + a_1 M_1 + a_2 M_2 + a_3 M_3 + a_4 M_4$$

مثال: عکس العمل های تکیه گاهی را به دست آورید.



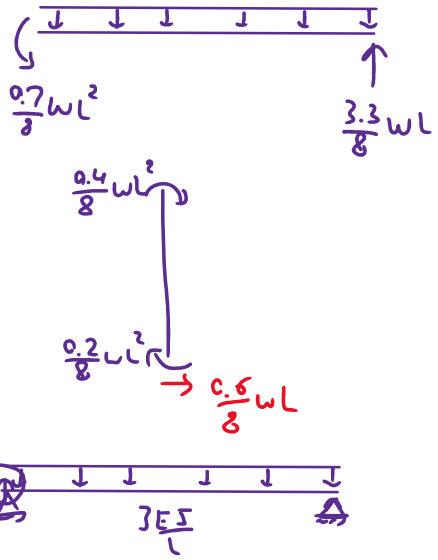
درجه آزادی Δ ندارد

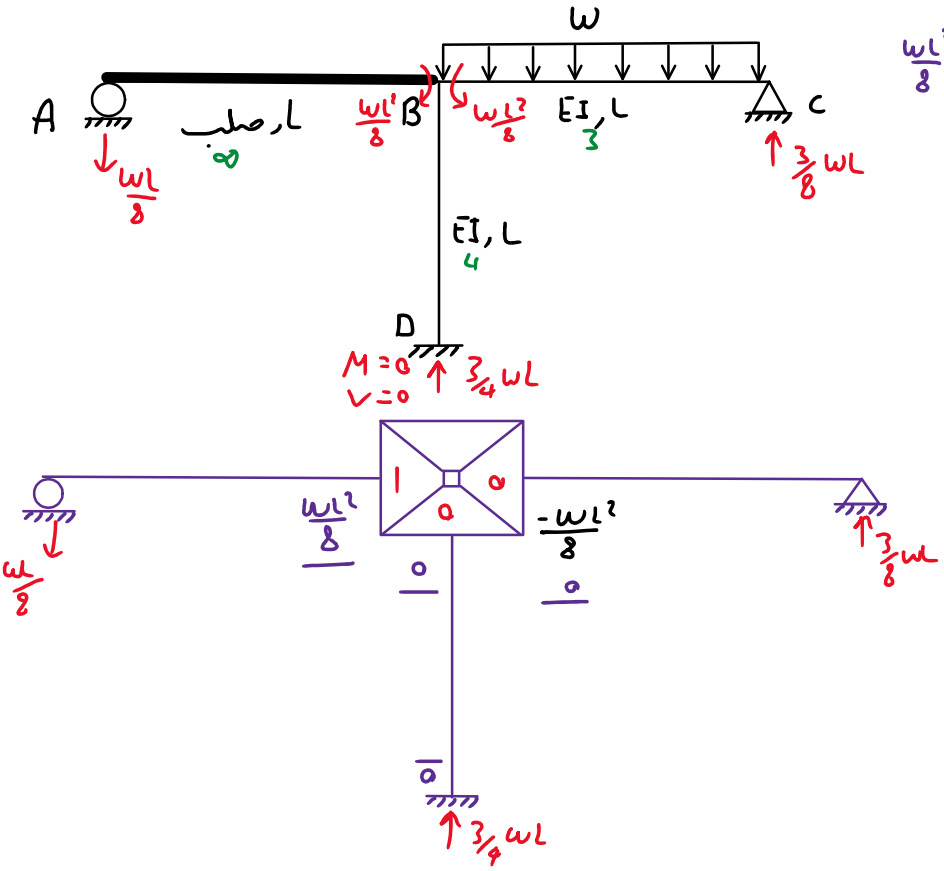


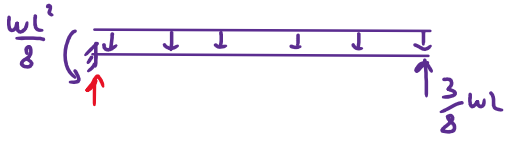
درجه آزادی ۵ دارد

۱۵

$FEM_{BC} = \frac{3}{2}\frac{wL^2}{12}$

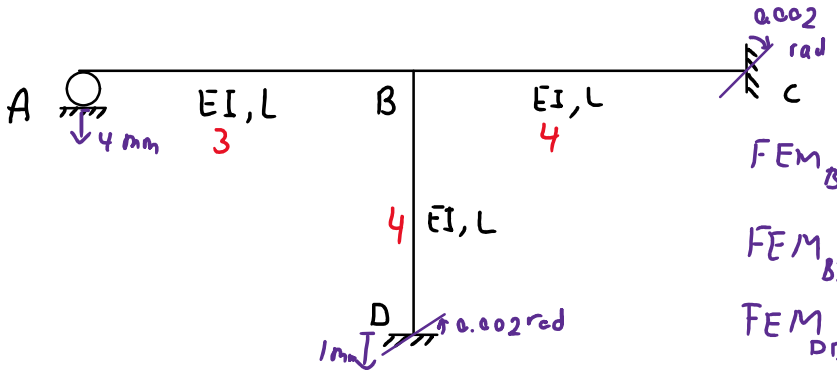






مثال : لنگرهای انتهایی اعضا را به دست آورید.

$EI = 1000\ t\cdot m^2$, $L = 3\ m$



Δ ندارد و یک θ دارد.

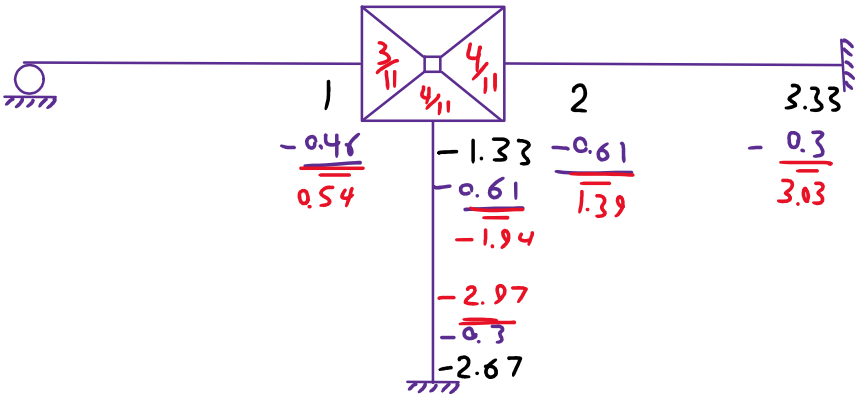
$$FEM_{BA} = \frac{3EI}{L}\left(-\frac{\Delta}{L}\right) = \frac{3\times1000}{3}\left(\frac{0.003}{3}\right) = 1\ tm$$

$$FEM_{BD} = \frac{2EI}{L}(\theta_D) = \frac{2\times1000}{3}(-0.002) = -1.33$$

$$FEM_{DB} = \frac{2EI}{L}(2\theta_D) = -2.67$$

$$FEM_{BC} = \frac{2EI}{L}\left(\theta_C - \frac{3\Delta}{L}\right) = \frac{2\times1000}{3}\left(0.002 + 3\times\frac{0.001}{3}\right) = 2$$

$$FEM_{CB} = \frac{2EI}{L}\left(2\theta_C - 3\frac{\Delta}{L}\right) = \frac{2\times1000}{3}\left(2\times0.002 + 3\times\frac{0.001}{3}\right) = 3.33$$

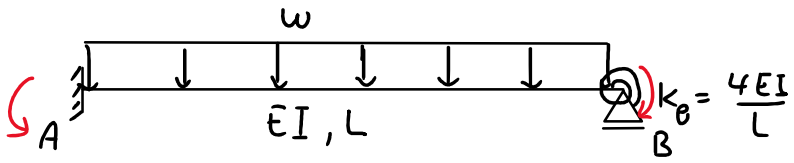


Video 21

Moment Distribution 9

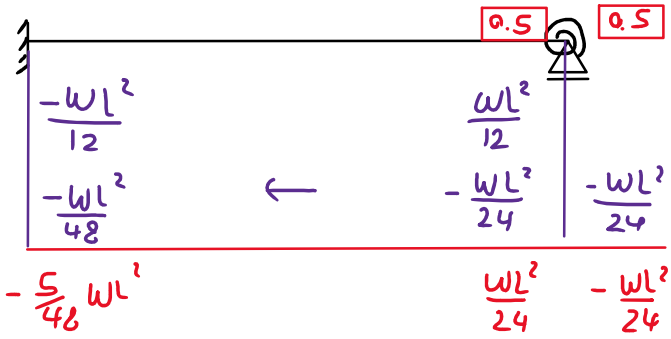
Monday, April 29, 2024 22:54

مثال: لنگرهای دو سر تیر را به دست آورید.



Δ ندارد

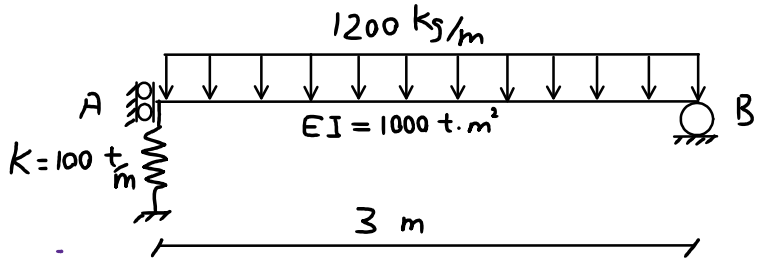
$1\ \theta$



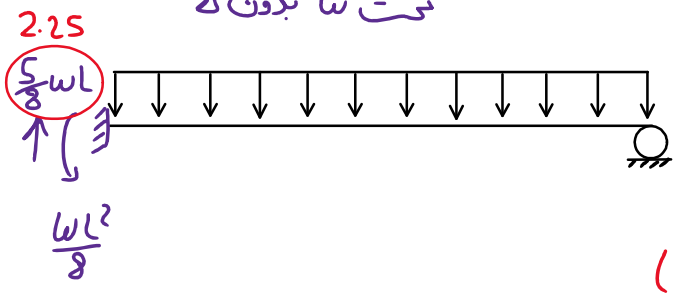
مثال : نیروی فنر را به دست آورید.

$1\ \Delta$

θ نداریم ← اصلاح شده



Δ بدون w تیر



+

Δ تیر

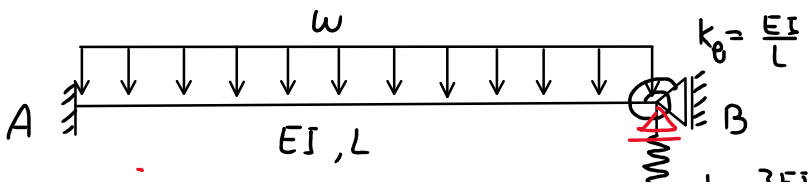
$211.1\Delta = 2.25 \rightarrow \Delta = 0.0107\ m$

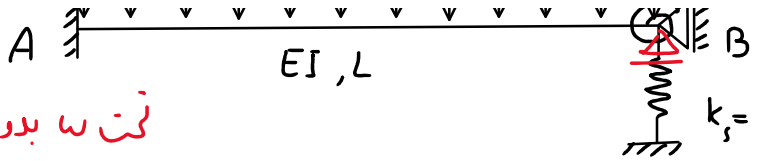
$F = k\Delta = 1.07\ ton$

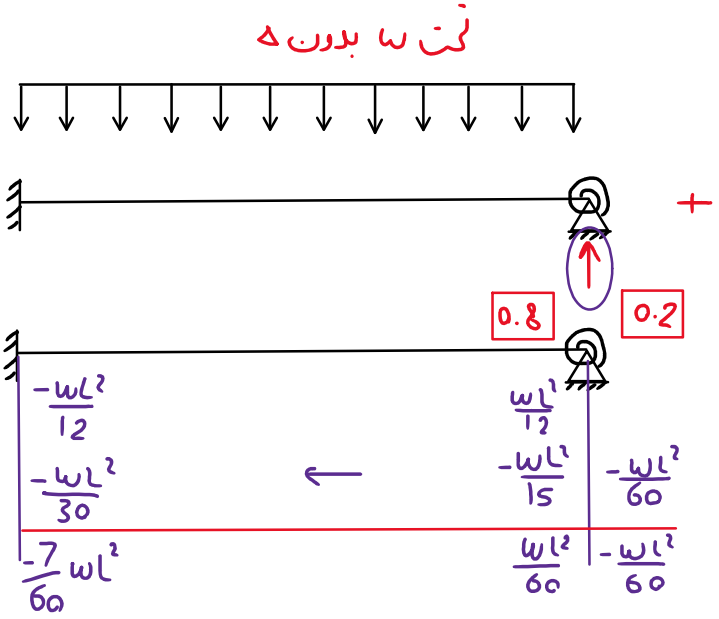
مثال : نیروهای فنرها را به دست آورید.

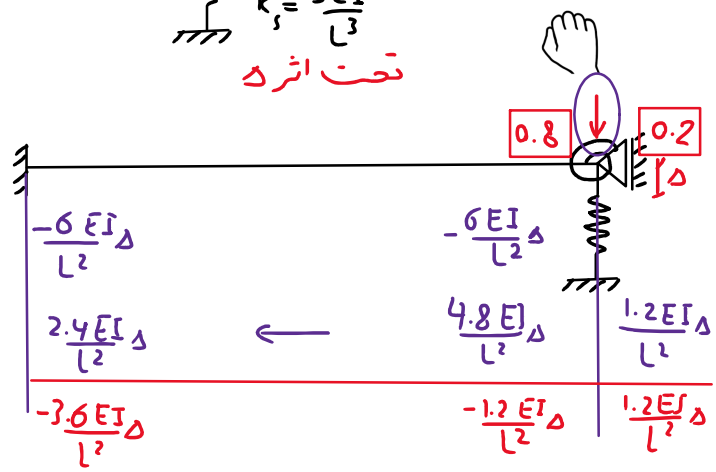
$1\ \Delta$

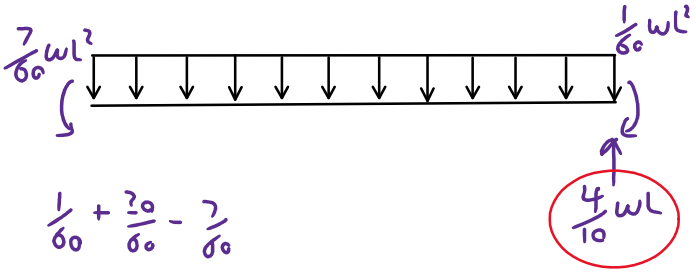
$1\ \theta$

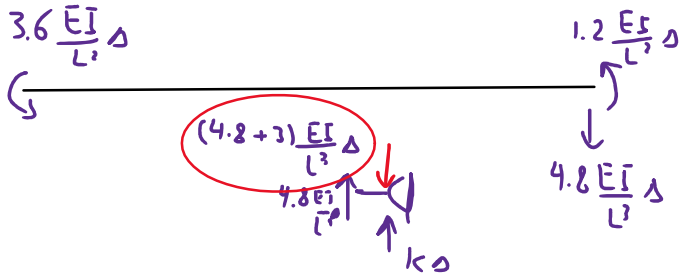










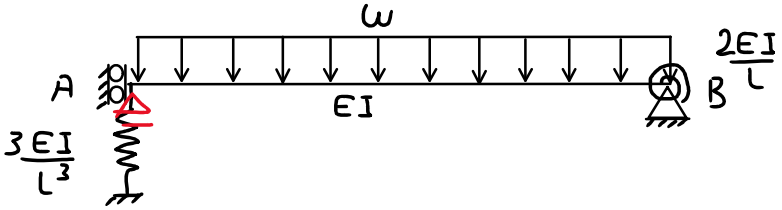


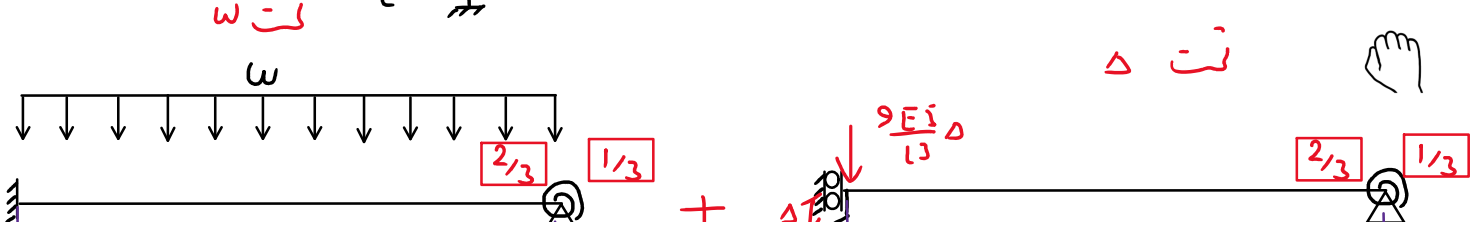
$$\frac{4}{10}wL = 7.8\,\frac{EI}{L^2}\Delta \rightarrow \boxed{EI\Delta = \frac{2}{39}wL^4}$$

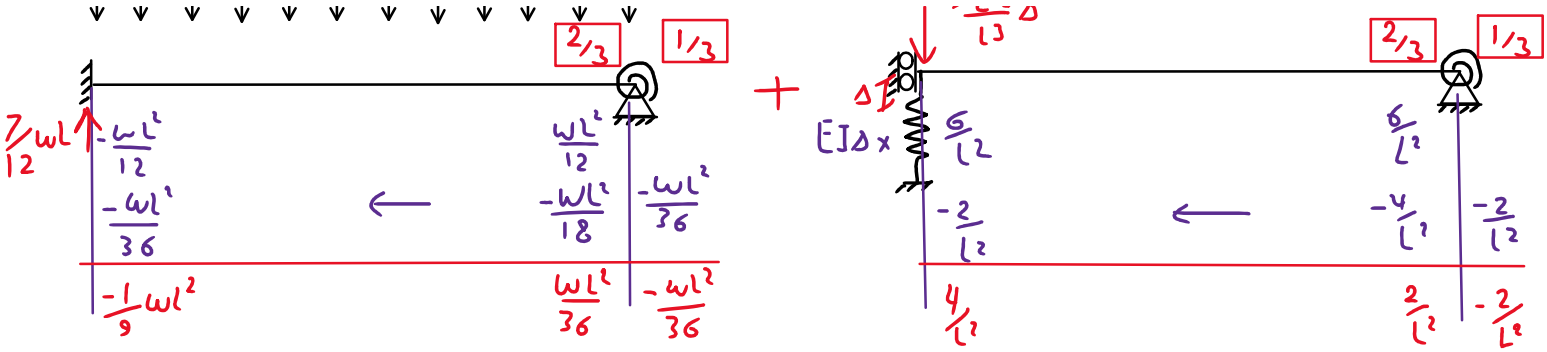
$$F_s = k\Delta = \frac{3EI}{L^3}\Delta = \frac{3}{L^3}\cdot\frac{2}{39}wL^4 = \boxed{\frac{2}{13}wL}$$

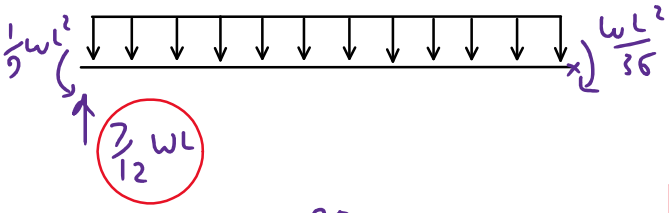
$$M_s = \frac{-wL^2}{60} + \frac{1.2EI}{L^2}\Delta = \left(-\frac{1}{60} + \frac{4}{10}\times\frac{2}{13}\right)wL^2 = \frac{-13+46}{5\times12\times13}wL^2 = \boxed{\frac{7}{156}wL^2}$$

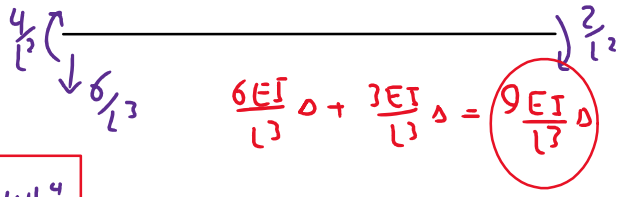
مثال : نیروهای منزها را به دست آورید.





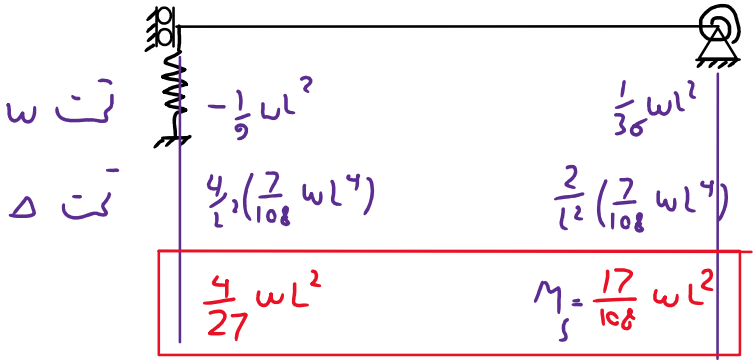






$$\frac{9EI}{L^3}\Delta = \frac{7}{12}wL \rightarrow \boxed{EI\Delta = \frac{7}{108}wL^4}$$

$$F_s = k\Delta = \frac{3EI}{L^3}\Delta = \frac{7}{36}wL$$

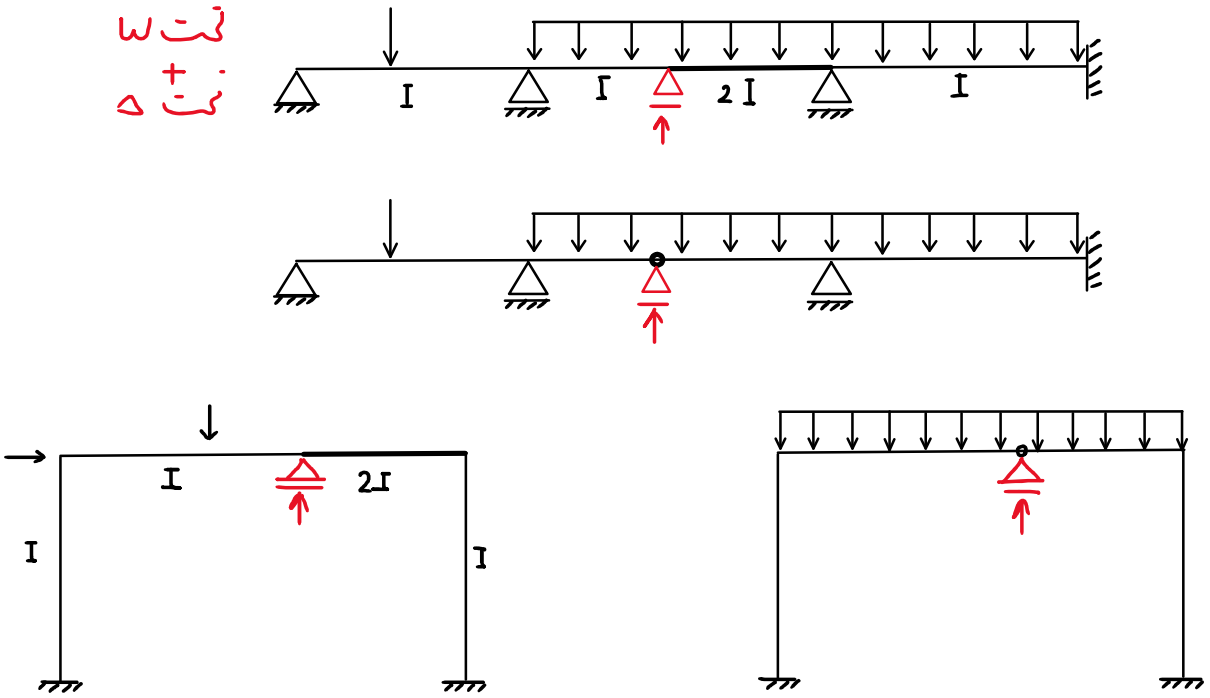


Video 22

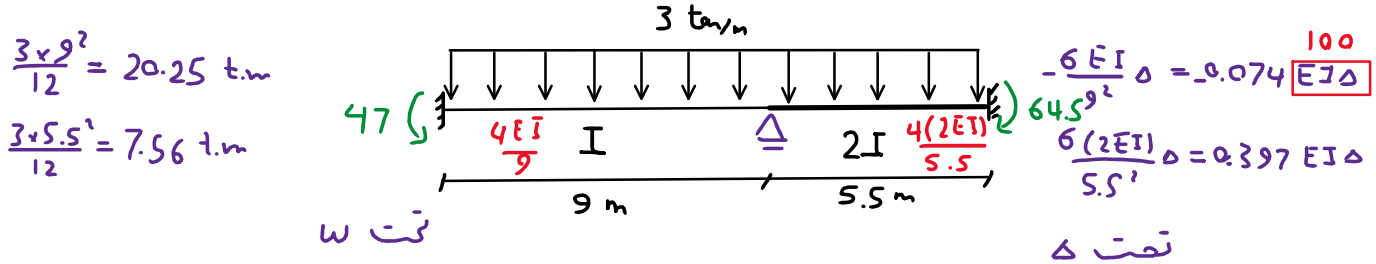
Moment Distribution 10

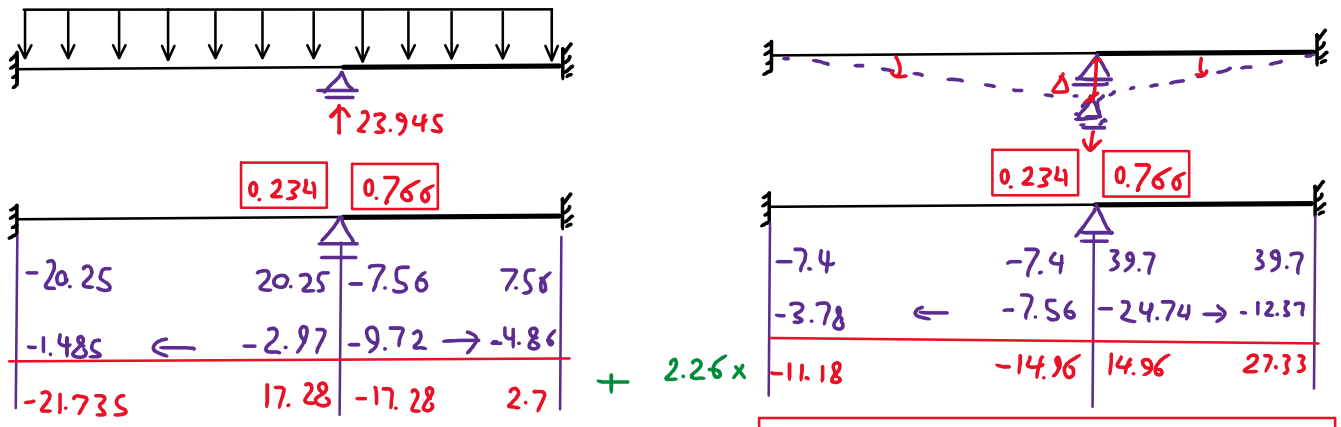
Monday, April 29, 2024 22:54

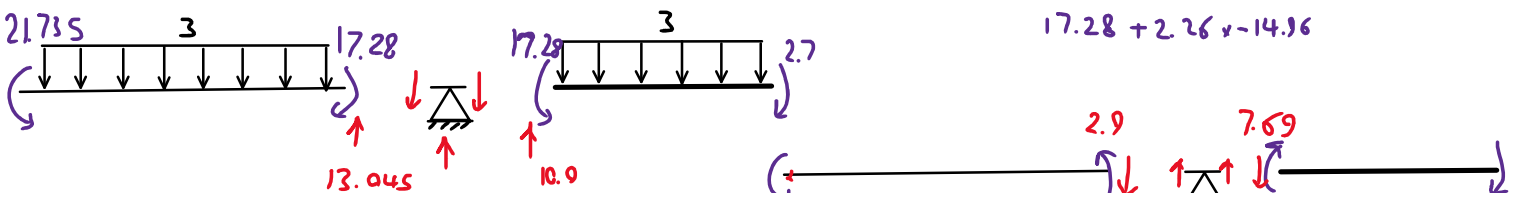
تغییر مقطع یا وجود مفصل در طول عضو

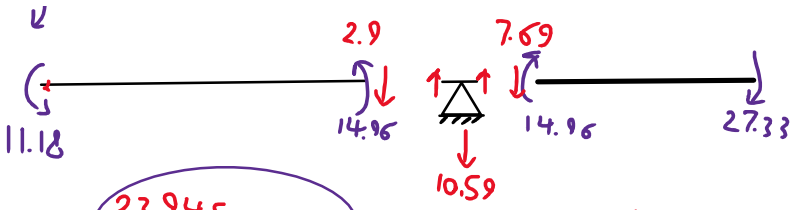


مثال : لنگرهای انتهایی اعضا را بدست آورید.









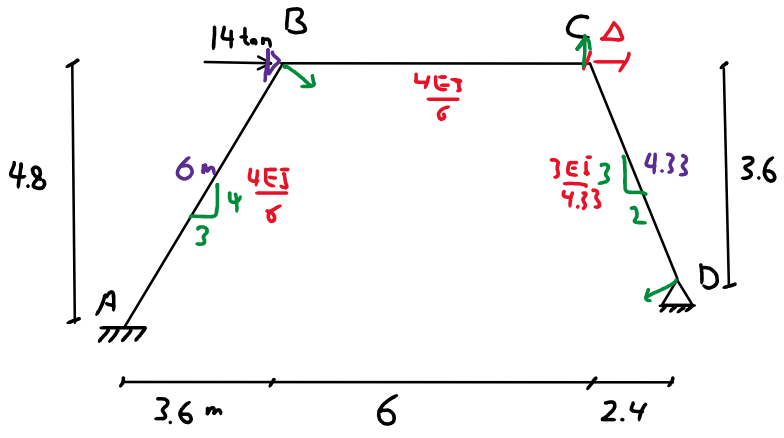
$\frac{23.945}{10.59} = 2.26$ ضریب متناسب برای سازه ی ت Δ

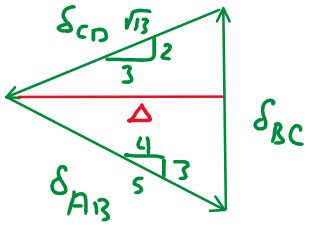
$EI\Delta = 226$

قاب شیبدار

مثال: لنگرهای انتهایی اعضا را به دست آورید.

درجه آزادی
1 Δ
2 θ





1000

$$FEM_{AB} = FEM_{BA} = -\frac{6EI}{6^2} \times 1.25\Delta = -0.208\,EI\Delta$$

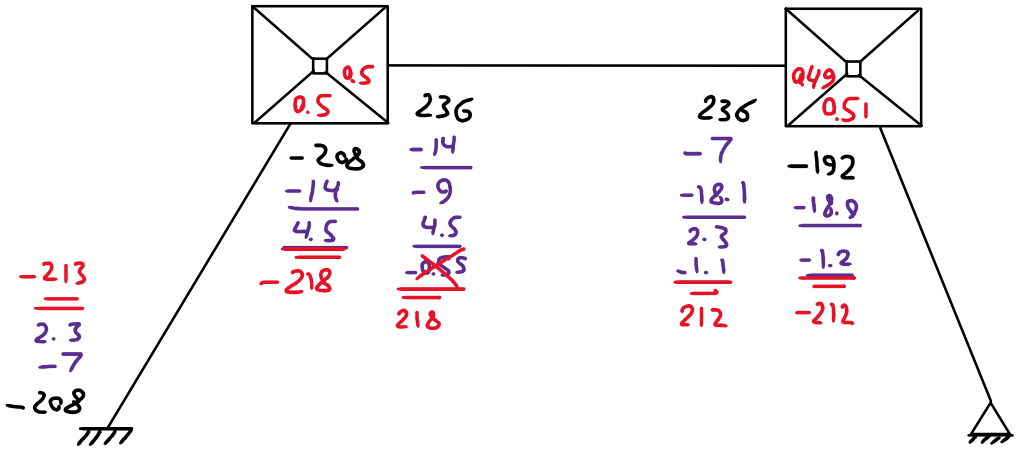
$$FEM_{BC} = FEM_{CB} = \frac{6EI}{6^2} \times 1.417\Delta = 0.236\,EI\Delta$$

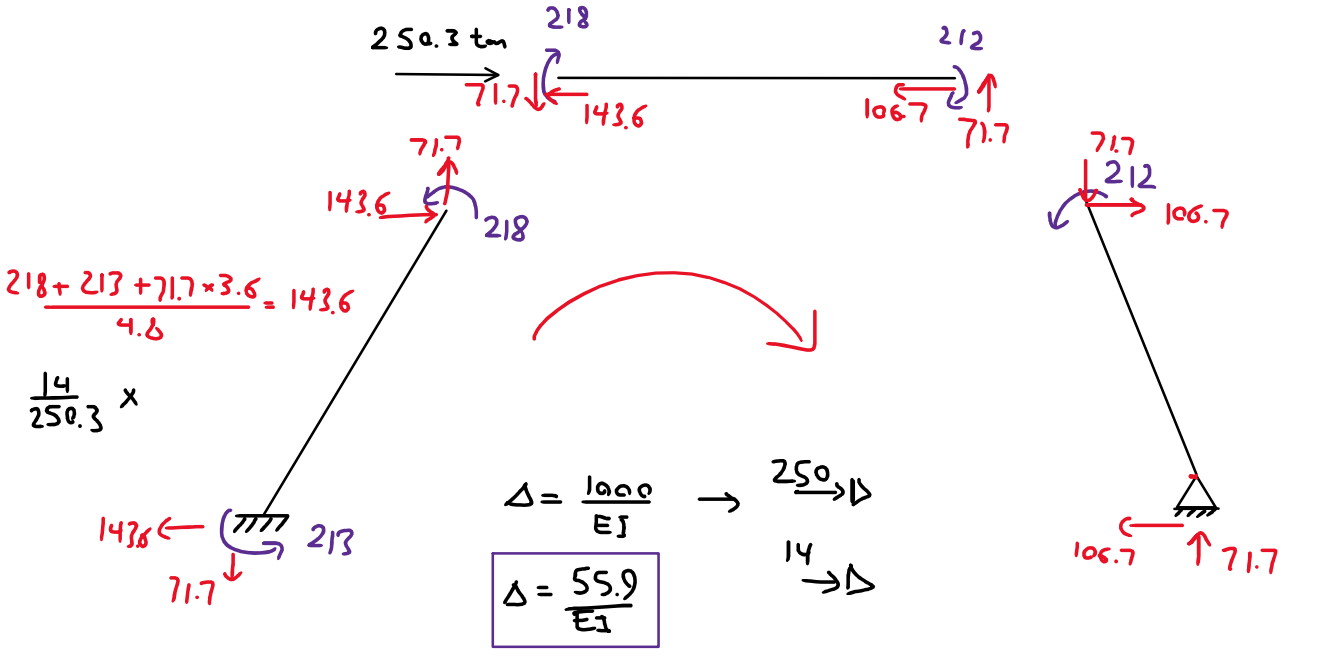
$$FEM_{CD} = -\frac{3EI}{4.33^2} \times 1.202\Delta = -0.192\,EI\Delta$$

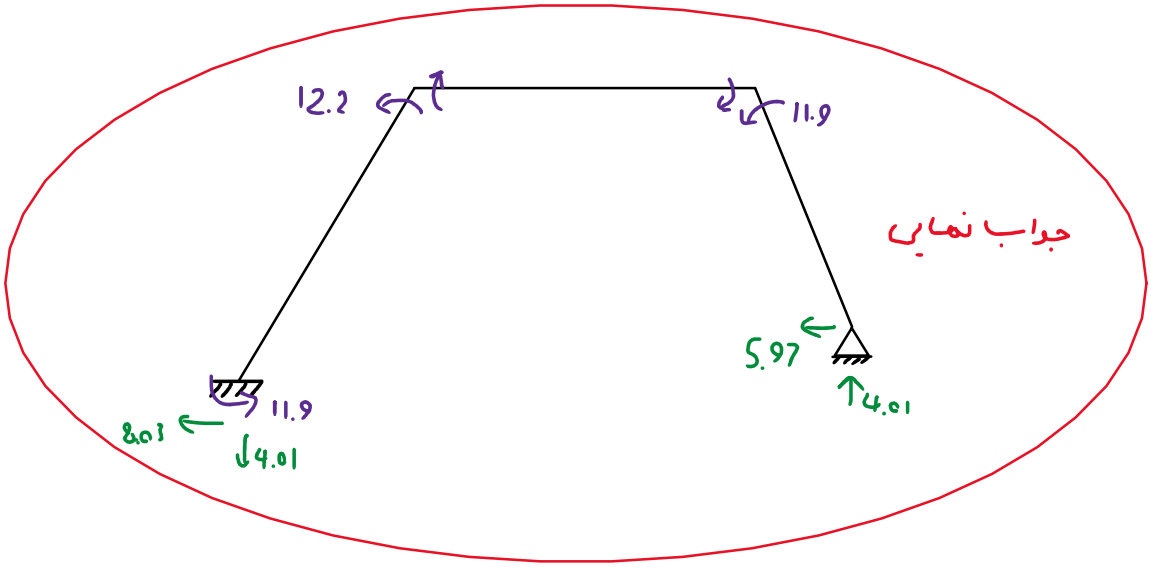
$$\delta_{AB} = \frac{5}{4}\Delta = 1.25\Delta$$

$$\delta_{BC} = \frac{3}{4}\Delta + \frac{2}{3}\Delta = 1.417\Delta$$

$$\delta_{CD} = \frac{\sqrt{13}}{3}\Delta = 1.202\Delta$$




250.3 tm
218
71.7
143.6
212
106.7
71.7
71.7
143.6
218
212
106.7
218+213+71.7×3.6 / 4.6 = 143.6
14 / 250.3 ×
143.6
213
71.7
Δ = 1000 / EI
250
14
Δ = 55.9 / EI
106.7
71.7

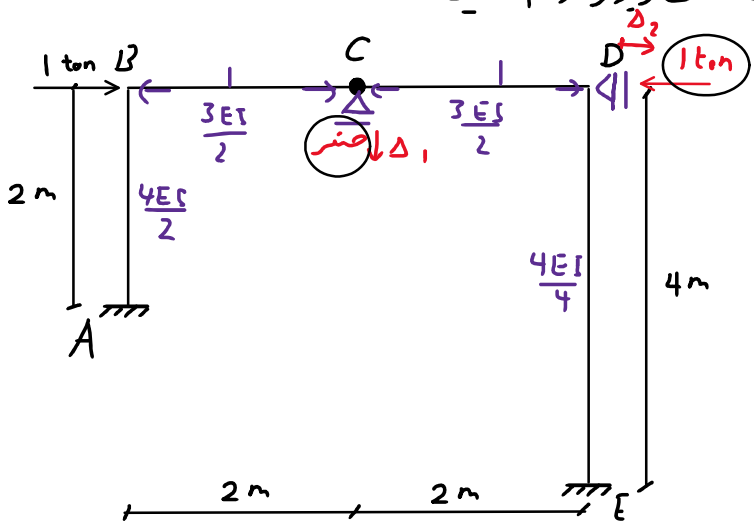

12.2
11.9
جواب نهایی
5.97
4.01
11.9
8.03
4.01

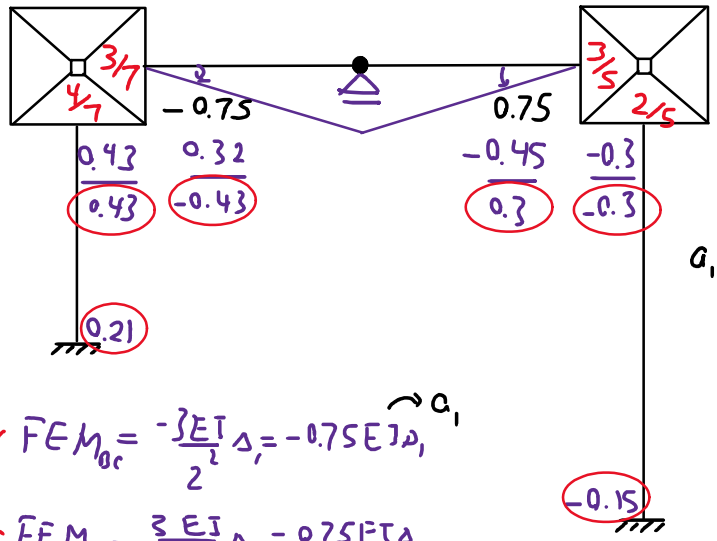
Video 23

Moment Distribution 11

Saturday, May 4, 2024 14:24

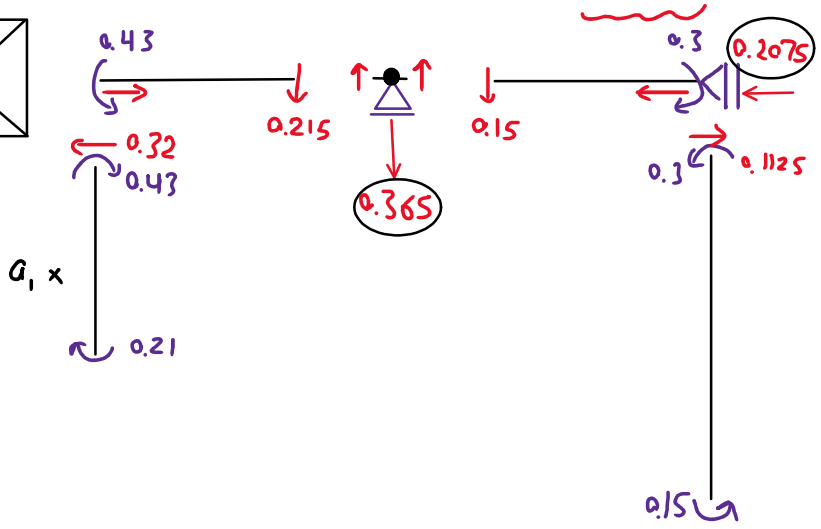
مثال: دیاگرام لنگر تا ب شکل زیر را رسم کنید.

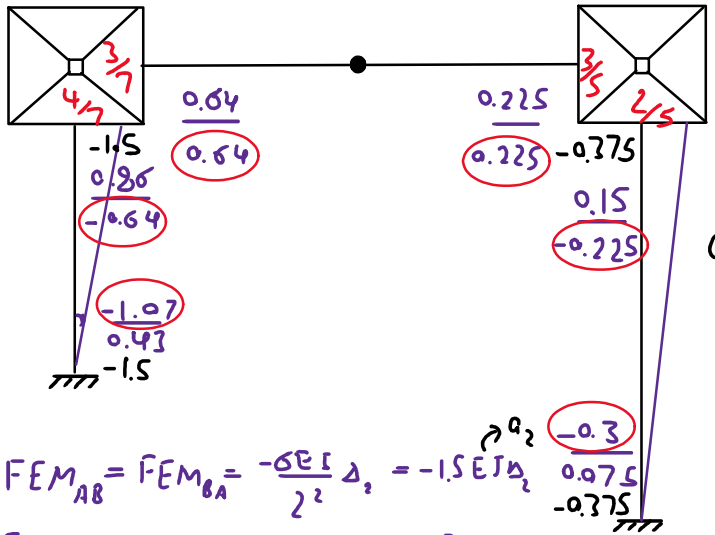




$FEM_{BC} = -\frac{3EI}{2^2}\Delta_1 = -0.75EI\Delta_1$ ↷ a_1

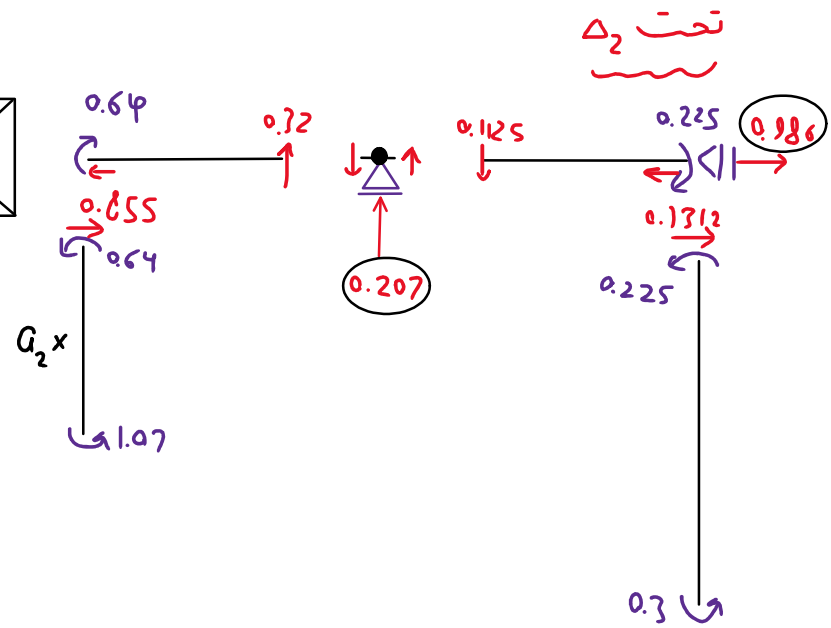
$FEM = \frac{3EI}{2^2}\Delta_1 = 0.75EI\Delta_1$





$FEM_{AB} = FEM_{BA} = \frac{-6EI}{2^2}\Delta_2 = -1.5EI\Delta_2$ ↷ a_2

$FEM_{CD} = FEM_{DE} = -\frac{6EI}{4^2}\Delta_2 = -0.375EI\Delta_1$



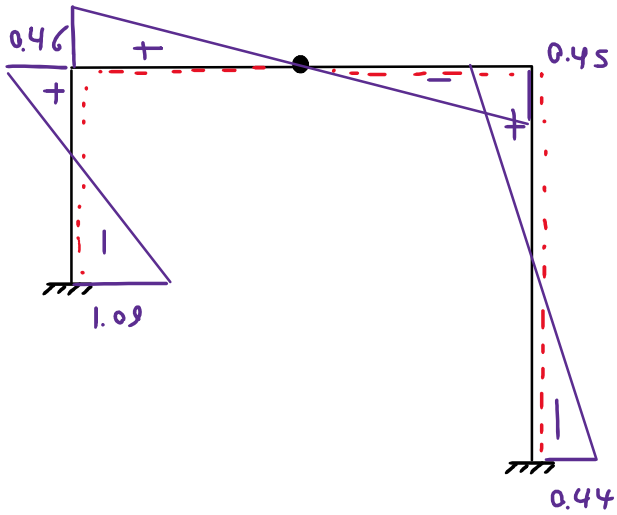
$$FEM_{CD} = FEM_{DE} = -\frac{6EI}{4^2}\Delta_2 = -0.375EI\Delta_2$$

$$\begin{cases} -0.385a_1 + 0.207a_2 = 0 \\ -0.2075a_1 + 0.986a_2 = 1 \end{cases} \rightarrow$$

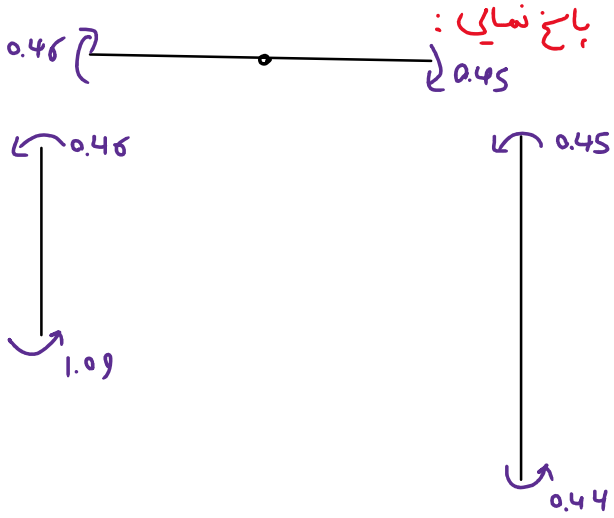
$$a_1 = EI\Delta_1 = 0.65$$

$$a_2 = EI\Delta_2 = 1.15$$

M(t.m)



پاسخ نهایی :



Video 24

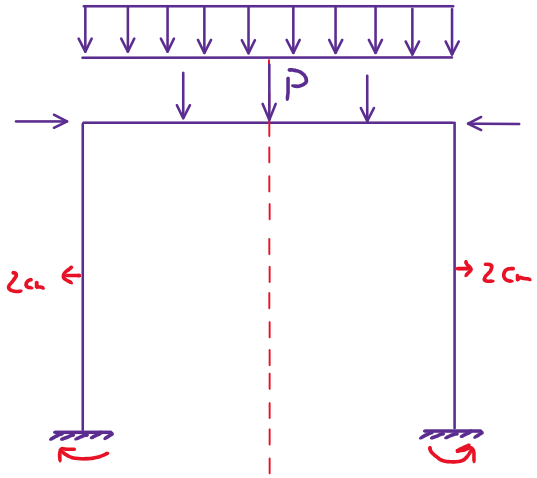
Symmetric Structures 1

Saturday, May 4, 2024 14:24

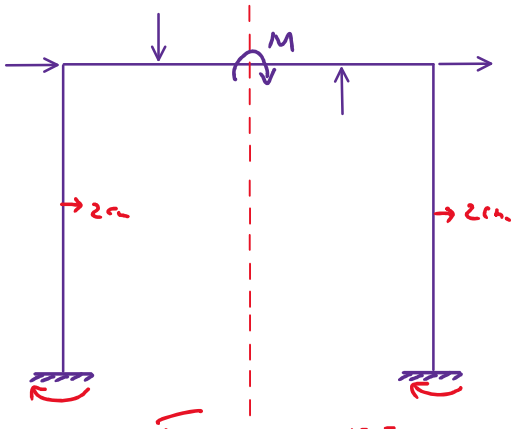
فصل سوم :

تحلیل سازه های نامعین

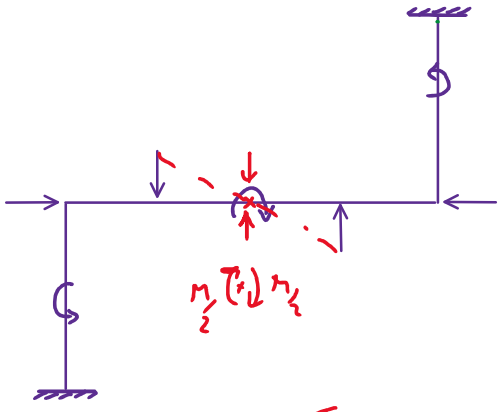
انواع تقارن
- محوری
  - مستقیم
  - معکوس
- نقطه ای (مرکزی)
  - مستقیم
  - معکوس



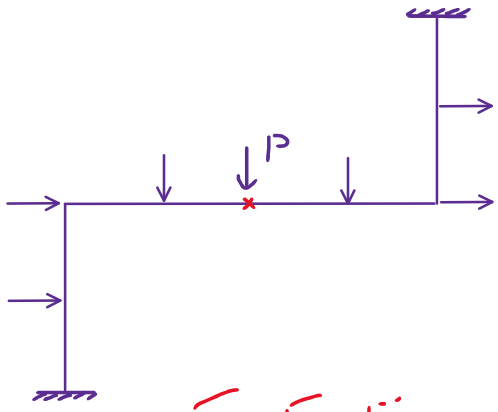
تقارن محوری مستقیم



تقارن محوری معکوس



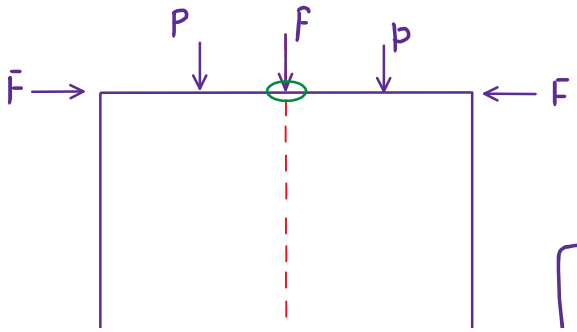
تقارن مرکزی مستقیم

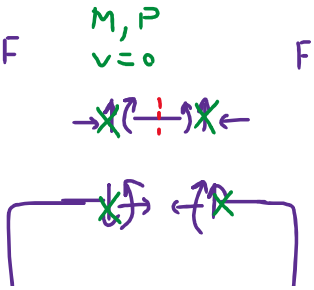


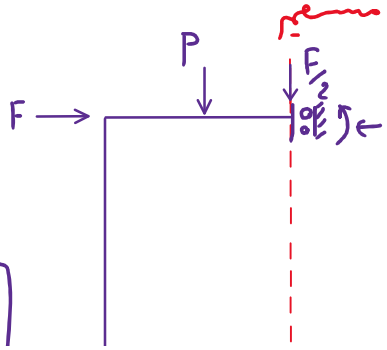
تقارن مرکزی معکوس

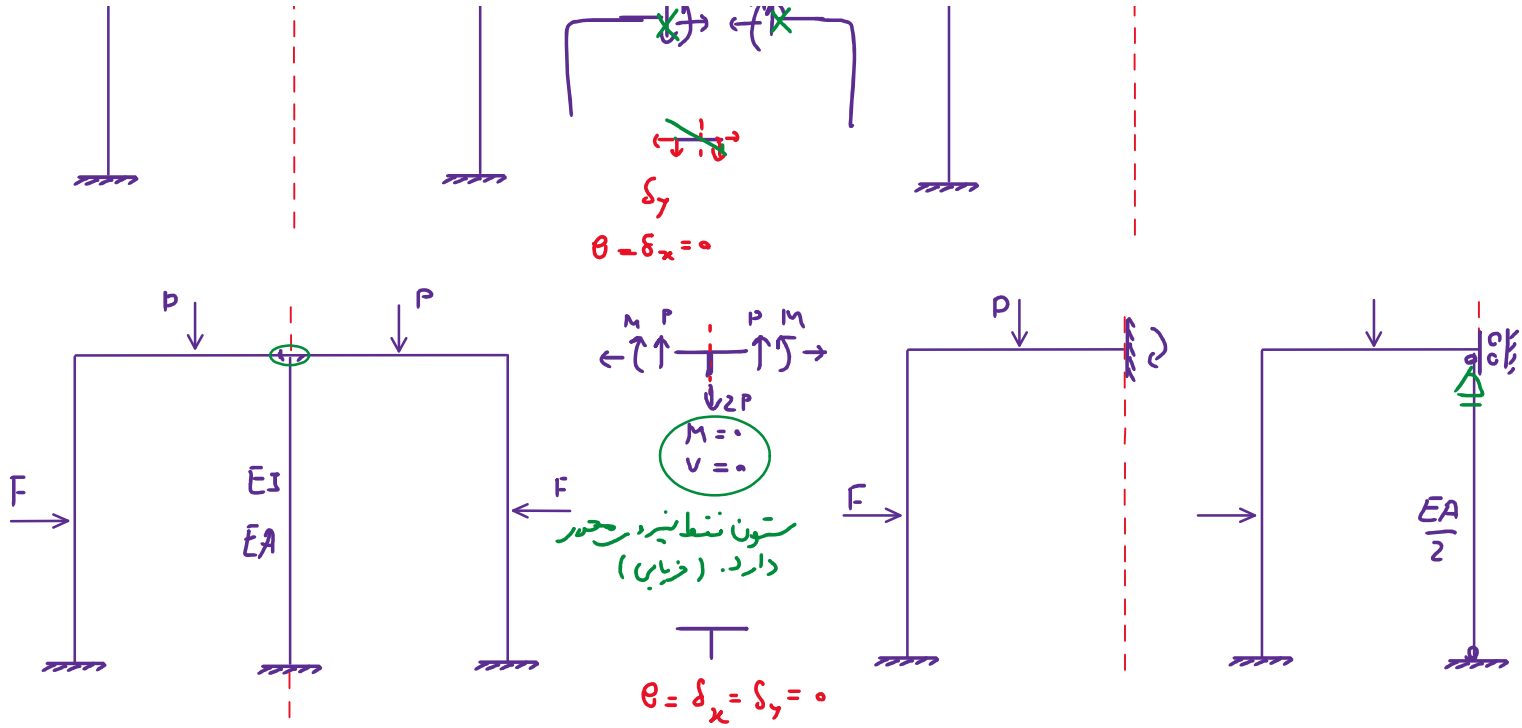
رسم نصف سازه

① تقارن محوری مستقیم

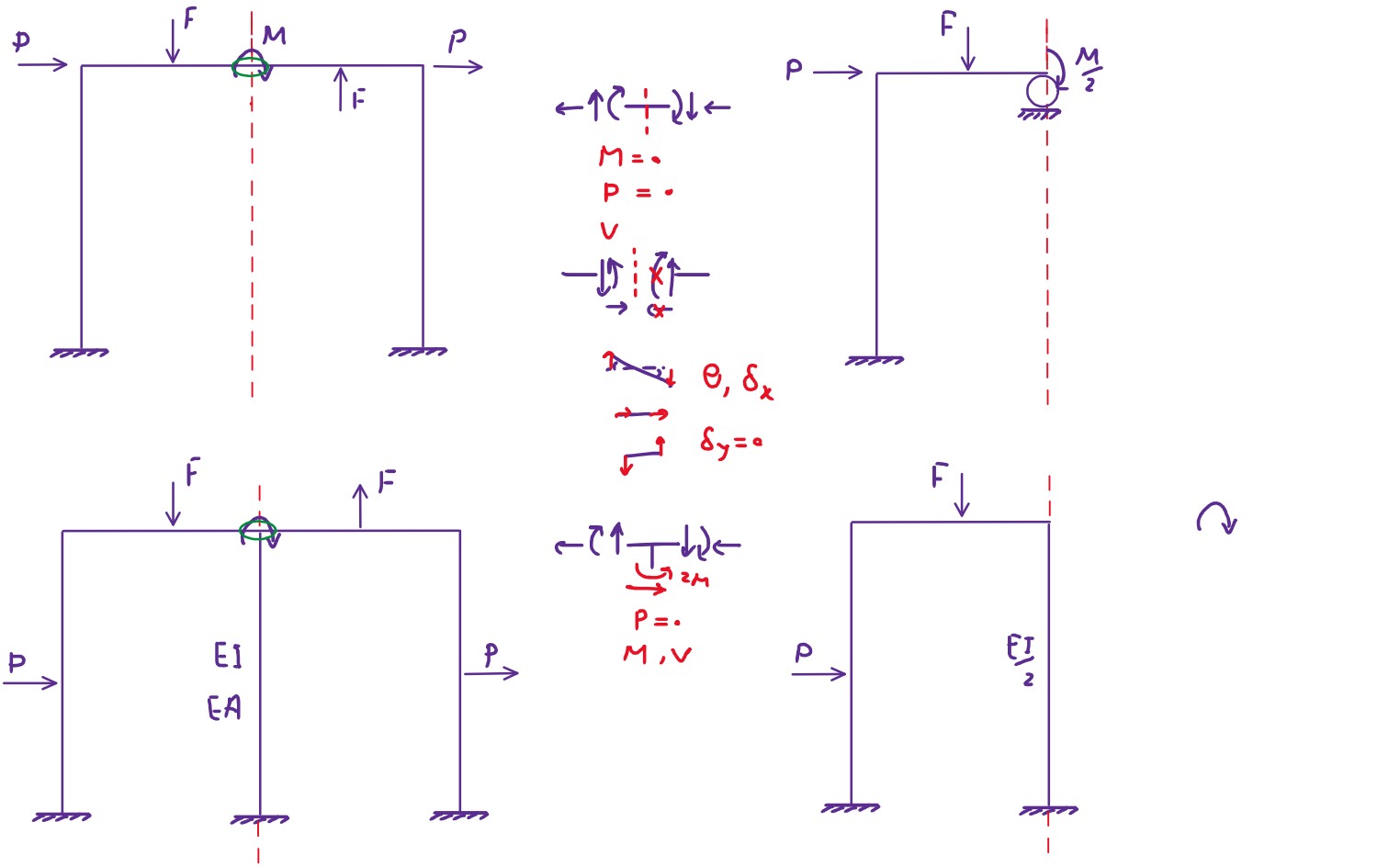






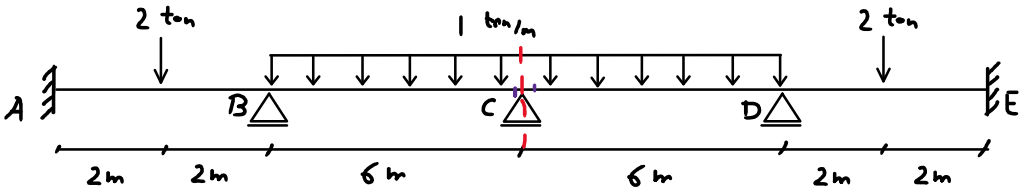


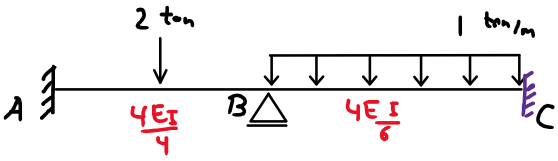
② تمارین محوری معکوس

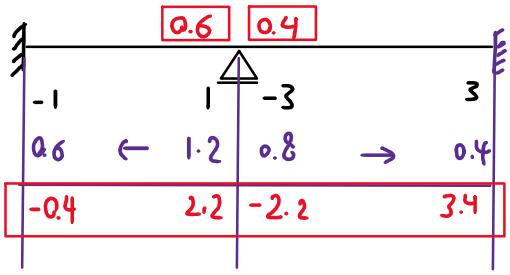


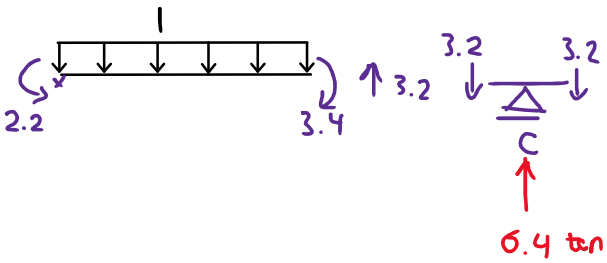
مثال : لنگرهای انتهایی اعضا و عکس العمل تکیه‌گاه c را به دست آورید.

2 ton 1 ton/m 2 ton

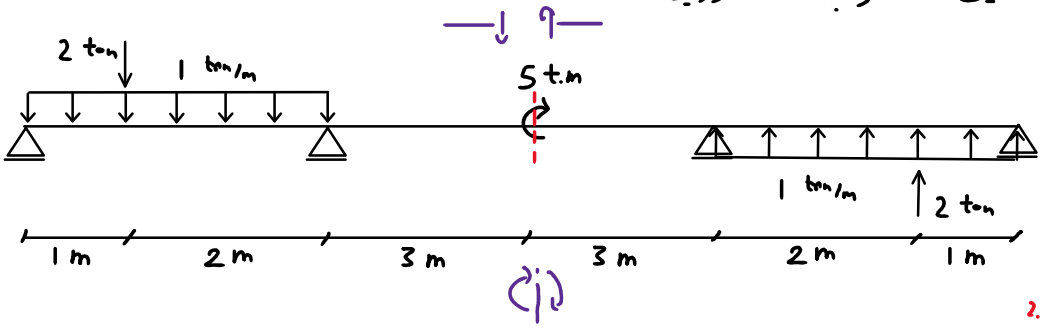


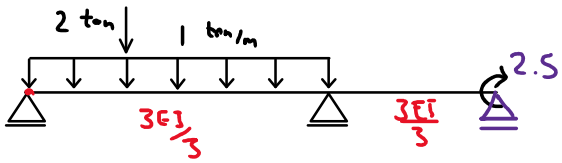


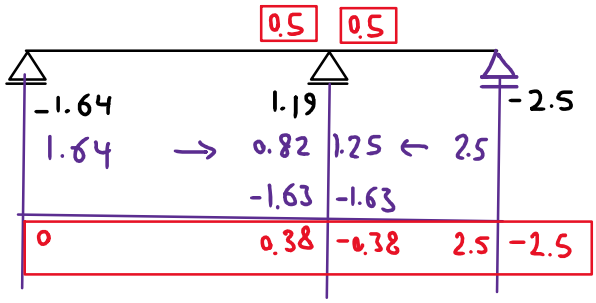




مثال : لنگرهای انتهایی اعضا را به دست آورید.







$$-\frac{1\times 3^2}{12}-\frac{2\times 1\times 2^2}{3^2}=-1.64$$

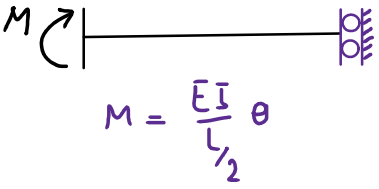
$$\frac{1\times 3^2}{12}+\frac{2\times 2\times 1^2}{3^2}=1.19$$

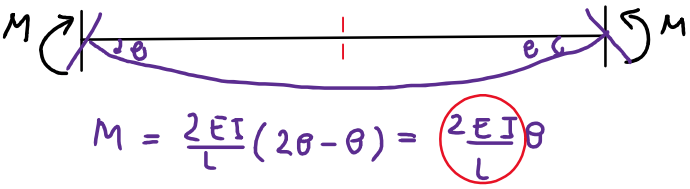
Video 25

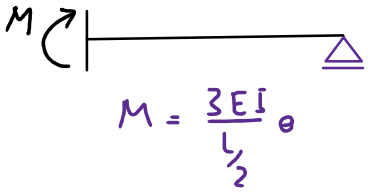
Symmetric Structures 2

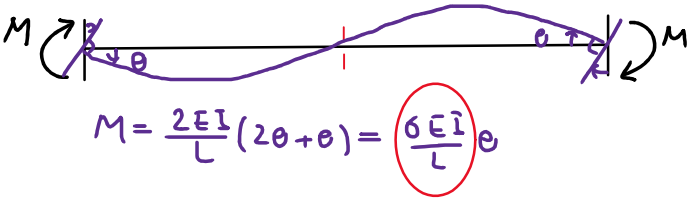
Saturday, May 4, 2024 14:24

سختی اعضای متقارن



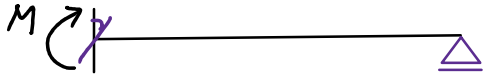




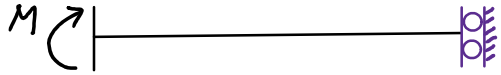


مقایسه سختی اعضا با شرایط متفاوت تکیه گاهی

$k = \frac{4EI}{L}$

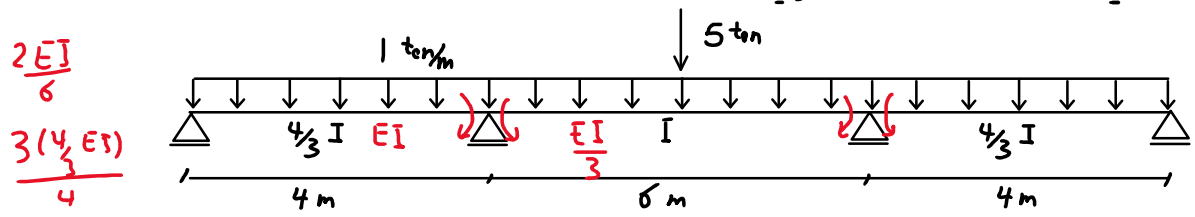


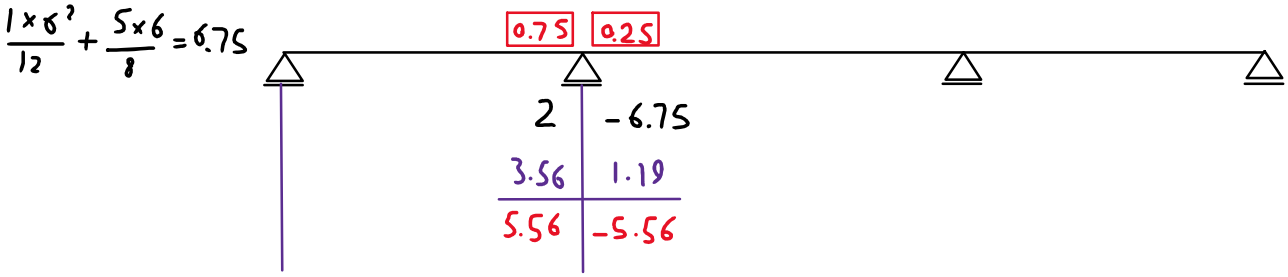
$k = \frac{3EI}{L}$

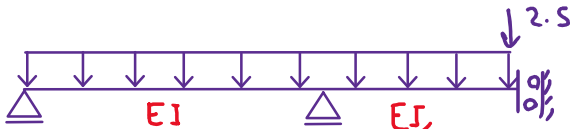


$k = \frac{EI}{L}$

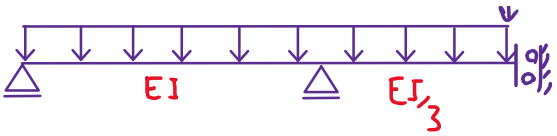
مثال : لنگرهای انتهایی اعضا را به دست آورید.

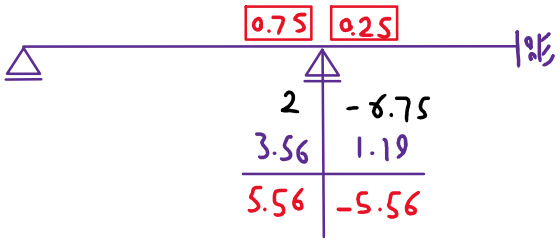




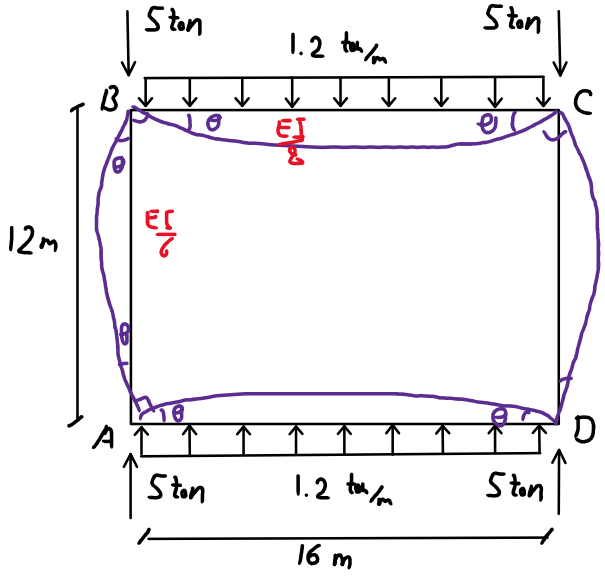


و یا اینکه :



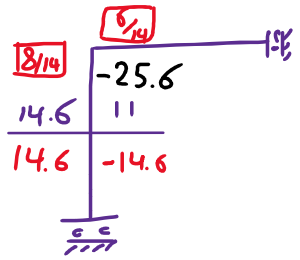


مثال: لنگر در گره B و چرخش گره B را به دست آورید.



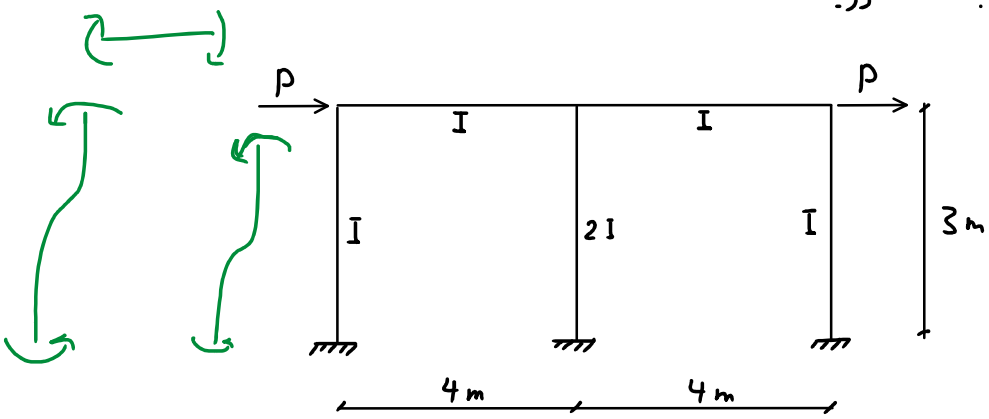
$$\theta_B = \frac{M}{\sum k_i} = \frac{\frac{1.2\times16^2}{12} 25.6}{\frac{2EI}{16} + \frac{2EI}{12}} = \frac{87.8}{EI}$$

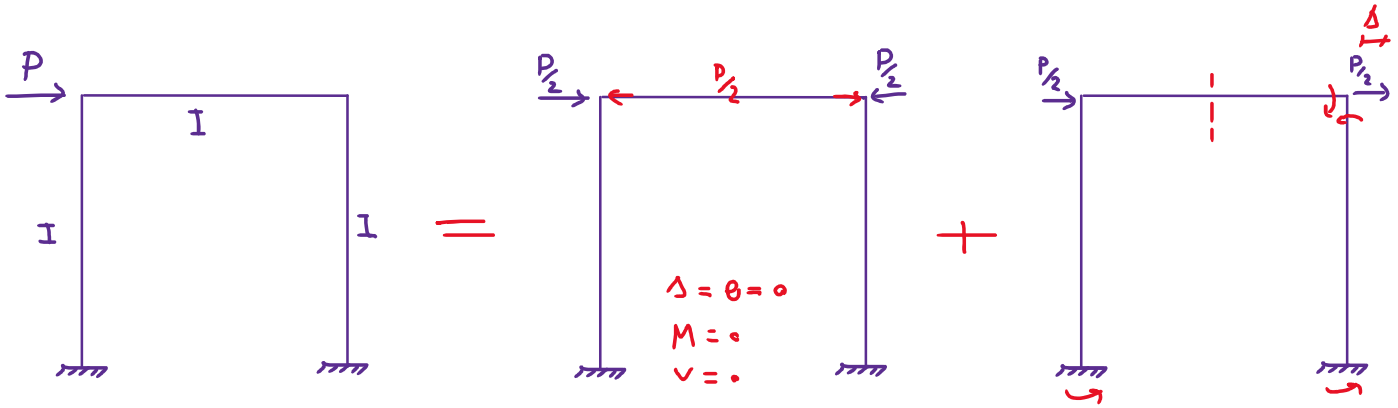
$$M_{BA} = 25.6 \times \frac{8}{14} = 14.6$$

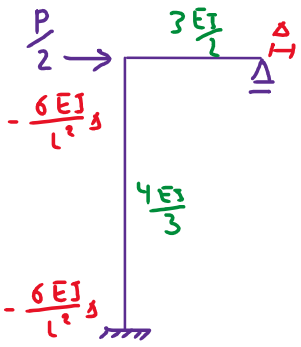


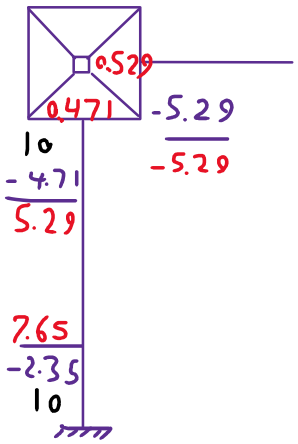
$$14.6 = \frac{2EI}{12}\theta \rightarrow \theta = \frac{87.6}{EI}$$

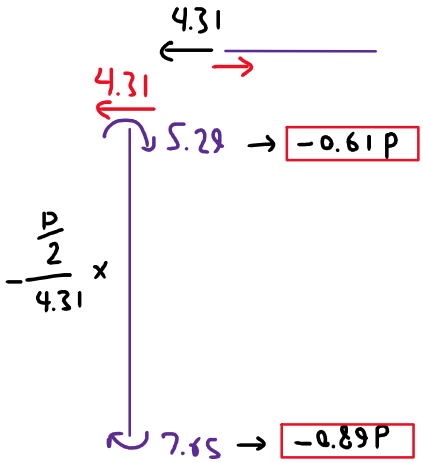
مثال: لنگرهای انتهایی اعضا را به دست آورید.

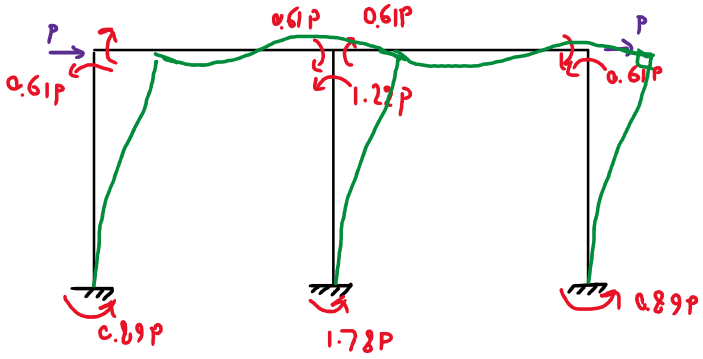


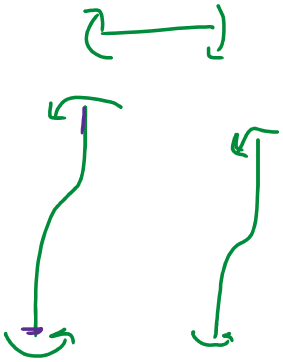






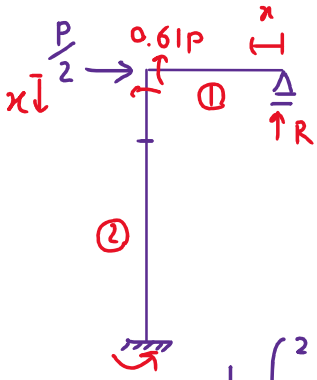






$$M = \frac{2EJ}{L}(2\theta + \theta - 3\frac{\Delta}{L})$$

$$U = \frac{1}{2}\int \frac{M^2}{EI}\,dx$$



$$\delta_y = 0 \rightarrow \frac{\partial U}{\partial R} = 0 \rightarrow \int \frac{M}{EI}\left(\frac{\partial M}{\partial R}\right) = 0$$

① $M = Rx \qquad \frac{\partial M}{\partial R} = x$

② $M = 2R - \frac{P}{2}x \qquad \frac{\partial M}{\partial R} = 2$

$$\frac{1}{EI}\int_0^2 Rx^2\,dx + \int_0^3 (4R - Px)\,dx = 0$$

$$R\frac{x^3}{3}\Big|_0^2 + \left(4Rx - \frac{Px^2}{2}\right)\Big|_0^3 = 0$$

$$\frac{8}{3}R + 12R - 4.5P = 0 \rightarrow \boxed{R = 0.307P} \qquad \boxed{M = 0.61P}$$

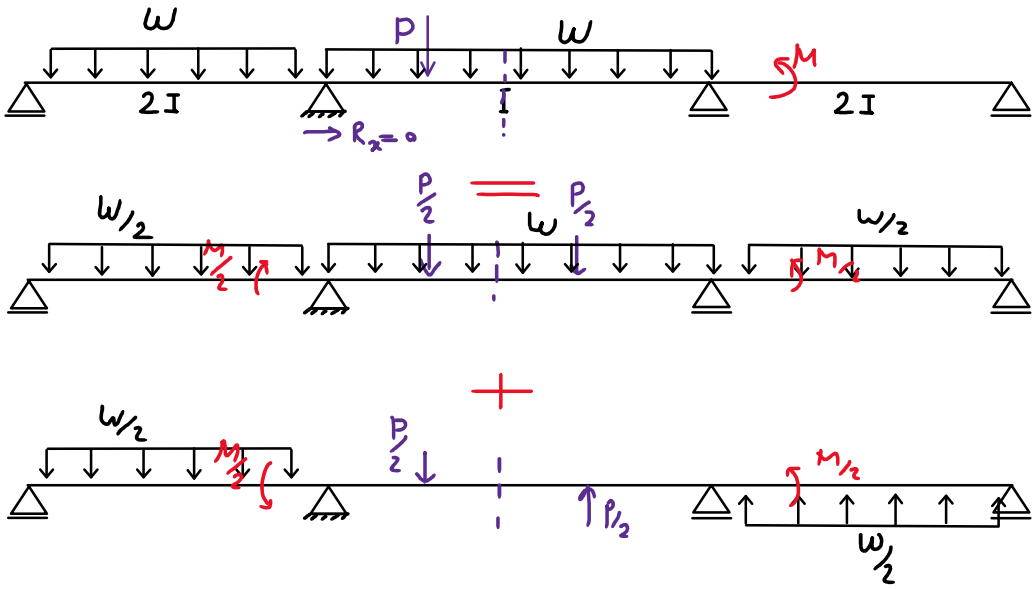
Video 26

Symmetric Structures 3

Saturday, May 4, 2024 14:24

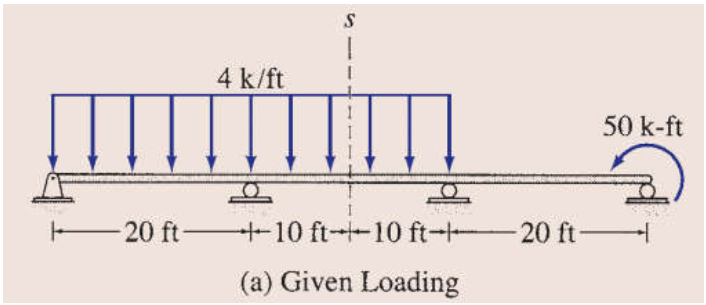
تفکیک یک بارگذاری کلی به یک بارگذاری متقارن و پادمتقارن

سازه متقارن با بارگذاری کلی

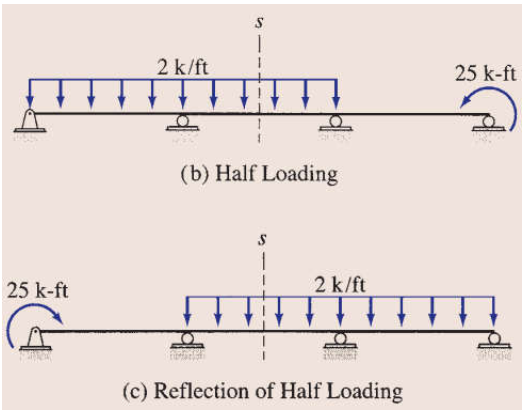


بارگذاری کلی = بارگذاری متقارن + بارگذاری پادمتقارن

مثال ①

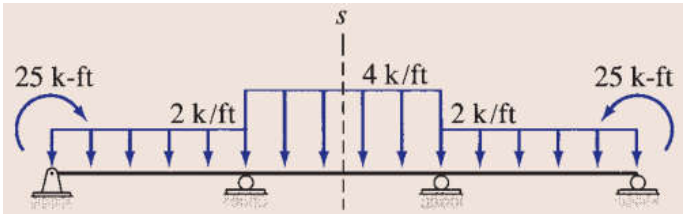


(a) Given Loading



(b) Half Loading

(c) Reflection of Half Loading



(d) Symmetric Loading Component

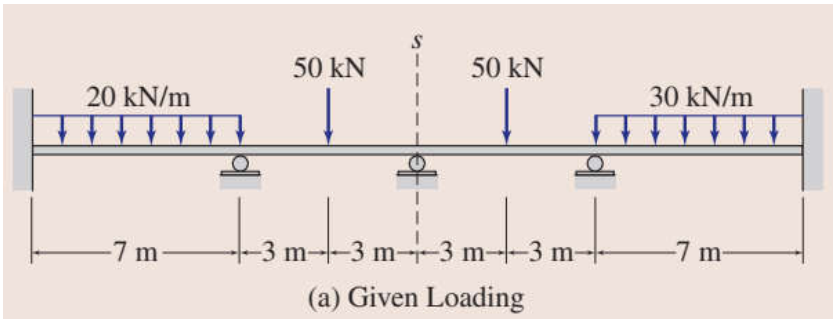
(e) Antisymmetric Loading Component

① رسم سازه با نصف بار و تصویر آینه‌ای آن نسبت به محور تقارن ② ③

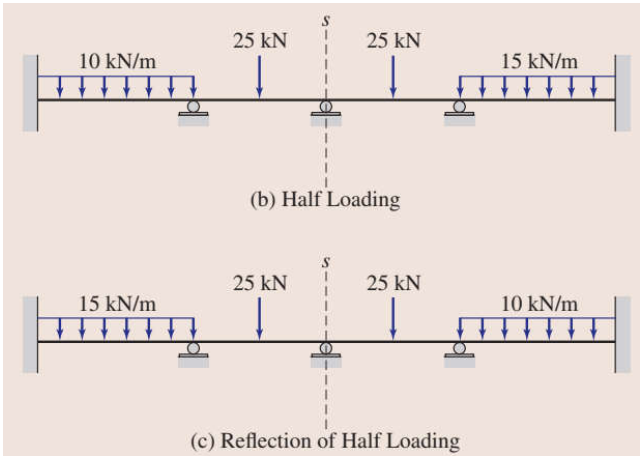
② جمع بارها ① + ② تقارن

③ تفریق بارها ② - ① پادمتقارن

مثال ②

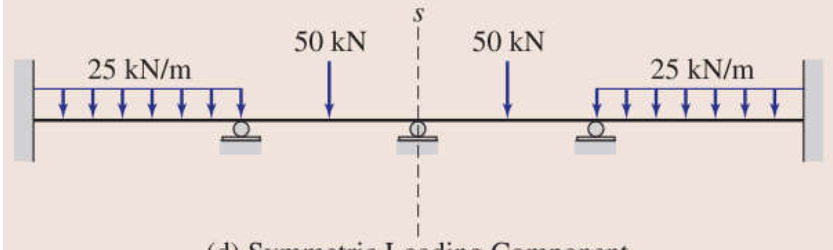


(a) Given Loading

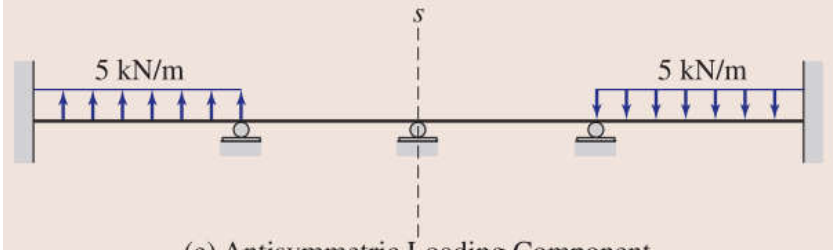


(b) Half Loading

(c) Reflection of Half Loading



(d) Symmetric Loading Component



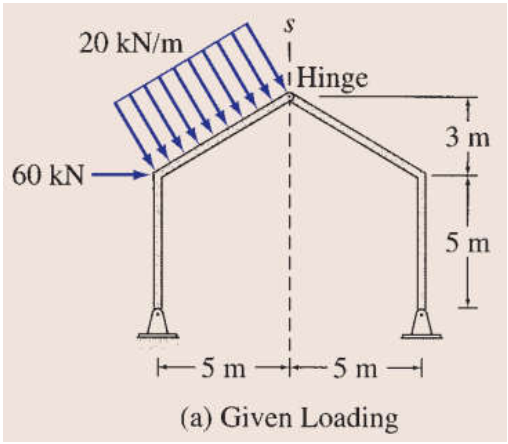
(e) Antisymmetric Loading Component

20 10

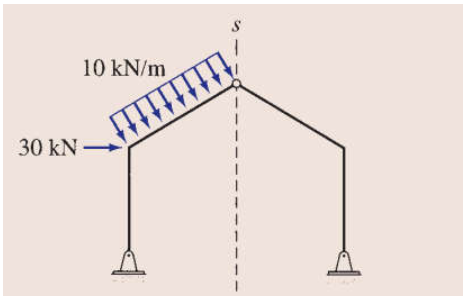
15 15

5 -5

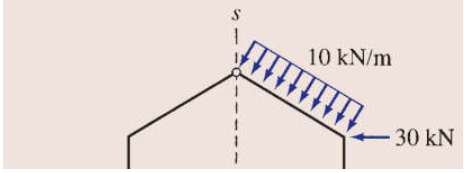
مثال ۳

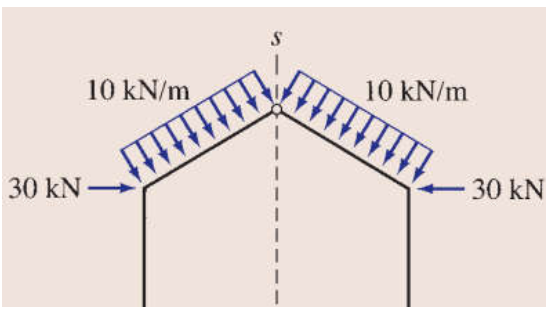


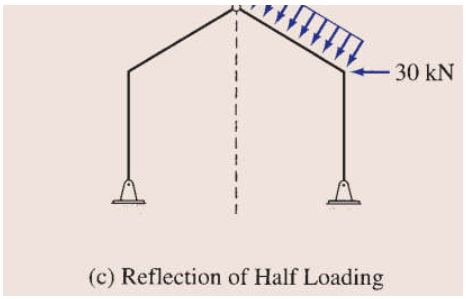
(a) Given Loading



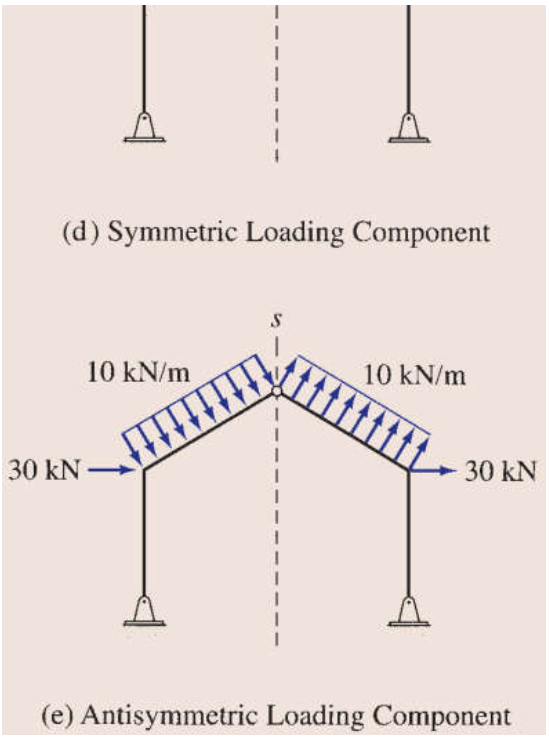
(b) Half Loading







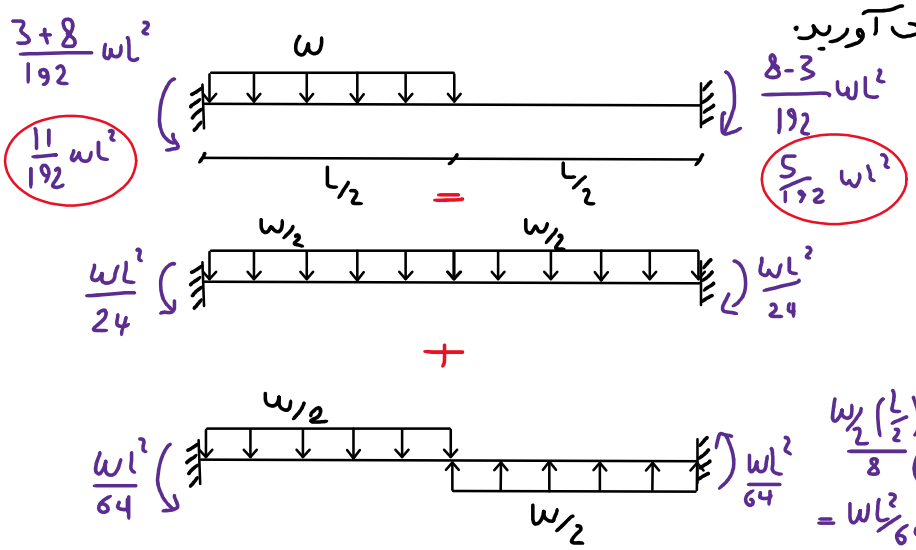
(c) Reflection of Half Loading



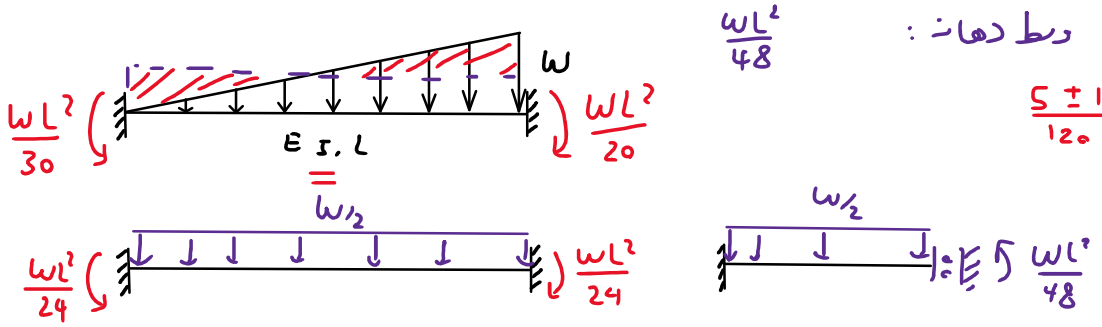
(d) Symmetric Loading Component

(e) Antisymmetric Loading Component

مثال: لنگرهای گیرداری دوسرتیر را به دست آورید.



مثال: لنگرهای گیرداری دوسرتیر را به دست آورید.



$$\delta_y = \frac{\partial U}{\partial R} = 0 \rightarrow U = \int \frac{1}{2}\frac{M^2}{EI}dx \rightarrow \frac{1}{EI}\int M\left(\frac{\partial M}{\partial R}\right)dx = 0$$

$$\int_0^{L/2}\left(Rx + \frac{w}{L}\frac{x^3}{6}\right)(x)\,dx = 0$$

$$\int_0^{L/2}\left(Rx^2 + \frac{w}{L}\frac{x^4}{6}\right)dx = 0$$

$$\left(R\frac{x^3}{3} + \frac{w}{L}\frac{x^5}{30}\right)\Big|_0^{L/2} = 0$$

$$R\left(\frac{L^3}{24}\right) = -\frac{w}{L}\frac{L^5}{30\times 32}$$

$$R = -\frac{wL^2}{40}$$

$M = Rx + \frac{1}{2}\left(\frac{w}{L}x\right)x\left(\frac{x}{3}\right)$

$M = Rx + \frac{w}{L}\frac{x^3}{6}$

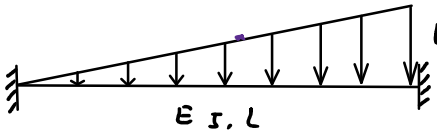
$\frac{\partial M}{\partial R} = x$

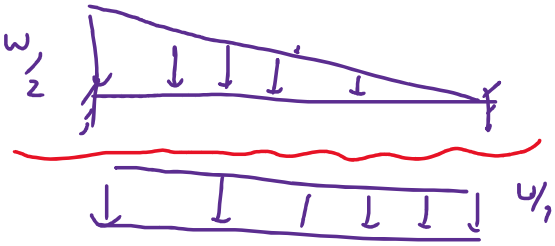
$-\frac{wL^2}{40}\left(\frac{L}{2}\right) +$

$\frac{1}{2}\left(\frac{w}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) =$

$\left(\frac{1}{80} + \frac{1}{48}\right)$

$\frac{-3+5}{5\times 16\times 3}$



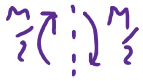


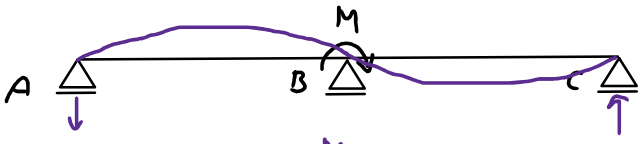
Video 27

Symmetric Structures 4

Sunday, May 12, 2024 8:58

تحلیل سازه های متقارن





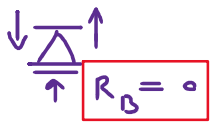
مثال: θ_B , θ_C , R_B = ?

$M_{BC} = \frac{3EI}{L}(\theta_B - \frac{\Delta}{L}) + \text{F-EM}$

$\frac{M}{2} = \frac{3EI}{L}\theta_B \rightarrow \theta_B = \frac{ML}{6EI}$

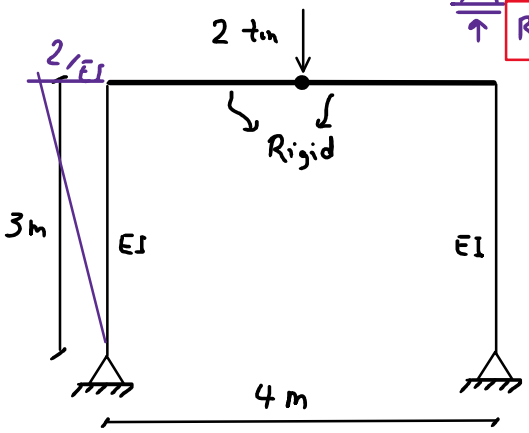
$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B) = 0 \rightarrow \theta_C = -\frac{\theta_B}{2}$

$\theta_C = \frac{ML}{12EI}$

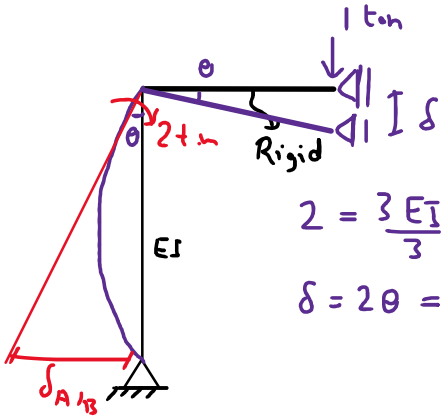


$R_B = 0$

مثال: تغییرمکان تائم زیر بارمتمرکز را محاسبه کنید.



$EI = 100\ t.m^2$

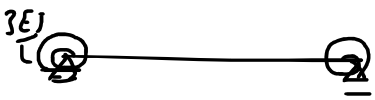


$\delta_{A/B} = \frac{1}{2}(2)(3)(2) = \frac{6}{EI}$

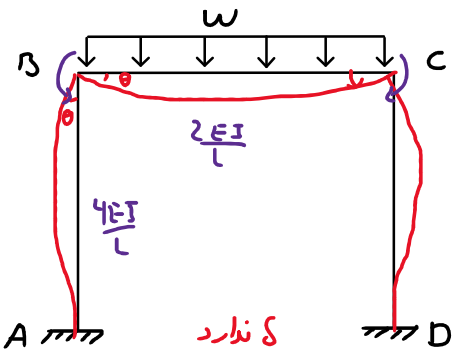
$\theta = \frac{\delta_{A/B}}{3} = \frac{2}{EI}$

$2 = \frac{3EI}{3}\theta \rightarrow \theta = \frac{2}{EI} = 0.002$

$\delta = 2\theta = \frac{4}{EI} = 0.004\ m$

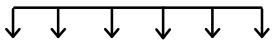


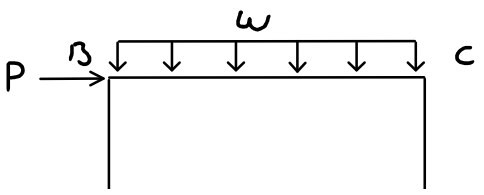
مثال: θ_B , M_B = ?

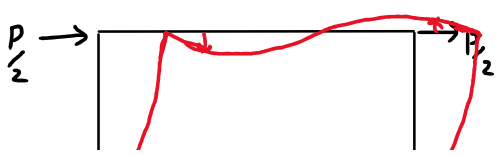


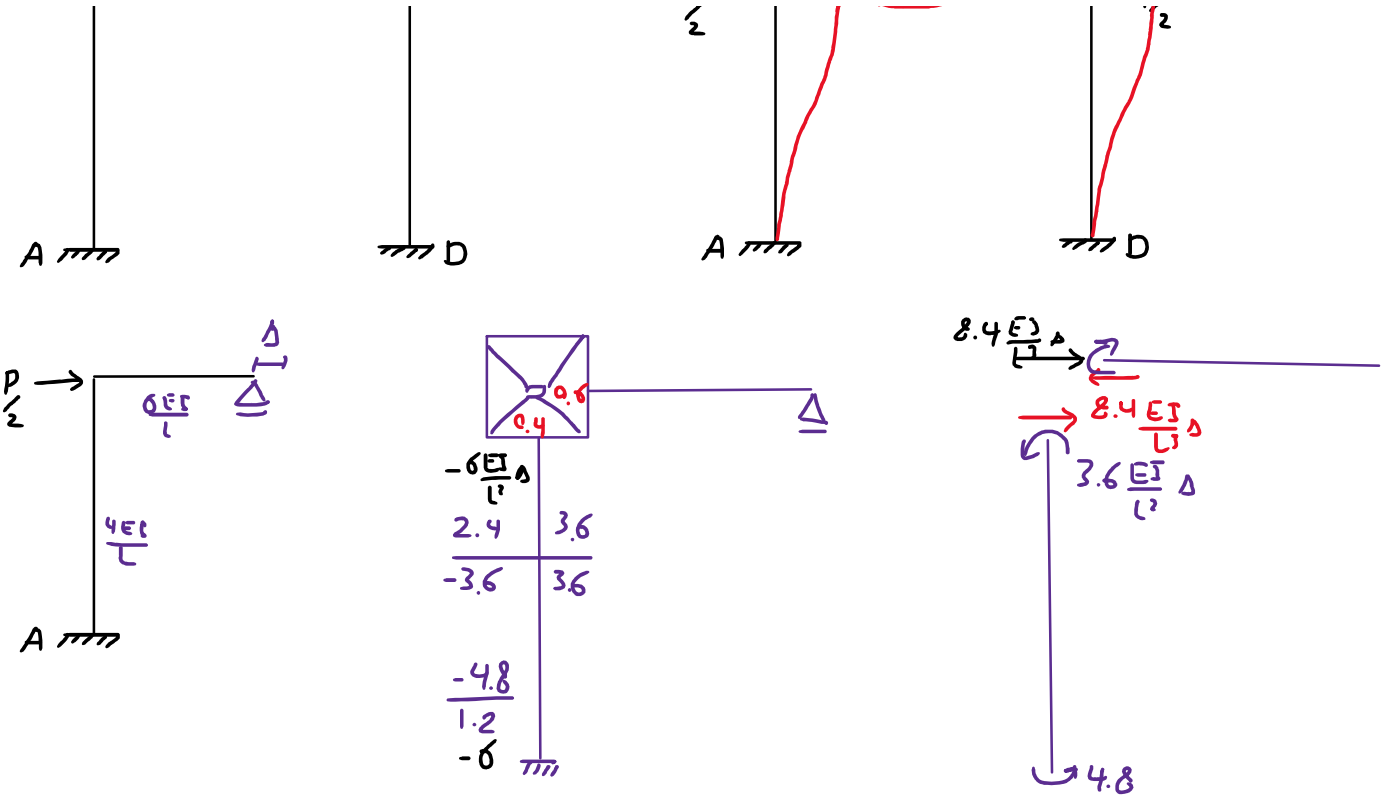
$\theta_B = \frac{\frac{wL^2}{12}}{\frac{6EI}{L}} = \frac{wL^3}{72EI}$

$M_B = \frac{2}{3} \times \frac{wL^2}{12} = \frac{wL^2}{18}$









$$\frac{P}{2} = 8.4\,\frac{EI}{L^3}\Delta \rightarrow \Delta = \frac{1}{16.8}\frac{PL^3}{EI} = \frac{5}{84}\frac{PL^3}{EI}$$

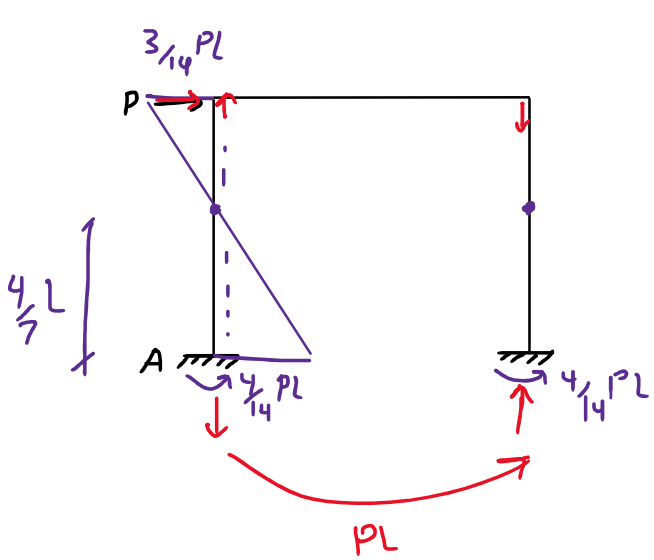
$$3.6\,\frac{EI}{L^2}\Delta = 6\,\frac{EI}{L}\theta \rightarrow \theta = 0.6\,\frac{\Delta}{L} = \frac{3}{5}\times\frac{5}{84}\frac{PL^2}{EI} = \frac{1}{28}\frac{PL^2}{EI}$$

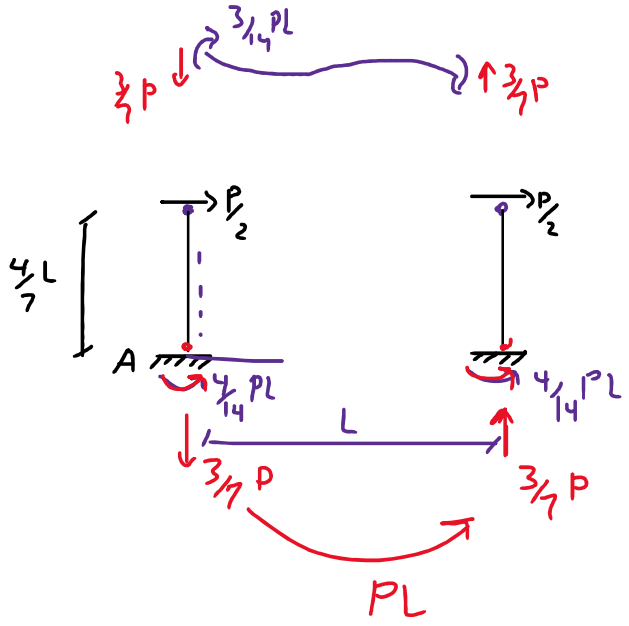
$$\theta = \frac{6\frac{EI}{L^2}\Delta}{10\frac{EI}{L}}$$

$$\theta_{\Delta C} = -\frac{wL^3}{72EI} + \frac{PL^2}{28EI}$$

$$M_B = 3.6\,\frac{EI}{L^2}\Delta = \frac{3.6}{2}\times\frac{5}{14}PL = \frac{3}{14}PL$$

$$M_B = -\frac{wL^2}{12} + \frac{3}{14}PL$$

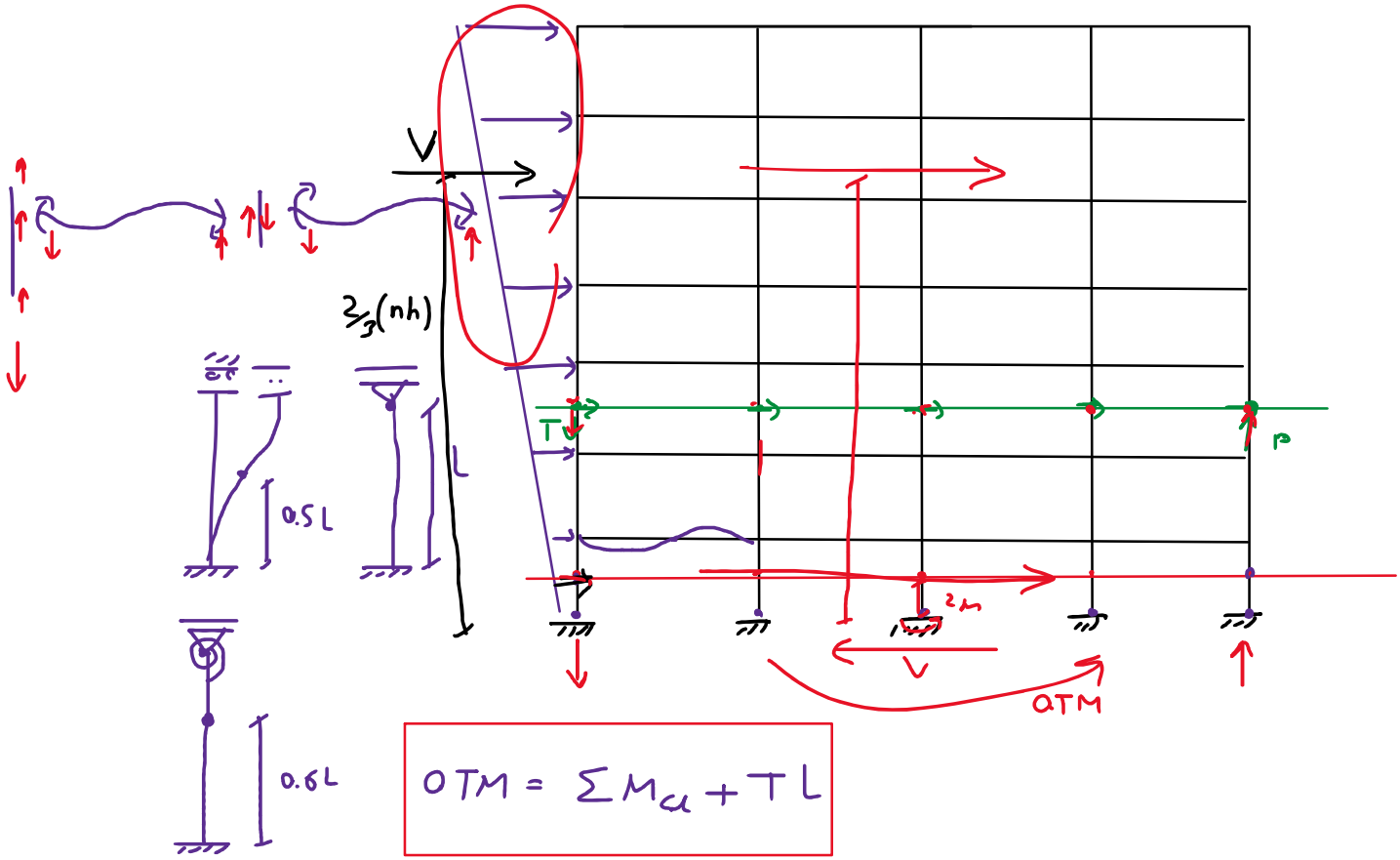




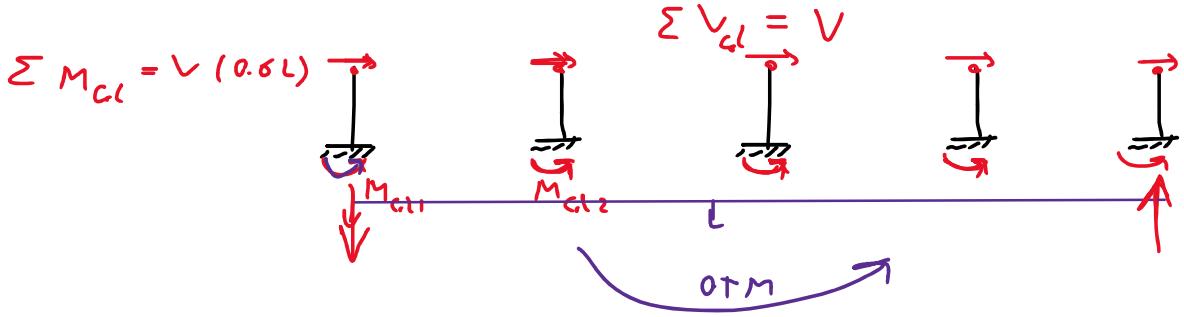
$TL = OTM - \Sigma M_{cl}$

$TL = PL - 2 \times \frac{2}{7} PL \rightarrow$ $\boxed{T = \frac{3}{7}P}$

سهم از OTM { لوبل نیرد $\frac{3}{7}PL$ ، تکیه‌گاه ستون $\frac{4}{7}PL$



$\boxed{OTM = \Sigma M_{cl} + TL}$



$V\left(\frac{2}{3}nh\right) = V(0.6h) + TL$

$* \; TL = V\left(\frac{2}{3}n - 0.6\right)h$

↓ 10

سهم از OTM { لوبل نیرد $0.1Vh$ ، تکیه‌گاه ستون $0.6Vh$

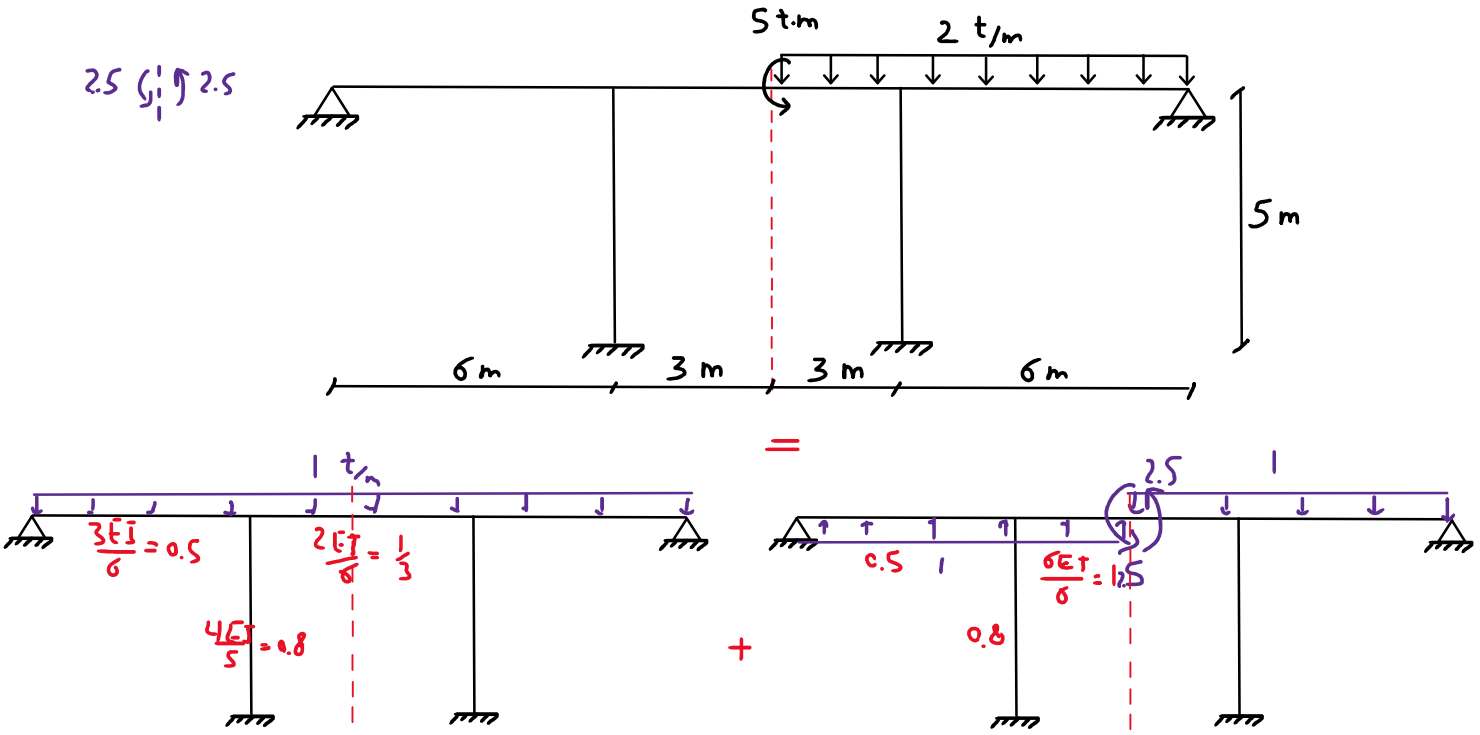
Video 28

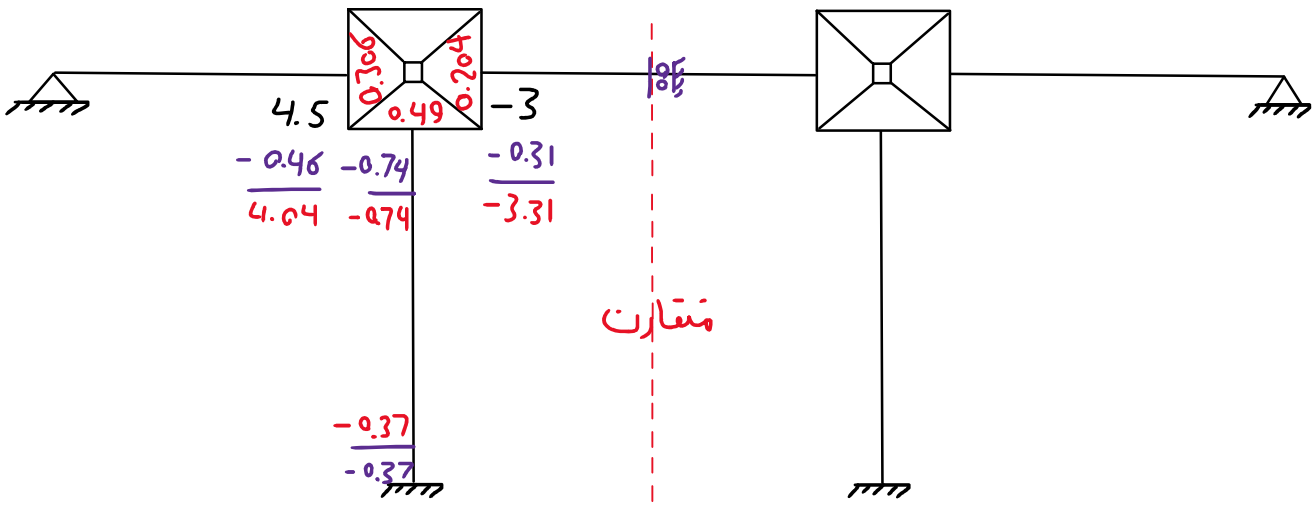
Symmetric Structures 5

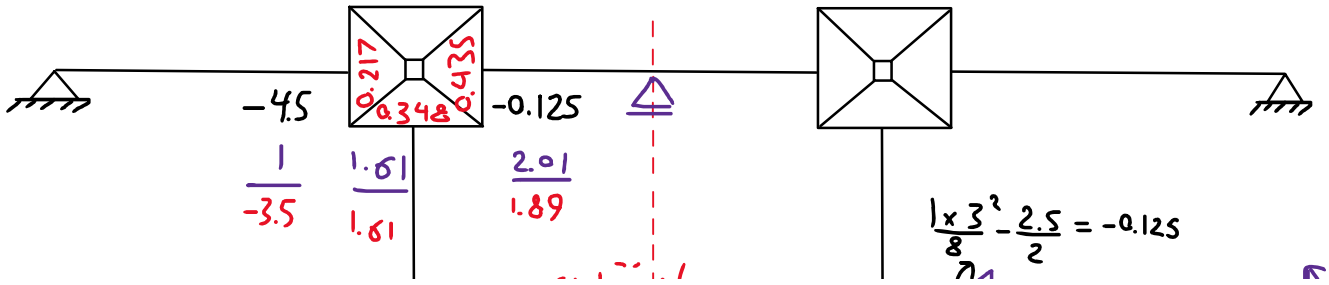
Sunday, May 12, 2024 8:58

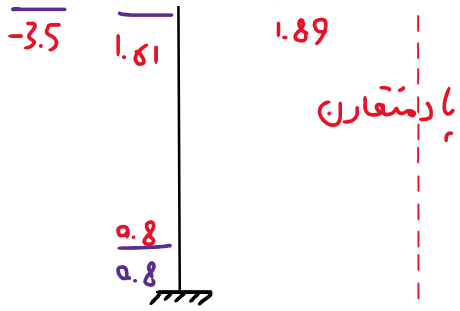
تحلیل سازه های نامتقارن

مثال: لنگرهای انتهایی اعضا را به دست آورید.

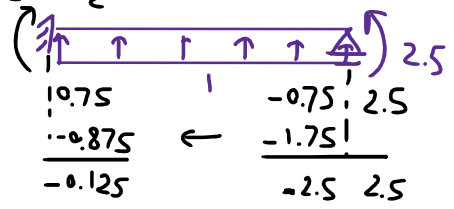


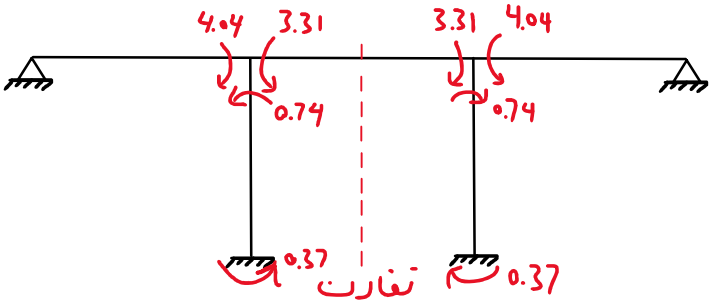




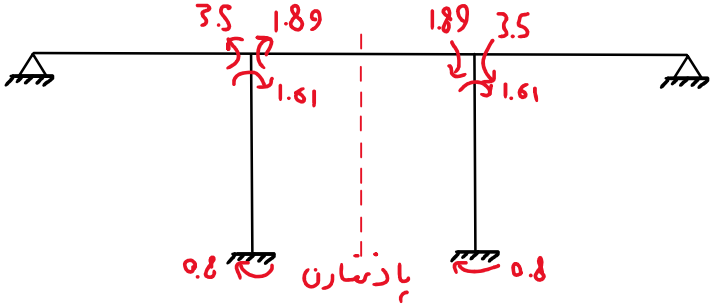


$\frac{1\times 3^2}{8} - \frac{2.5}{2} = -0.125$

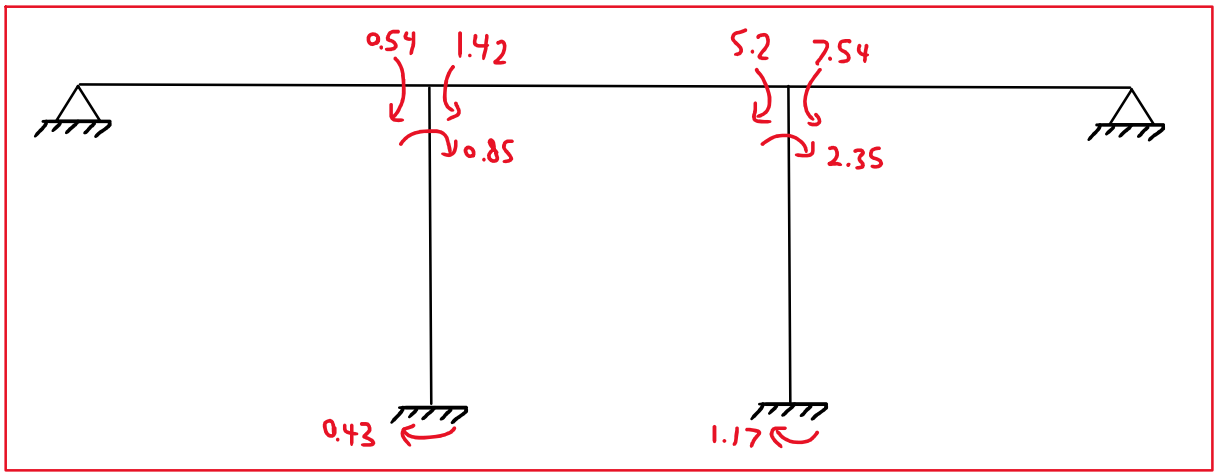




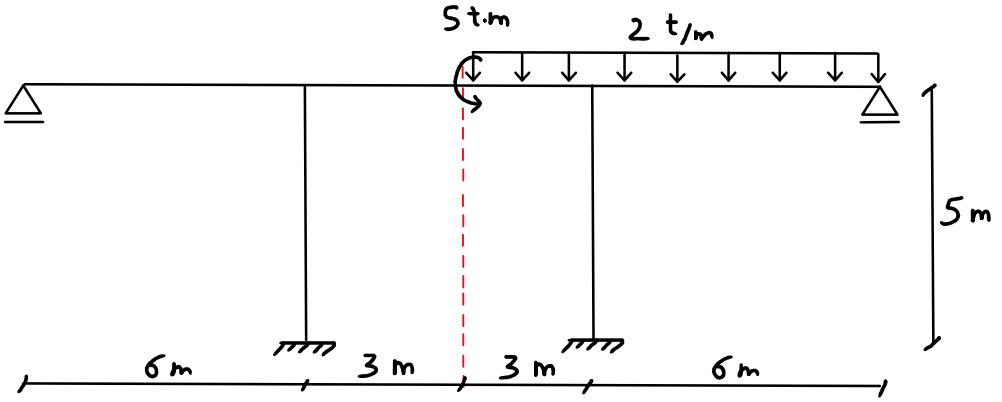
+



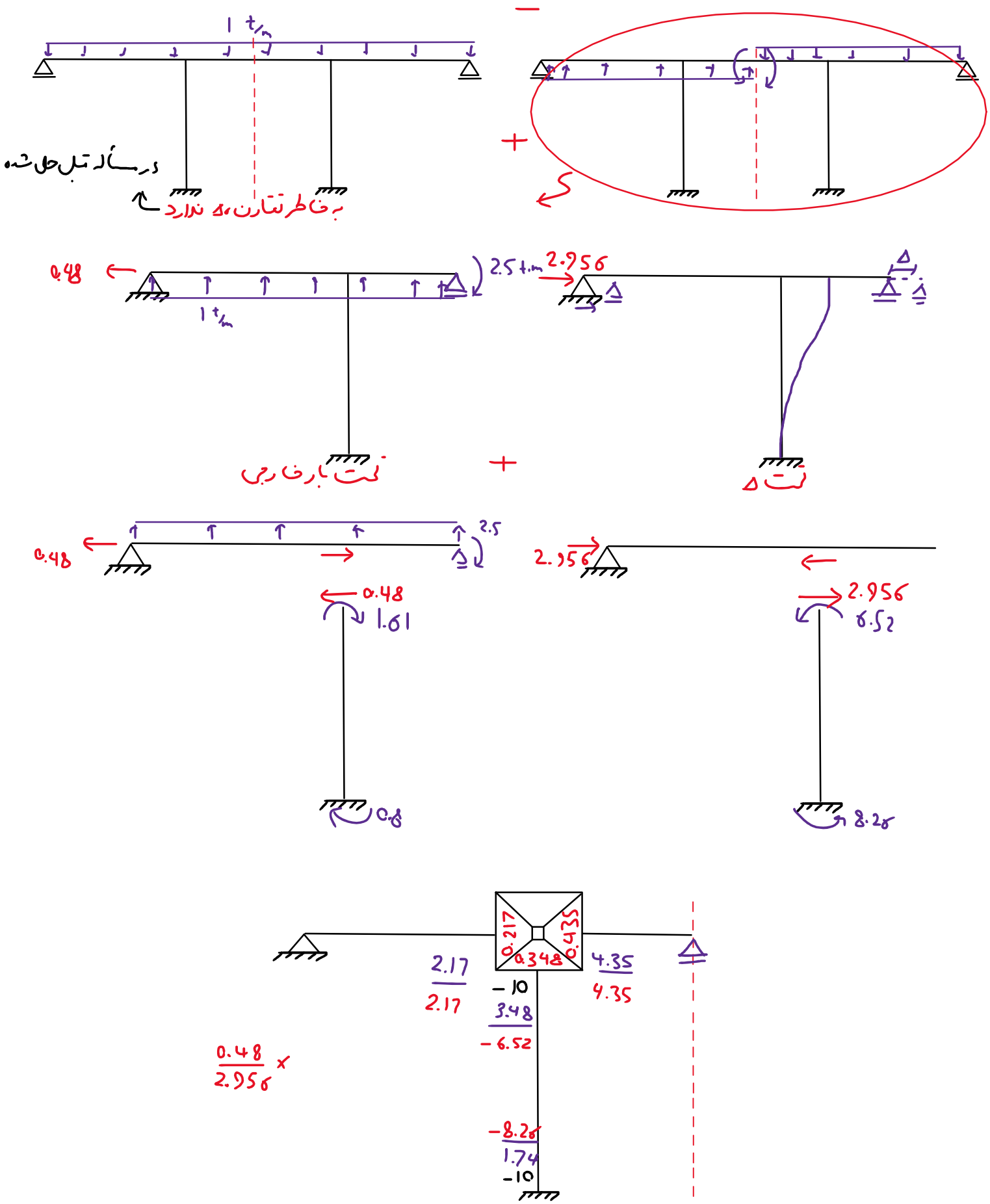
=

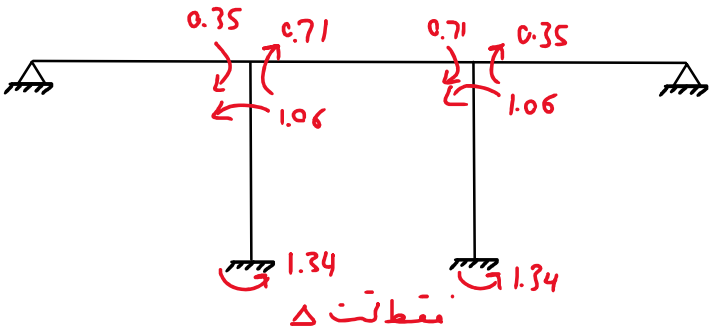


مثال: لنگرهای انتهایی اعضا را به دست آورید.

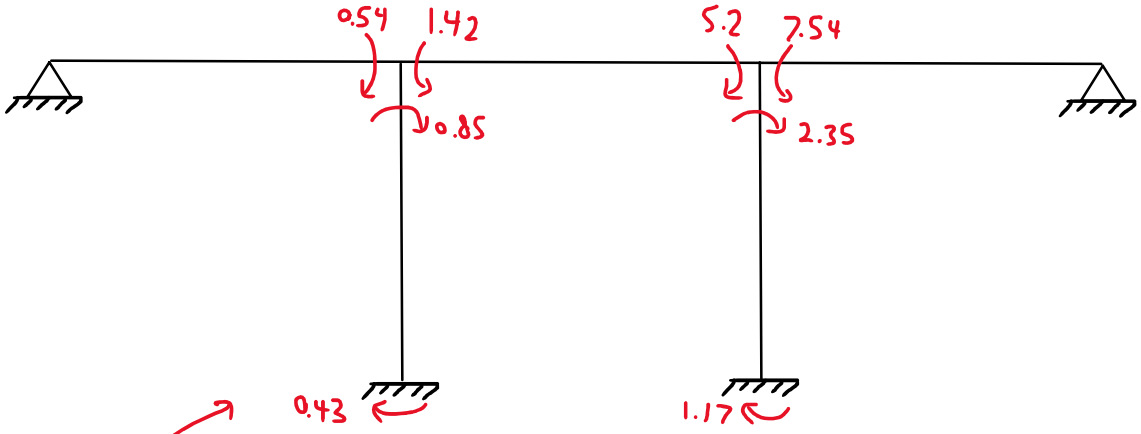


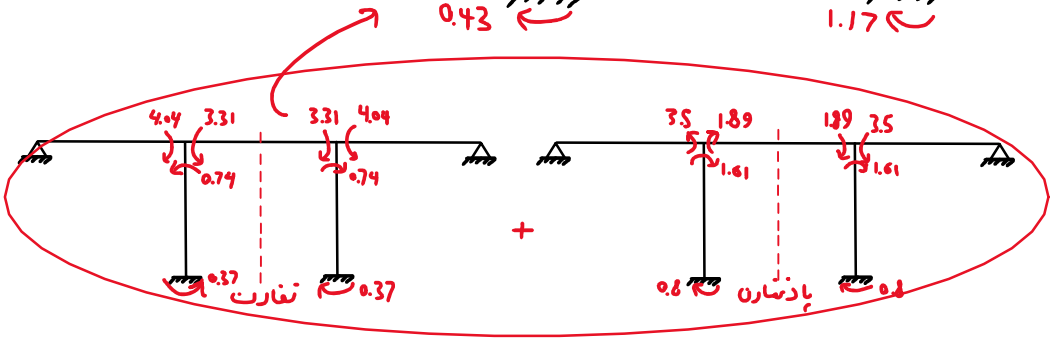
=

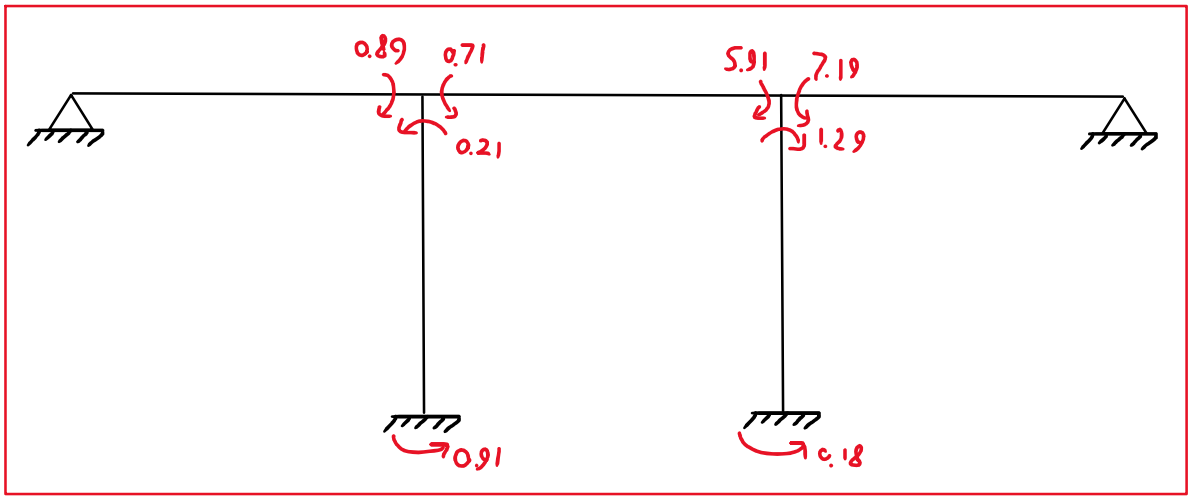

1 t/m
در مسأله قبل حل شده
به خاطر تقارن ها، ندارد
0.48
2.5 t.m
1 t/m
2.956
تست بار خارجی
تست Δ
0.48
2.5
0.48
1.61
0.8
2.956
2.956
0.52
8.28
0.217
0.348
0.435
2.17
2.17
4.35
4.35
-10
3.48
-6.52
0.48
2.956
x
-8.28
1.74
-10


0.35
0.71
0.71
0.35
1.06
1.06
1.34
1.34
مقطعات

+


0.54
1.42
5.2
7.54
0.85
2.35
0.43
1.17


4.04
3.31
3.31
4.04
0.74
0.74
0.37
0.37
تقارب
3.5
1.89
1.89
3.5
1.61
1.61
0.8
0.8
پاذشارن
+


0.89
0.71
5.91
7.19
0.21
1.29
0.91
0.18

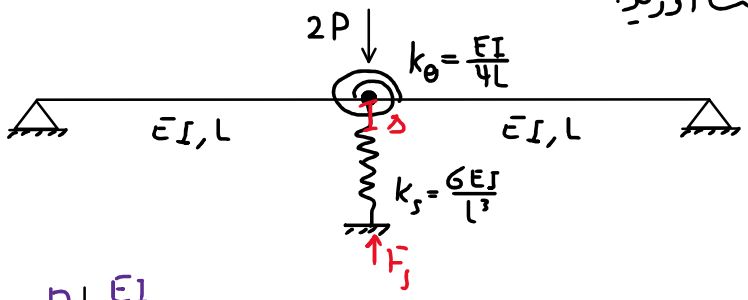
Video 29

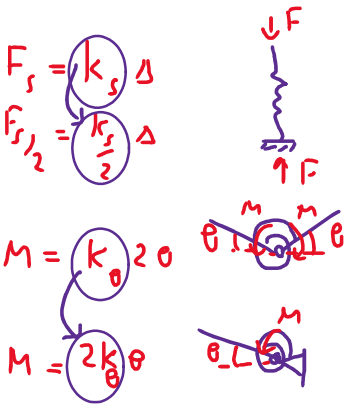
Symmetric Structures 6

Sunday, May 12, 2024 8:58

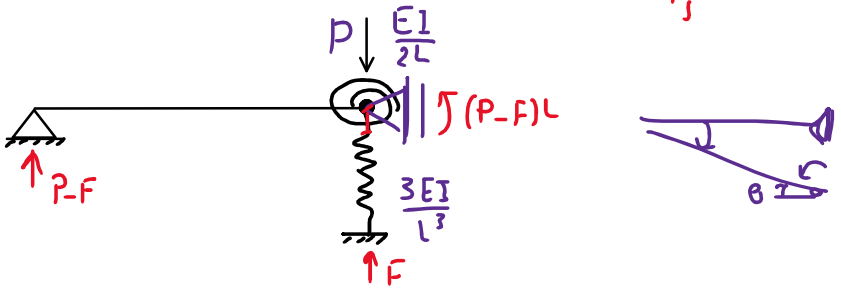
تحلیل سازه های متقارن

مثال: نیروی فنرها را به دست آورید.





روش شیب - انحنا



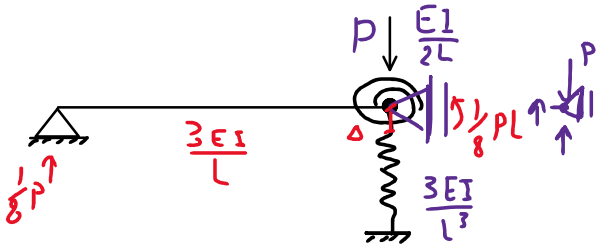
$$M = \frac{3EI}{L}\left(\theta - \frac{\Delta}{L}\right)$$

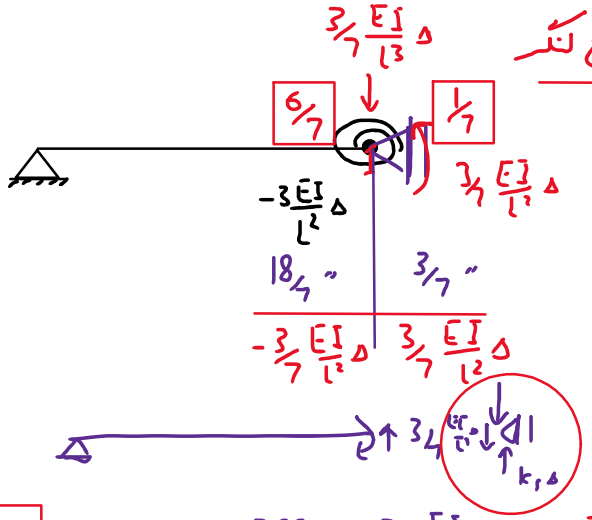
$$-(P-F)L = \frac{3EI}{L}\left(\frac{(P-F)L}{2EI} - \frac{F}{3EI \times L^3}\right) \rightarrow F - P = 3\left(2(P-F) - \frac{F}{3}\right)$$

$$F - P = 6P - 7F$$

$$\boxed{F_s = \frac{7}{8}P}, \quad \boxed{M_s = \frac{1}{8}PL}$$

روش توزیع لنگر





$$\frac{3}{7}\frac{EI}{L^3}\Delta + \frac{3EI}{L^3}\Delta = P \rightarrow \Delta = \frac{7}{24}\frac{PL^3}{EI}$$

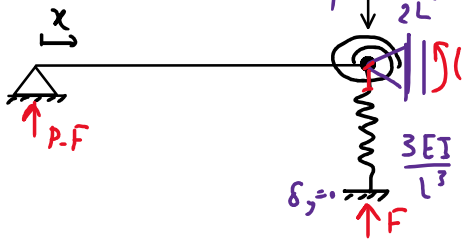
$$F_s = \frac{3EI}{L^3} \times \frac{7}{24}\frac{PL^3}{EI} = \boxed{\frac{7}{8}P}$$

$\Sigma F_y = 0$

$$\boxed{F_s = \frac{7}{8}P} \leftarrow \frac{3EI}{L^3}, \frac{3}{7}\frac{EI}{L^3}$$

مفهوم کنتی:

روش کاستیلیانو



$$U = \frac{1}{2}\int \frac{M^2}{EI}dx + \frac{1}{2}\frac{N^2}{k_s} + \frac{1}{2}\frac{M^2}{k_\theta}$$

$$\frac{\partial U}{\partial F} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial F}\right)dx + \frac{N}{k_s}\left(\frac{\partial N}{\partial F}\right) + \frac{M}{k_\theta}\left(\frac{\partial M}{\partial F}\right) = 0$$

$$\frac{1}{EI}\int_0^L (F-P)x^2\,dx + \frac{F}{k_s} + \frac{(F-P)L^2}{k_\theta} = 0$$

(1) $M = (P-F)x$ $\frac{\partial M}{\partial F} = -x$

$\delta_y = 0$ ↑F $\int \frac{M}{EI} \frac{\partial M}{\partial F} dx$ K_s $\frac{\partial M}{\partial F}$ K_θ $\frac{\partial F}{\partial F}$

$$\frac{1}{EI}\int_0^L (F-P)x^2\,dx + \frac{F}{k_s} + \frac{(F-P)L^2}{k_\theta} = 0$$

$$\frac{1}{EI}(F-P)\frac{x^3}{3}\Big|_0^L + \frac{FL^3}{3EI} + (F-P)\frac{2L^3}{EI} = 0$$

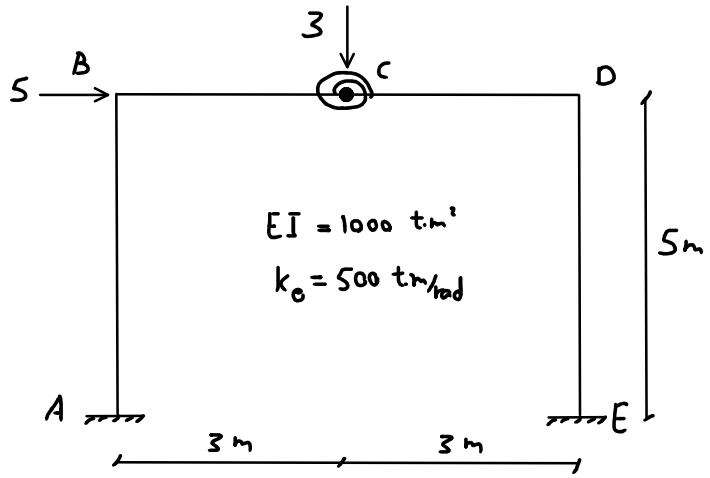
$$F = \frac{7}{8}P$$

① $M = (P-F)x$, $\frac{\partial M}{\partial F} = -x$

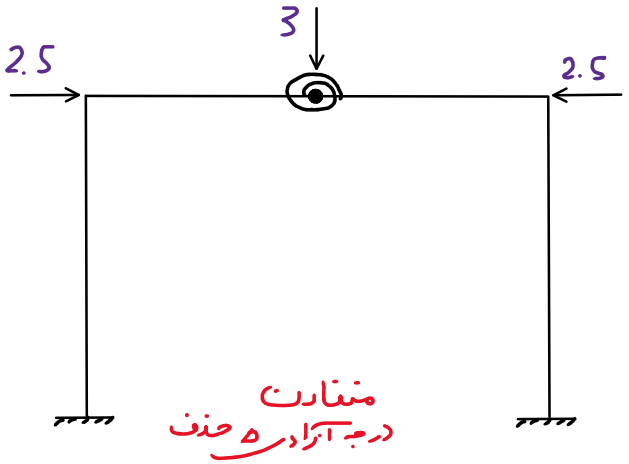
② $N = F$, $\frac{\partial N}{\partial F} = 1$

③ $M = (P-F)L$, $\frac{\partial M}{\partial F} = -L$

مثال: لنگرهای انتهایی اعضا را به دست آورید.



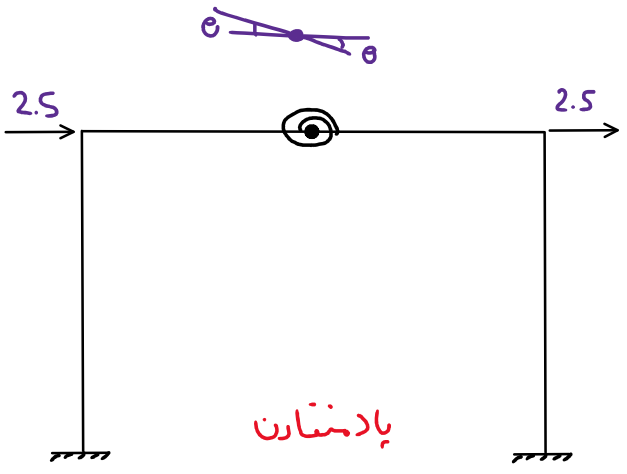
=



متقارن

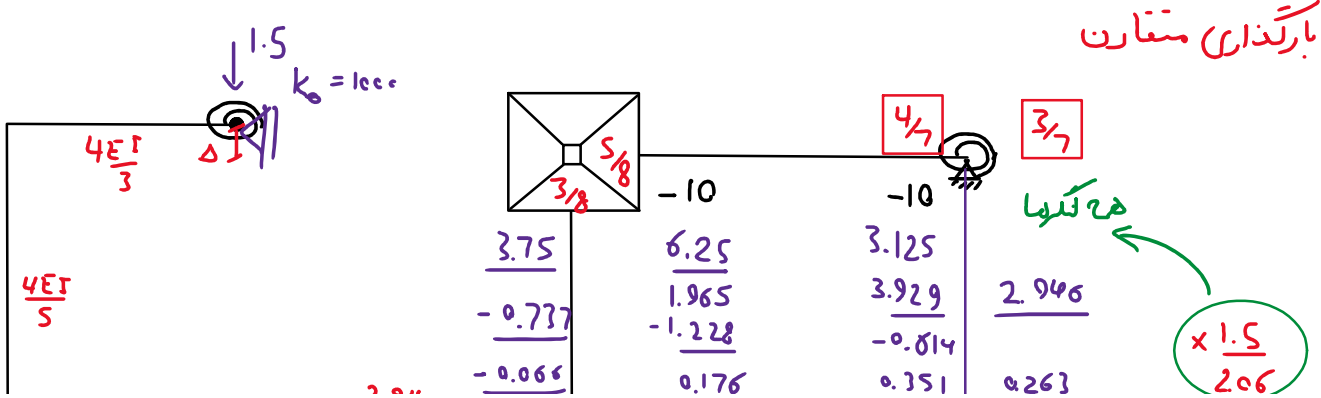
درجه آزادی حذف

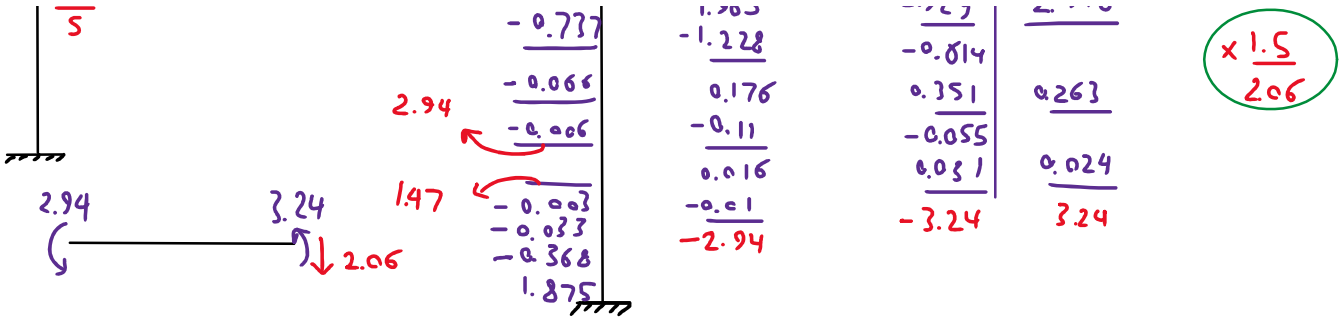
+



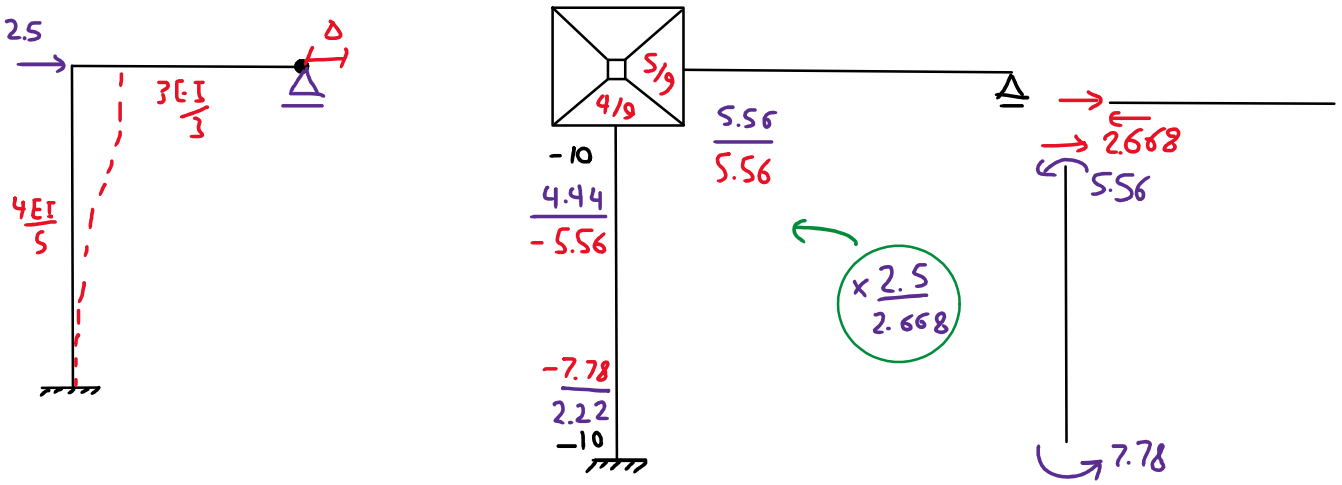
پادمتقارن

حل بارگذاری متقارن

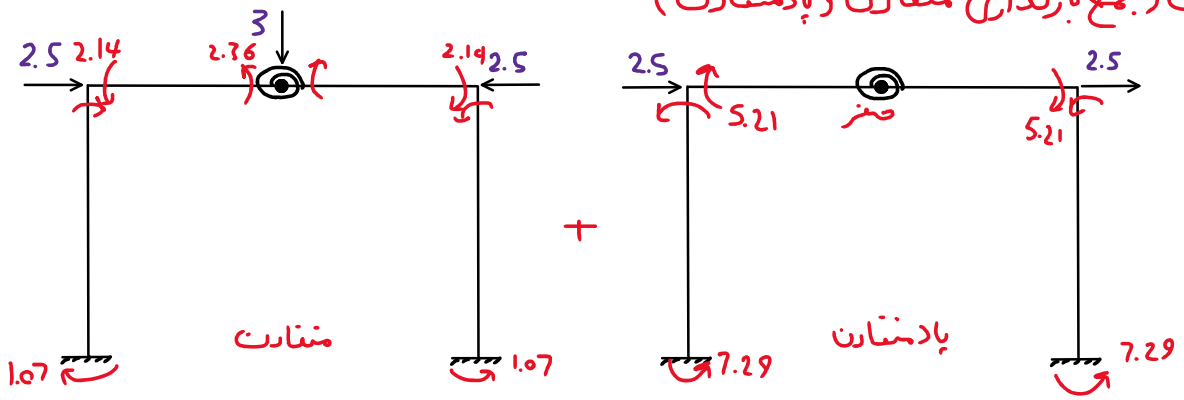


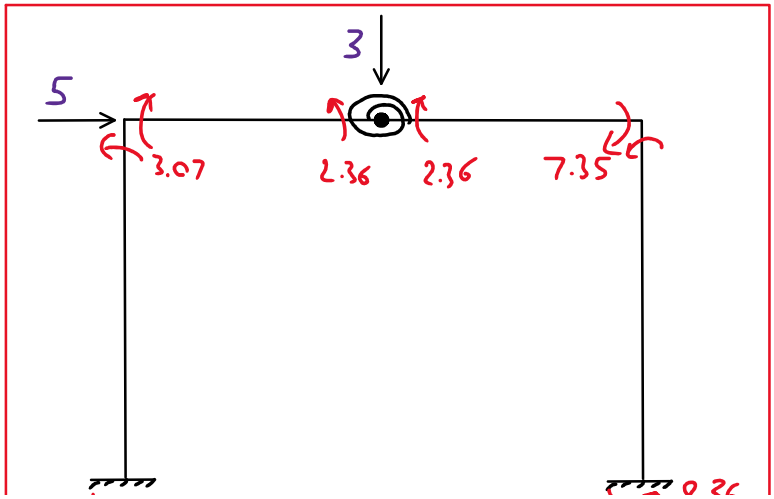


حل بارگذاری پادمتقارن



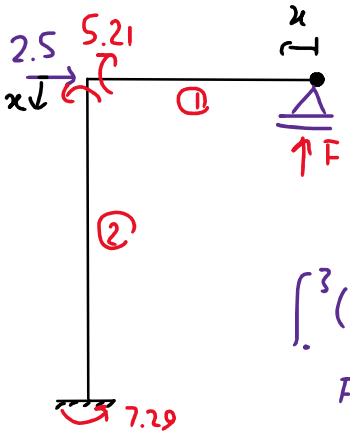
بارگذاری کل (جمع بارگذاری متقارن و پادمتقارن)





روش دیگر (روش کاستیلیانو)

حل بارگذاری یادمتقارن



$$\frac{\partial U}{\partial F} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial F}\right) dx = 0$$

① $M = Fx$ $\quad \frac{\partial M}{\partial F} = x$

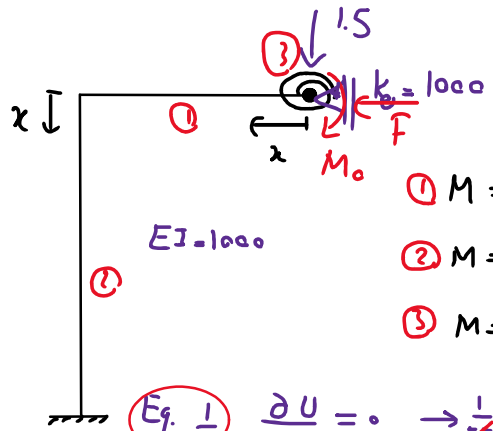
② $M = 3F - 2.5x$ $\quad \frac{\partial M}{\partial F} = 3$

$$\int_0^3 (Fx)(x)\,dx + \int_0^5 (3F - 2.5x)(3)\,dx = 0$$

$$F\frac{x^3}{3}\Big|_0^3 + \left(9Fx - \frac{7.5}{2}x^2\right)\Big|_0^5 = 0 \rightarrow 9F + 45F - 93.75 = 0$$

$$\boxed{F = 1.736}$$

حل بارگذاری متقارن



$$U = \frac{1}{2}\int \frac{M'}{EJ}dx + \frac{1}{2}\frac{M^2}{k_0}$$

$\begin{cases} \delta_k = 0 \rightarrow \frac{\partial U}{\partial F} = 0 \\ \theta = 0 \rightarrow \frac{\partial U}{\partial M_0} = 0 \end{cases}$

		$\frac{\partial M}{\partial M_0}$	$\frac{\partial M}{\partial F}$
①	$M = -M_0 - 1.5x$	$= -1$	$= 0$
②	$M = -M_0 - 4.5 + Fx$	$= -1$	$= x$
③	$M = M_0$	$= 1$	0

Eq. 1 $\quad \frac{\partial U}{\partial M_0} = 0 \rightarrow \frac{1}{EI}\int_0^3 (M_0 + 1.5x)\,dx + \frac{1}{EI}\int_0^5 (M_0 + 4.5 - Fx)\,dx + \frac{M_0}{k_0} = 0$

$$\left(M_0 x + 1.5\frac{x^2}{2}\right)\Big|_0^3 + \left(M_0 x + 4.5x - F\frac{x^2}{2}\right)\Big|_0^5 + M_0 = 0$$

$$3M_0 + 6.75 + 5M_0 + 22.5 - 12.5F + M_0 = 0 \rightarrow * \quad 9M_0 - 12.5F = -29.25$$

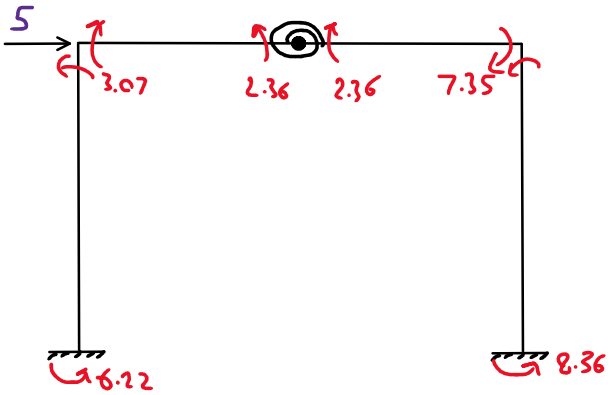
Eq. 2 $\quad \frac{\partial U}{\partial F} = 0 \rightarrow 0 + \frac{1}{EI}\int_0^5 (-M_0 x - 4.5x + Fx^2)\,dx + 0 = 0$

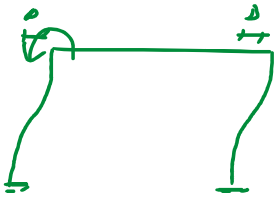
$$\left(-M_0\frac{x^2}{2} - \frac{4.5}{2}x^2 + F\frac{x^3}{3}\right)\Big|_0^5 = 0 \rightarrow * \quad -12.5M_0 + 41.67F = 56.25$$

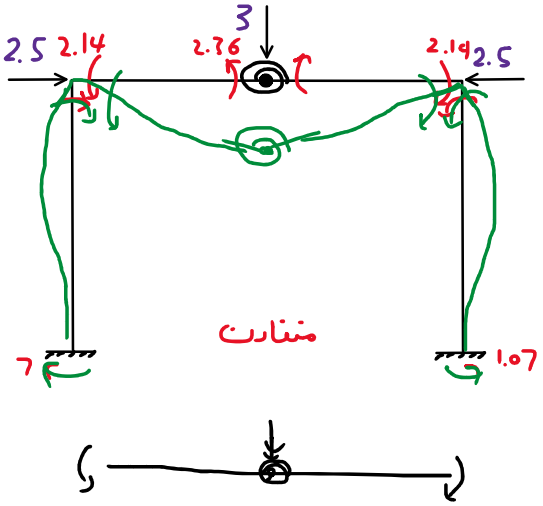
$$\begin{cases} 9M_0 - 12.5F = -29.25 \\ -12.5M_0 + 41.67F = 56.25 \end{cases} \quad (\times 12.5,\ \times 9)$$

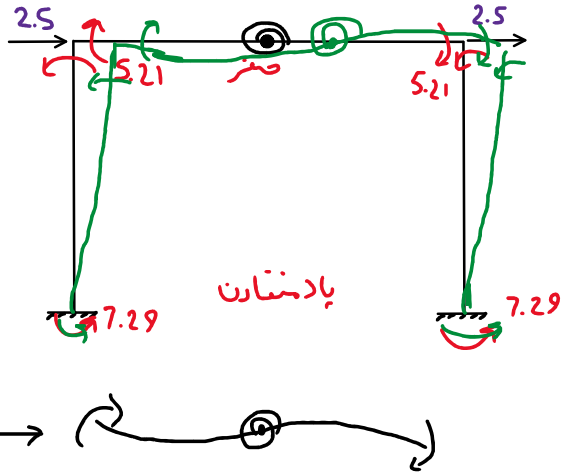
$$218.78F = 140.625 \rightarrow \boxed{F = 0.643}$$

$$\boxed{M_0 = -2.36}$$


5
3.07
2.36
2.36
7.35
6.22
8.36




3
2.5
2.14
2.36
2.14
2.5
7
1.07
منفارت

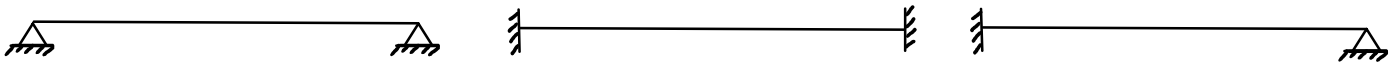

2.5
2.5
5.21
5.21
7.29
7.29
بادمنفارن

Video 30

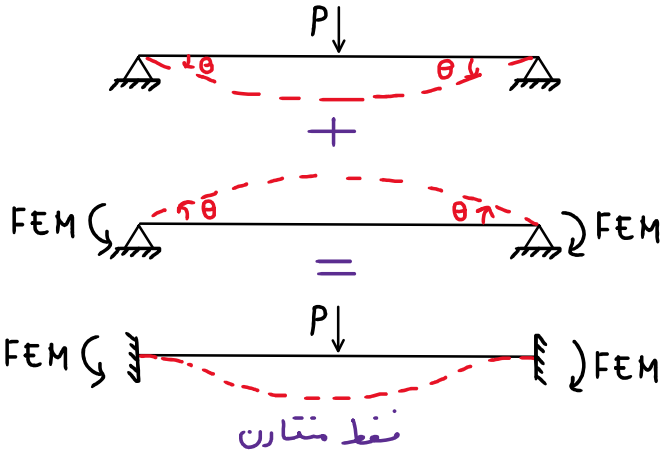
Symmetric Structures 7

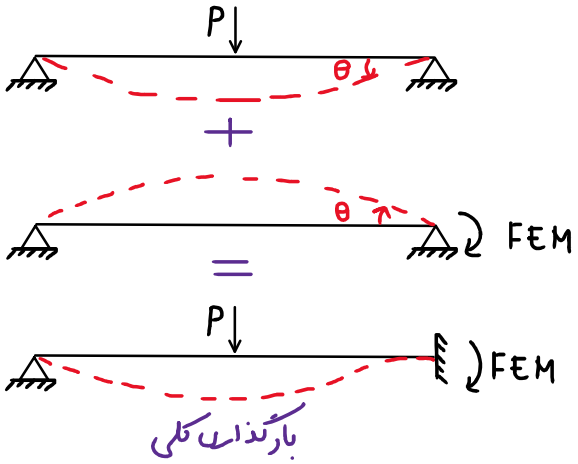
Sunday, May 12, 2024 8:58

محاسبه تغییرشکل θ و Δ در تیرهای متداول

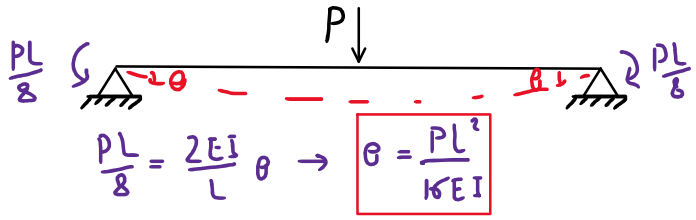


روش محاسبه θ

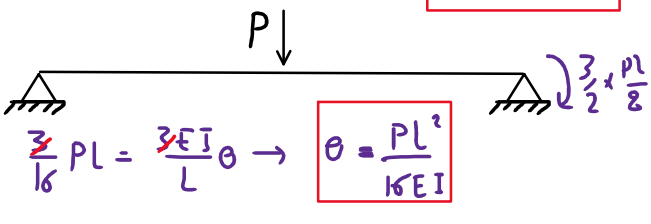




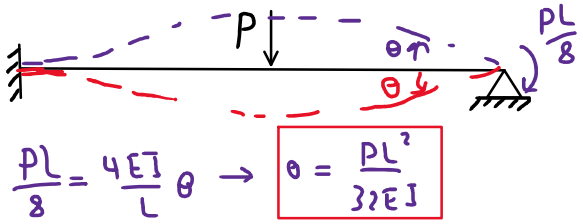
محاسبه θ



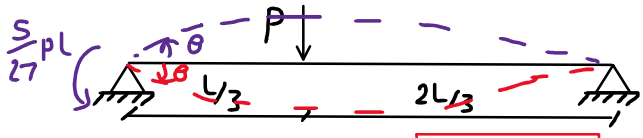
$$\frac{PL}{8} = \frac{2EI}{L}\theta \rightarrow \boxed{\theta = \frac{PL^2}{16EI}}$$



$$\frac{3}{16}PL = \frac{3EI}{L}\theta \rightarrow \boxed{\theta = \frac{PL^2}{16EI}}$$

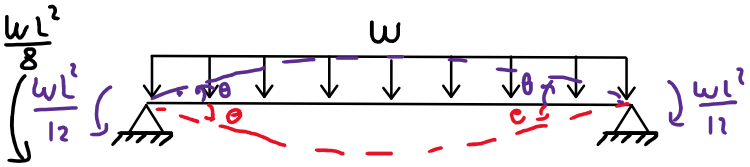


$$\frac{PL}{8} = \frac{4EI}{L}\theta \rightarrow \boxed{\theta = \frac{PL^2}{32EI}}$$



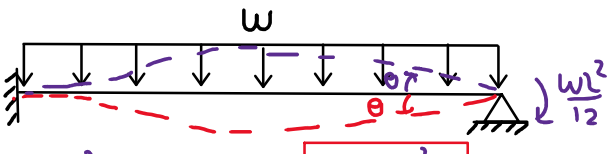
$$\frac{5}{27}PL = \frac{3EI}{L}\theta \rightarrow \boxed{\theta = \frac{5}{81}\frac{PL^2}{EI}}$$

$$\frac{P(\frac{L}{3})(\frac{2L}{3})^2}{L^2} = \frac{4}{27}PL \qquad \frac{P(\frac{2L}{3})(\frac{L}{3})^2}{L^2} = \frac{2}{27}PL$$

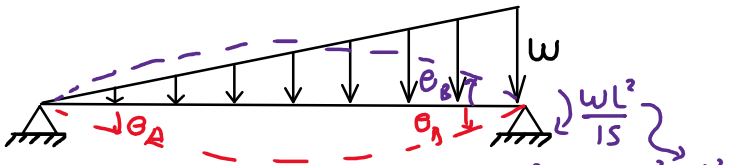


$$\frac{wL^2}{12} = \frac{2EI}{L}\theta \rightarrow \boxed{\theta = \frac{wL^3}{24EI}}$$

$$\frac{wL^2}{8} = \frac{3EI}{L}\theta \rightarrow \theta = \frac{wL^3}{24EI}$$

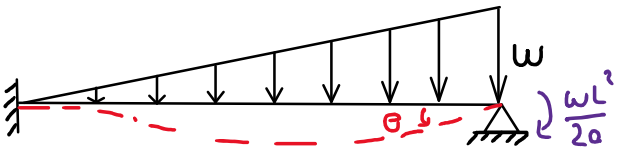


$$\frac{wL^2}{12} = \frac{4EI}{L}\theta \rightarrow \boxed{\theta = \frac{wL^3}{48EI}}$$



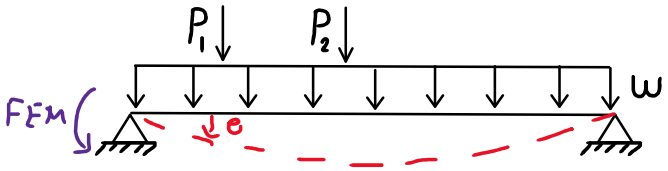
$\frac{wL^2}{15} = \frac{3EI}{L}\theta_B \rightarrow \theta_B = \frac{wL^3}{45EI}$

$\frac{wL^2}{20} + \frac{1}{2}\times\frac{wL^2}{30} = \frac{wL^2}{15}$



$\frac{wL^2}{20} = \frac{4EI}{L}\theta \rightarrow \theta = \frac{wL^3}{80EI}$

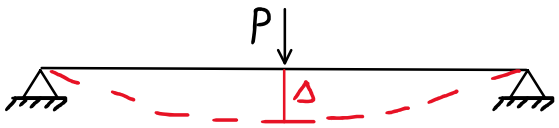
$\frac{7}{120}wL^2 = \frac{3EI}{L}\theta_A \rightarrow \theta_A = \frac{7}{360}\frac{wL^3}{EI}$



$FEM = \frac{3EI}{L}\theta$

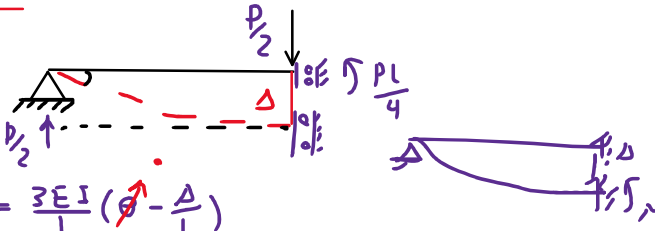
محاسبه Δ

روش محاسبه Δ



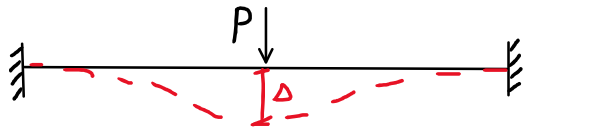
$\Delta = \frac{(\frac{P}{2})(\frac{L}{2})^3}{3EI} = \frac{PL^3}{48EI}$

$\theta = \frac{\frac{P}{2}(\frac{L}{2})^2}{2EI} = \frac{PL^2}{16EI}$

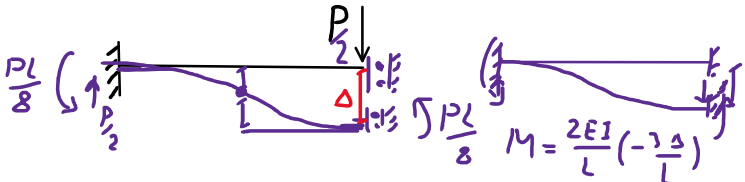


$M = \frac{3EI}{L}(\theta - \frac{\Delta}{L})$

$\frac{PL}{4} = \frac{3EI}{(\frac{L}{2})^2}\Delta \rightarrow \Delta = \frac{PL^3}{48EI}$

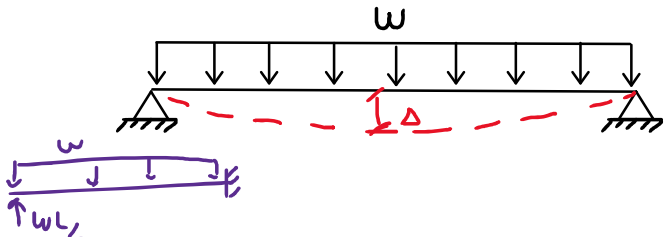


$\Delta = 2\times\frac{\frac{P}{2}(\frac{L}{4})^3}{3EI} = \frac{1}{192}\frac{PL^3}{EI}$

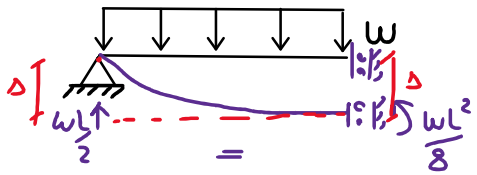


$M = \frac{2EI}{L}(-\frac{3\Delta}{L})$

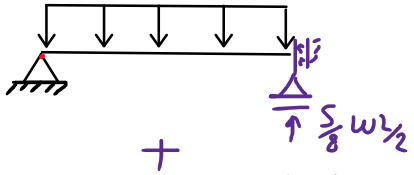
$\frac{PL}{8} = \frac{6EI}{(\frac{L}{2})^2}\Delta \rightarrow \Delta = \frac{1}{192}\frac{PL^3}{EI}$



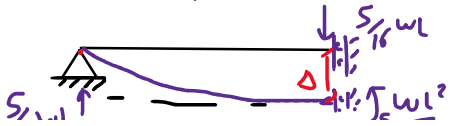
$\Delta = \frac{(\frac{wL}{2})(\frac{L}{2})^3}{3EI} - \frac{w(\frac{L}{2})^4}{8EI} = \frac{8-3}{16\times3\times8}\frac{wL^4}{EI} = \frac{5}{384}\frac{wL^4}{EI}$



=

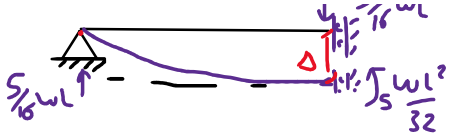


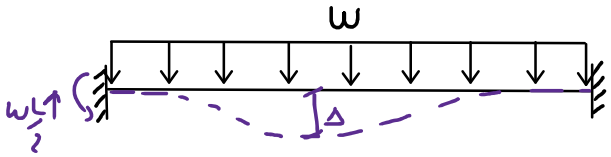
+

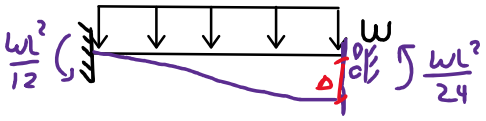


$\frac{5}{32}wL^2 = \frac{3EI}{(\frac{L}{2})^2}\Delta \rightarrow \Delta = \frac{5}{384}\frac{wL^4}{EI}$

$\frac{5wL^2}{32} = \frac{3EI}{(L/2)^2}\Delta \rightarrow \boxed{\Delta = \frac{5}{384}\frac{wL^4}{EI}}$



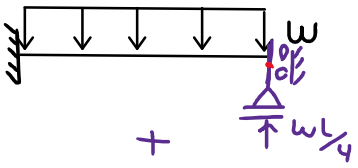




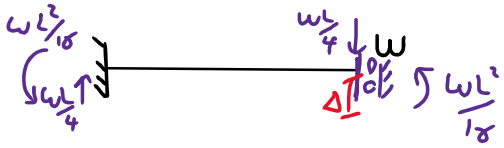
$\Delta = \frac{w(L/2)^4}{8EI} - \frac{wL^2}{24}\frac{(L/2)^2}{2EI} = \frac{1}{16\times 8} - \frac{1}{8\times 24} =$

$\frac{3-2}{16\times 24}$

$\boxed{\Delta = \frac{1}{384}\frac{wL^4}{EI}}$



$\frac{wL}{4} = \frac{12EI}{(L/2)^3}\Delta \rightarrow \boxed{\Delta = \frac{1}{384}\frac{wL^4}{EI}}$



$\frac{wL^2}{16} = \frac{6EI}{(L/2)^2}\Delta \rightarrow \boxed{\Delta = \frac{1}{384}\frac{wL^4}{EI}}$

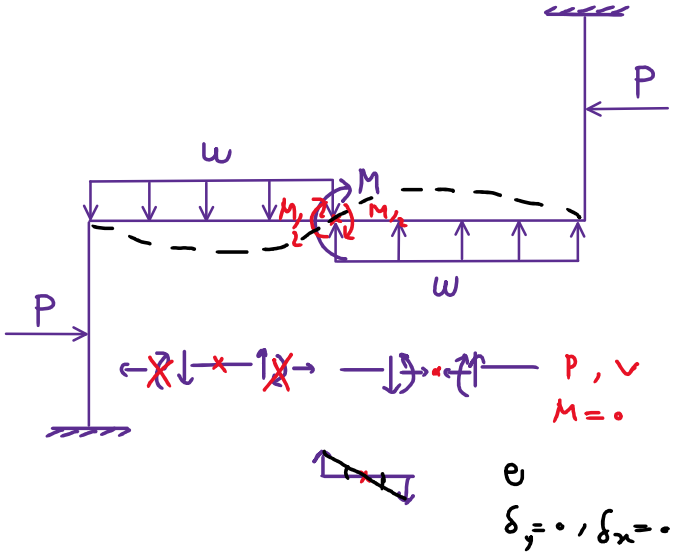
Video 31

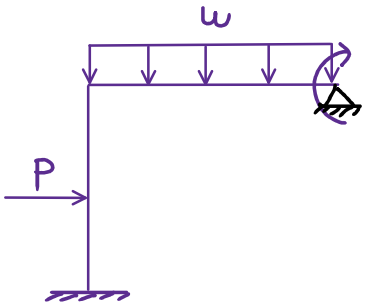
Symmetric Structures 8

Sunday, May 12, 2024 8:58

تقارن نقطه‌ای

تقارن مرکزی مستقیم

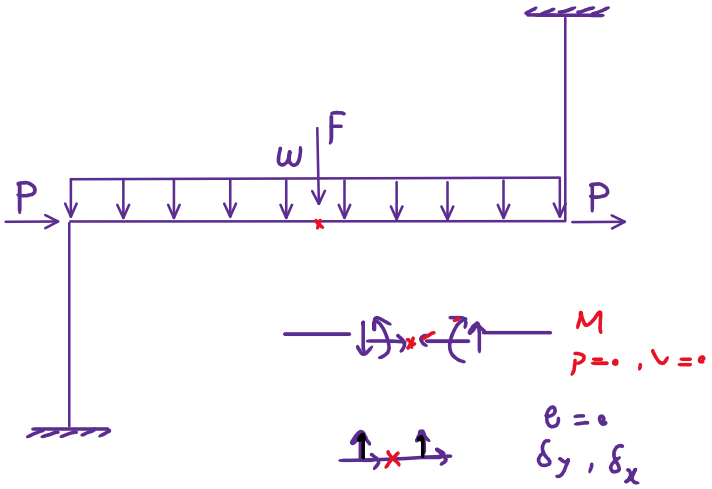


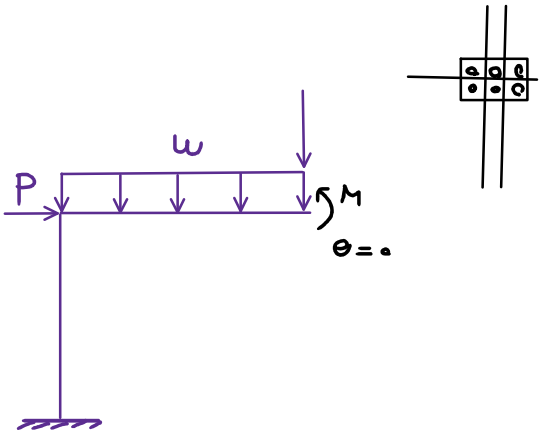


e

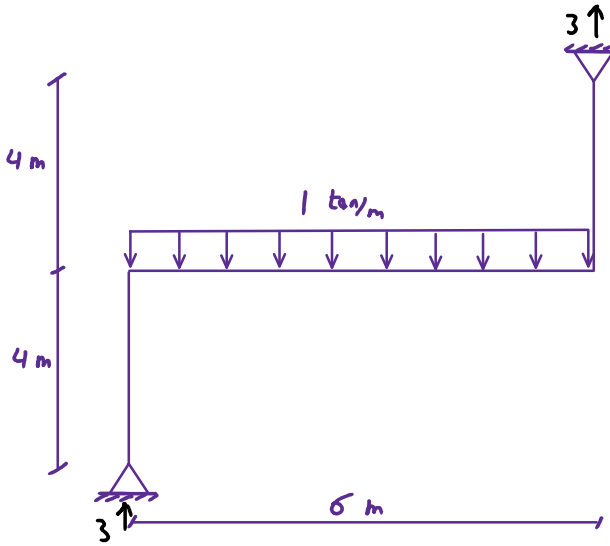
$\delta_y = 0 , \delta_x = 0$

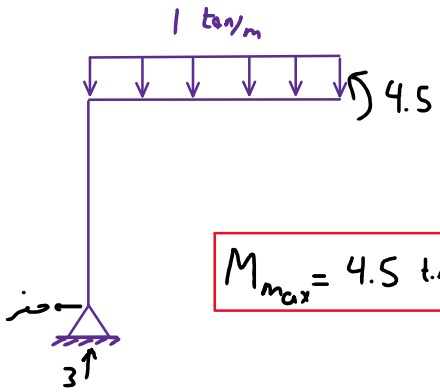
تقارن مرکزی معکوس





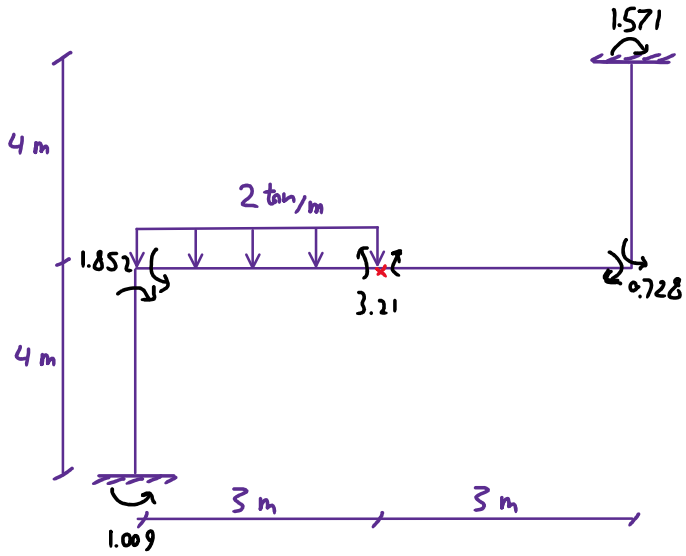
مثال : لنگر ماکزیمم در سازه شکل زیر را به دست آورید.



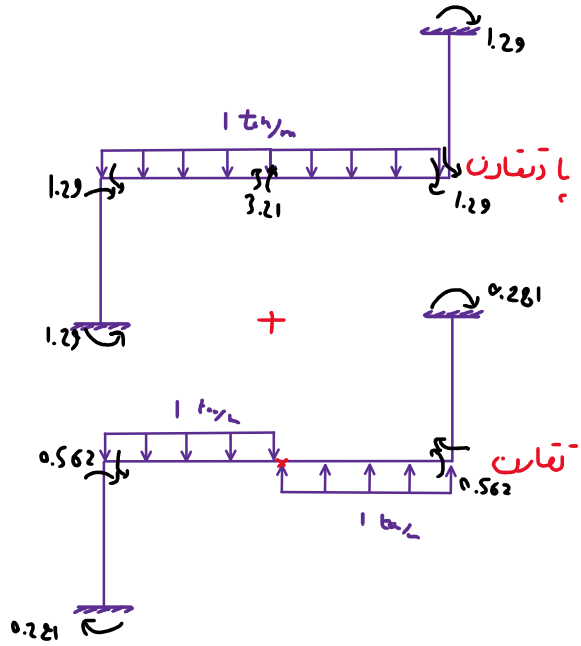


$M_{max} = 4.5$ t.m

مثال : لنگرهای انتهایی اعضا را به دست آورید.



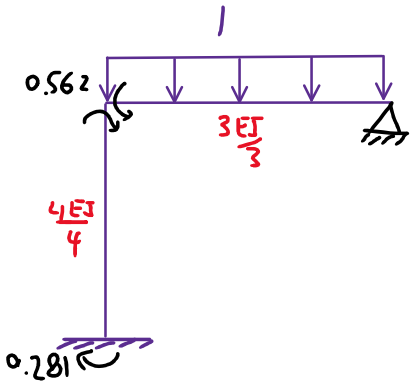
=

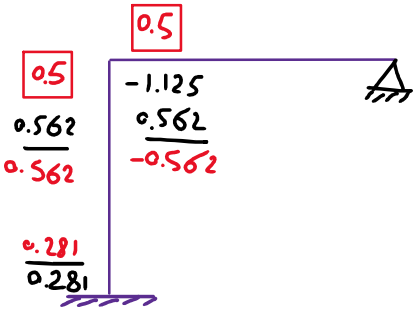


با تنانون

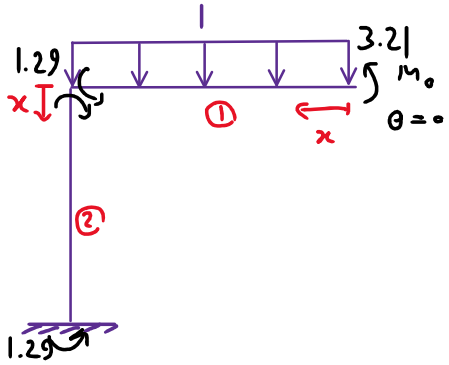
تنانون

تنانون نقطه‌ای مستقیم :





تنانون نقطه‌ای معکوس
روش کاستیانو



$$\theta = \frac{\partial U}{\partial M_0} = \int \frac{M}{EI}\left(\frac{\partial M}{\partial M_0}\right) dx = 0$$

① $M = M_0 - \frac{x^2}{2}$, $\frac{\partial M}{\partial M_0} = 1$

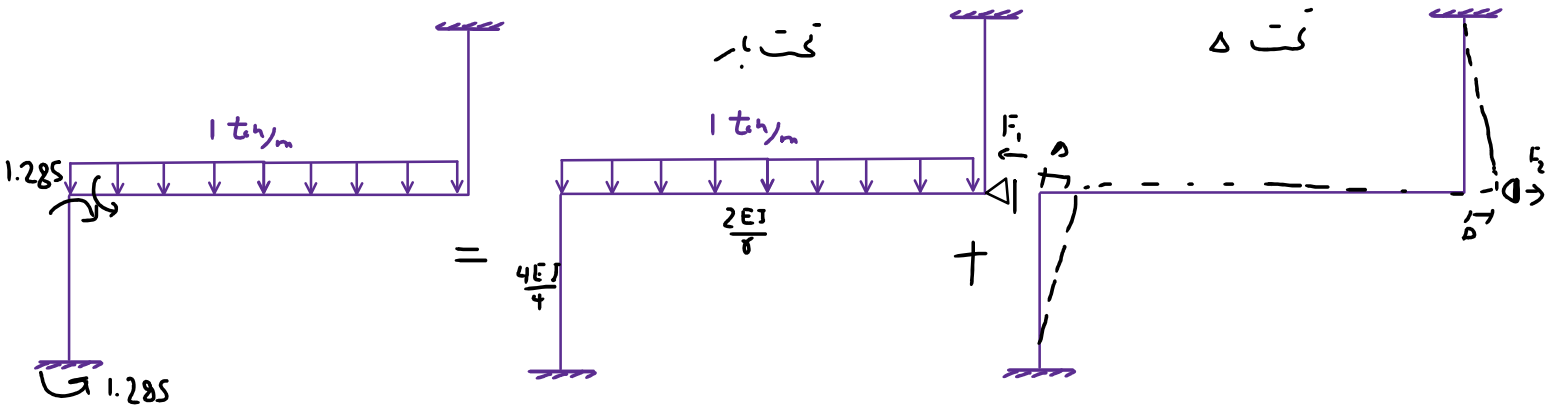
② $M = M_0 - 4.5$, $\frac{\partial M}{\partial M_0} = 1$

$$\int_0^3 \left(M_0 - \frac{x^2}{2}\right) dx + \int_0^4 (M_0 - 4.5)\, dx = 0$$

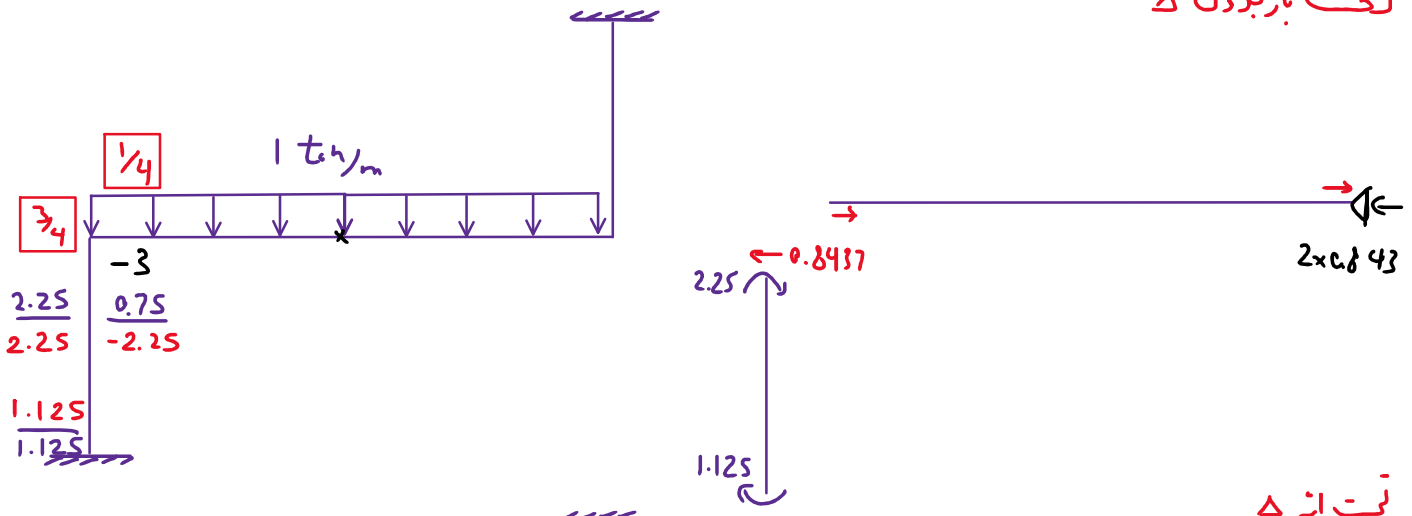
$$\left(M_0 x - \frac{x^3}{6}\right)\Big|_0^3 + (M_0 x - 4.5x)\Big|_0^4 = 0$$

$$3M_{\circ} - 4.5 + 4M_{\circ} - 18 = 0 \rightarrow 7M_{\circ} = 22.5 \rightarrow \boxed{M_{\circ} = 3.21\ t.m}$$

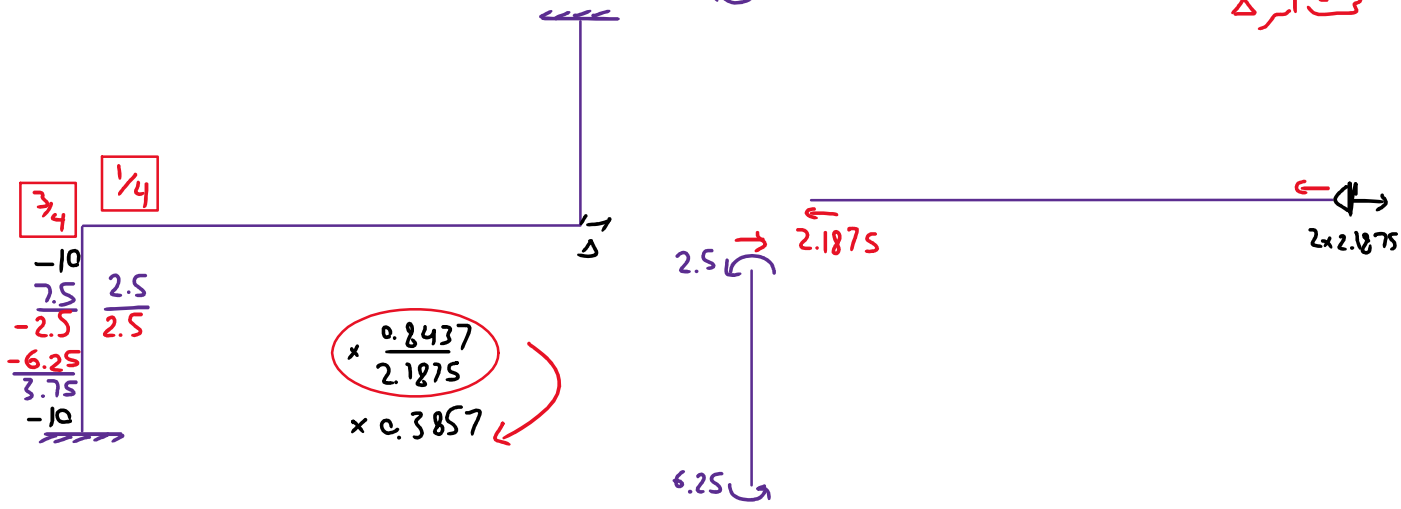
روش کرانی



تحت باربردن Δ



تحت اثر Δ

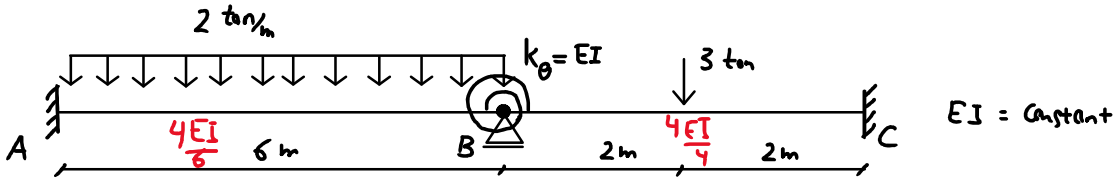


Video 32

Symmetric Structures 9

Sunday, May 12, 2024 8:58

مثال : نیروی منزها را به دست آورید.



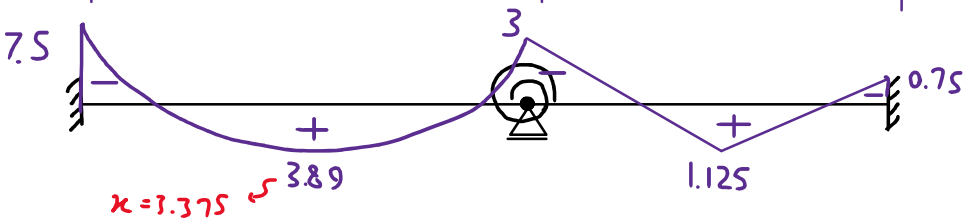
$\frac{2}{3}$, 1

1 , 1

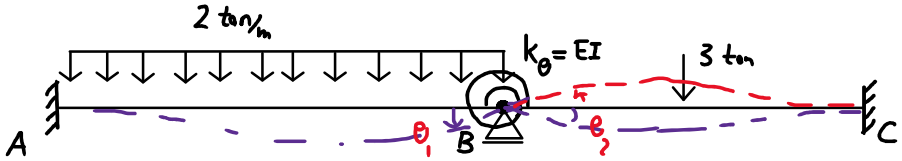
① روش توزیع لنگر

تیر	ستون	ستون	تیر
0.4	0.6	0.5	0.5

	-6	6		-1.5	1.5
	-1.2	-2.4	-3.6	3.6	
		1.05		-1.05	-1.05 → -0.53
Cycle 1					
	-0.21	-0.42	-0.63	0.63	
		0.31		-0.31	-0.31 → -0.16
Cycle 2					
	-0.06	-0.12	-0.19	0.19	
		0.1		-0.1	-0.1 → -0.05
Cycle 3					
	-0.02	-0.04	-0.06	0.06	
		0.03		-0.03	-0.03 → -0.02
Cycle 4					
	-0.01	-0.01	-0.02	0.02	
		0.01		-0.01	-0.01 → -0.01
Cycle 5					
	-7.5	3	-3	3	-3 0.75



② روش شیب - انحنا



2θ

$k_\theta \Delta\theta$

$$\begin{cases} M_{BA} + M_S = 0 \rightarrow \frac{2EI}{6}(2\theta_1) + 6 + EI(\theta_1 - \theta_2) = 0 \\ M_{BC} + M_S = 0 \rightarrow \frac{2EI}{4}(2\theta_2) - 1.5 + EI(\theta_2 - \theta_1) = 0 \end{cases} \rightarrow 2 \begin{cases} \frac{5}{3}\theta_1 - \theta_2 = \frac{-6}{EI} \\ -\theta_1 + 2\theta_2 = \frac{1.5}{EI} \end{cases}$$

$$\begin{cases} M_{BC} + M_s = 0 \\ \frac{2EI}{4}(2\theta_2) - 1.5 + EI(\theta_2 - \theta_1) = 0 \end{cases} \rightarrow \begin{cases} -\theta_1 + 2\theta_2 = \frac{1.5}{EI} \end{cases}$$

$$\frac{7}{3}\theta_1 = \frac{-10.5}{EI} \rightarrow \boxed{\theta_1 = \frac{-4.5}{EI}} \quad \boxed{\theta_2 = \frac{-1.5}{EI}}$$

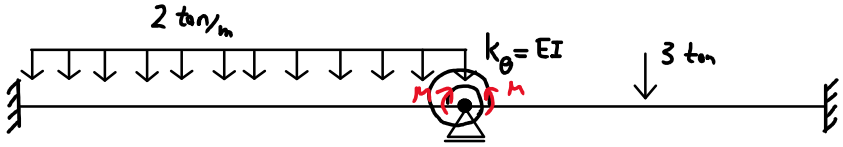
$$M_s = k_\theta \Delta\theta = EI(-4.5 + 1.5)/EI = 3$$

$$M_{AB} = \frac{2EI}{6}\left(\frac{-4.5}{EI}\right) - 6 = -7.5$$

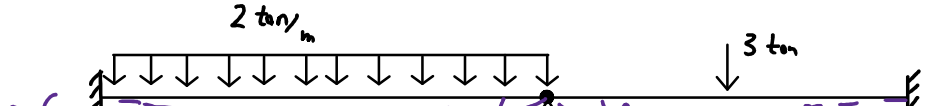
$$M_{CB} = \frac{2EI}{4}\left(\frac{-1.5}{EI}\right) + 1.5 = 0.75$$

$$\boxed{M_s = 3} \quad \boxed{M_{AB} = -7.5} \quad \boxed{M_{CB} = 0.75}$$

③ روش سازگاری تغییر شکل ها



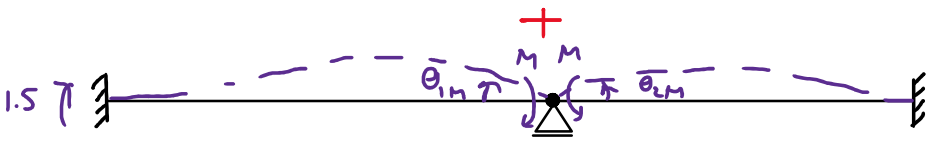
=



$$\frac{2EI}{6}(2\theta_{1P}) + 6 = 0 \rightarrow \theta_{1P} = \frac{-9}{EI}$$

$$\frac{2EI}{4}(2\theta_{2P}) - 1.5 = 0 \rightarrow \theta_{2P} = \frac{1.5}{EI}$$

+



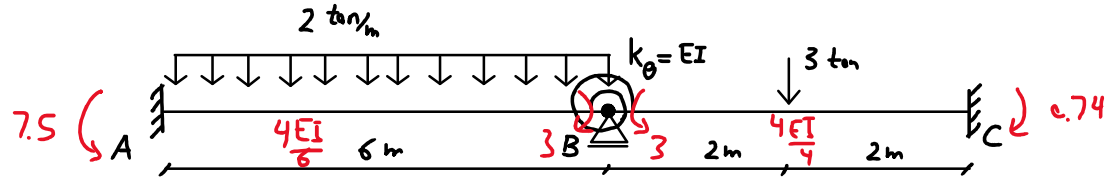
$$M = \frac{2EI}{6}(2\theta_{1M}) \rightarrow \theta_{1M} = \frac{1.5M}{EI}$$

$$\theta_{2M} = \frac{M}{EI}$$

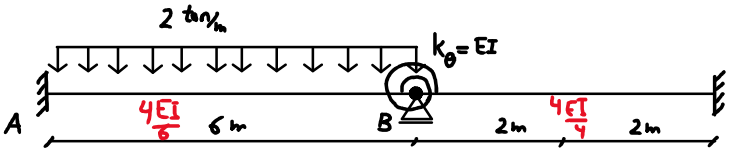
$\Delta\theta = \frac{M}{k_\theta}$ معادله سازگاری

$$\Delta\theta = \Delta\theta_P + \Delta\theta_M = \frac{M}{k_\theta} \rightarrow \frac{10.5}{EI} - \frac{2.5M}{EI} = \frac{M}{EI} \rightarrow \boxed{M = 3}$$

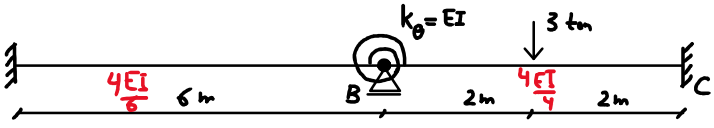
④ روش توزیع لنگر برای حالت خاص



=



+



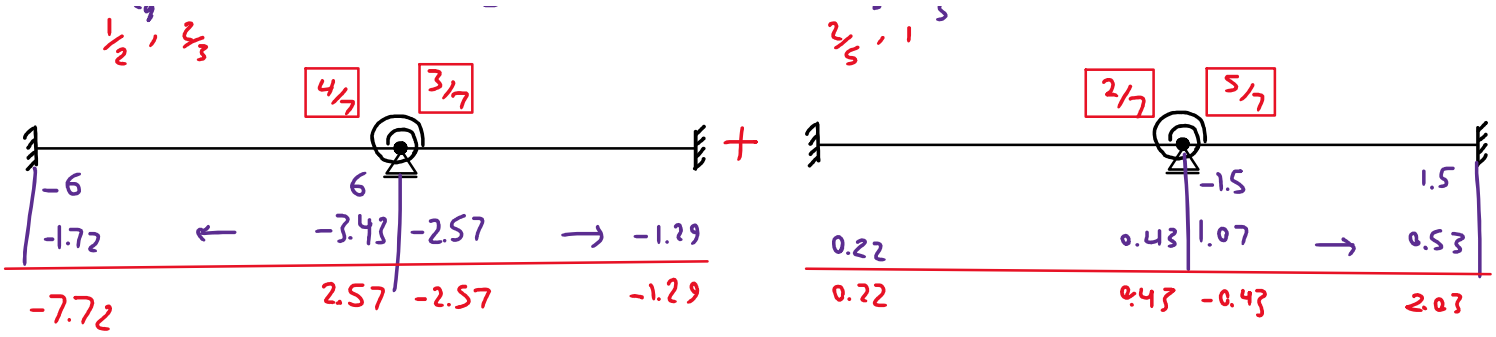
$$\frac{1}{k_{eq}} = \frac{1}{EI} + \frac{1}{EI} \rightarrow k_{eq} = \frac{EI}{2}$$

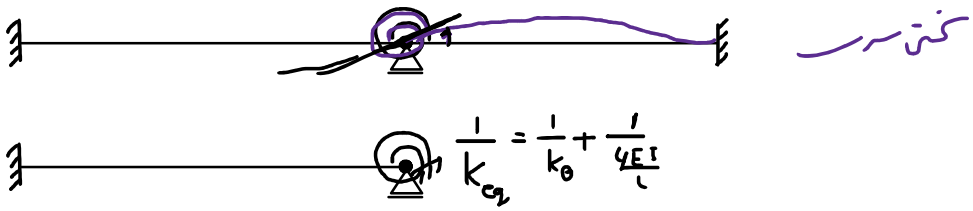
$\frac{1}{2}$, $\frac{2}{3}$

$$\frac{1}{k_{eq}} = \frac{1}{\frac{2}{3}EI} + \frac{1}{EI} \rightarrow k_{eq} = \frac{2}{5}EI$$

$\frac{2}{5}$, 1

4/ 3/ 2/ 5/


1/2 , 2/3
4/7
3/7
-6
6
-1.72
-3.43
-2.57
-1.29
-7.72
2.57
-2.57
-1.29
+
2/5 , 1
2/7
5/7
-1.5
1.5
0.22
0.43
1.07
0.53
0.22
0.43
-0.43
2.03


$\frac{1}{k_{eq}} = \frac{1}{k_\theta} + \frac{1}{\frac{4EI}{L}}$

Video 33

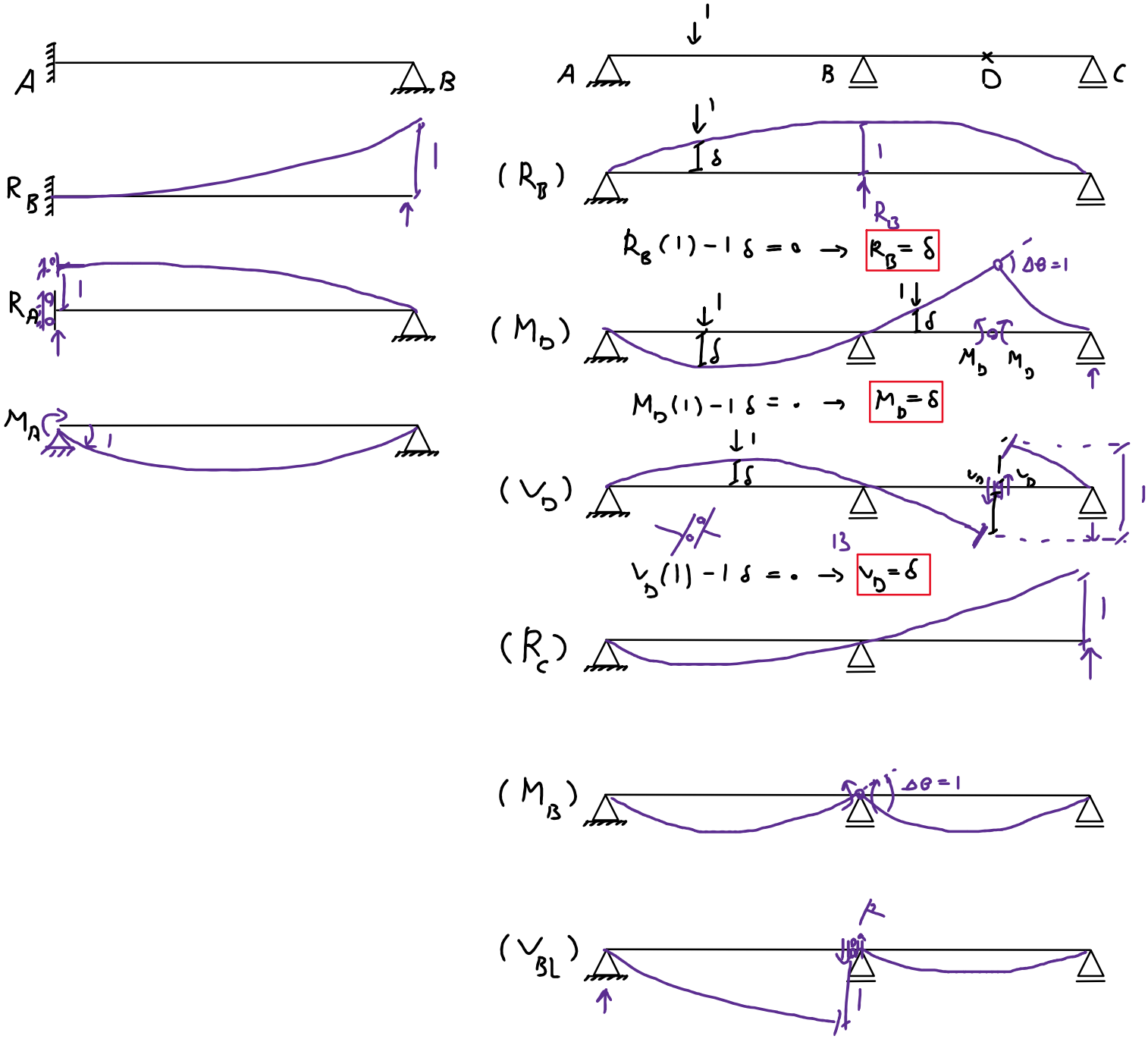
Influence Line 1

Sunday, May 12, 2024 8:58

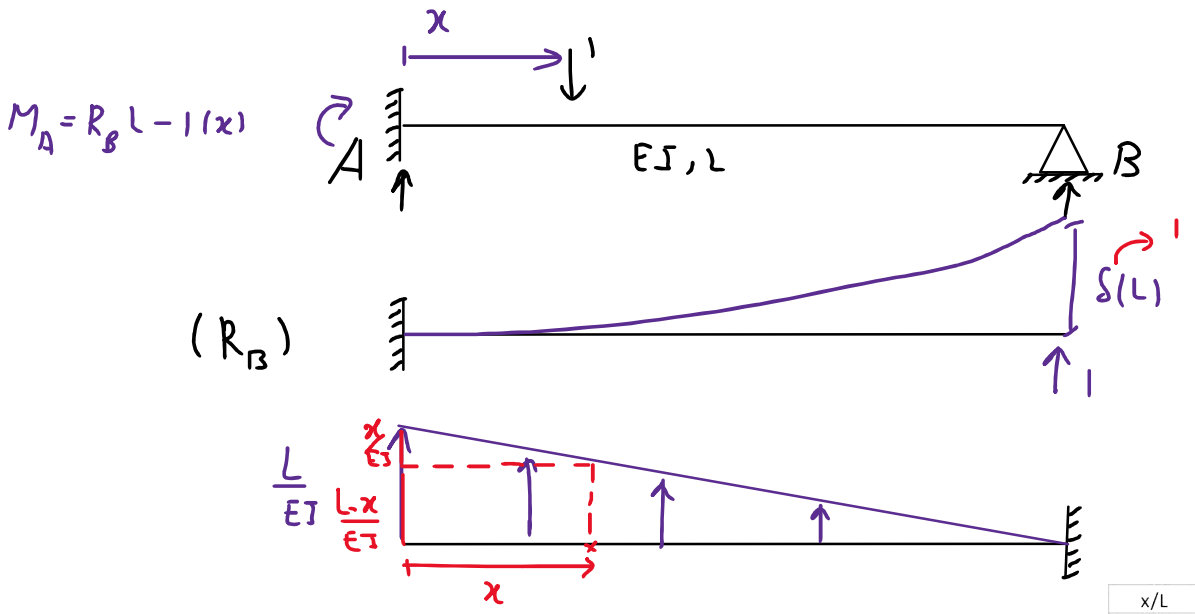
خط تأثیر سازه های نامعین

اصل مولر - برسلاو (یادآوری)

خط تأثیر یک نیرو برابر است با تابع تغییر شکل سازه وقتی نیرو مورد نظر از سازه حذف شده و سازه آزاد شده تحت اثر همان نیرو تغییر شکل دهد.



مثال: تابع خط تأثیر R_A ، R_B و M_A را به دست آورید.



x/L	RB	RA	MA/L
0	0	1	0
0.1	0.0145	0.9855	-0.0855
0.2	0.056	0.944	-0.144
0.3	0.1215	0.8785	-0.1785
0.4	0.208	0.792	-0.192
0.5	0.3125	0.6875	-0.1875
0.6	0.432	0.568	-0.168
0.7	0.5635	0.4365	-0.1365
0.8	0.704	0.296	-0.096
0.9	0.8505	0.1495	-0.0495
1	1	0	0

$\delta(x) = \frac{M}{EI} = \frac{1}{EI}\left[(L-x)x\left(\frac{x}{2}\right) + \frac{1}{2}(x)(x)\left(\frac{2}{3}x\right)\right] = \frac{1}{EI}\left(\frac{-x^3}{6} + \frac{L}{2}x^2\right)$

اصلی — مزدوج

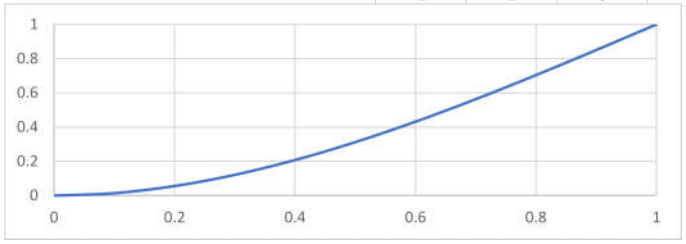
$\delta(L) = \frac{L^3}{3EI}$

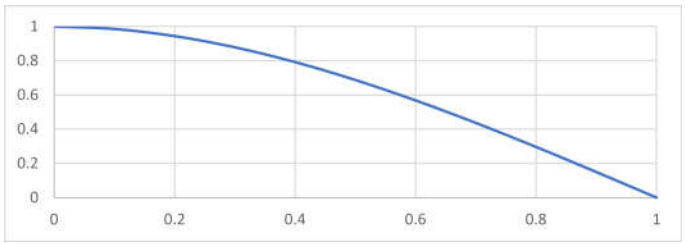
$$R_B = \frac{\delta(x)}{\delta(L)} = -\frac{1}{2}\left(\frac{x}{L}\right)^3 + \frac{3}{2}\left(\frac{x}{L}\right)^2$$

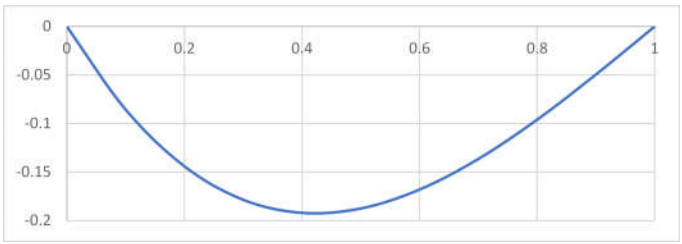
$$R_A = 1 - R_B = 1 + \frac{1}{2}\left(\frac{x}{L}\right)^3 - \frac{3}{2}\left(\frac{x}{L}\right)^2$$

$M_A = R_B L - x$

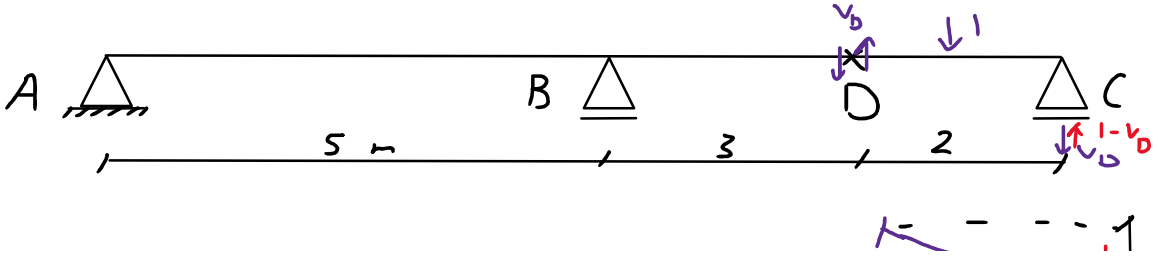
$$\frac{M_A}{L} = -\frac{1}{2}\left(\frac{x}{L}\right)^3 + \frac{3}{2}\left(\frac{x}{L}\right)^2 - \frac{x}{L}$$





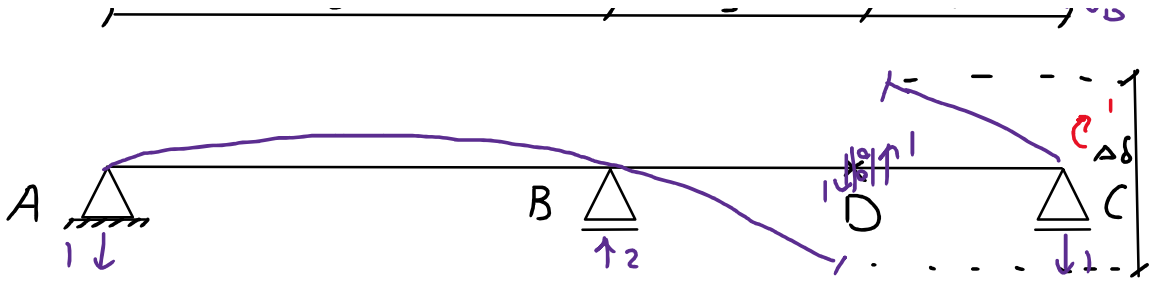


مثال: تابع خط تأثیر V_D و M_D را به دست آورید.



e

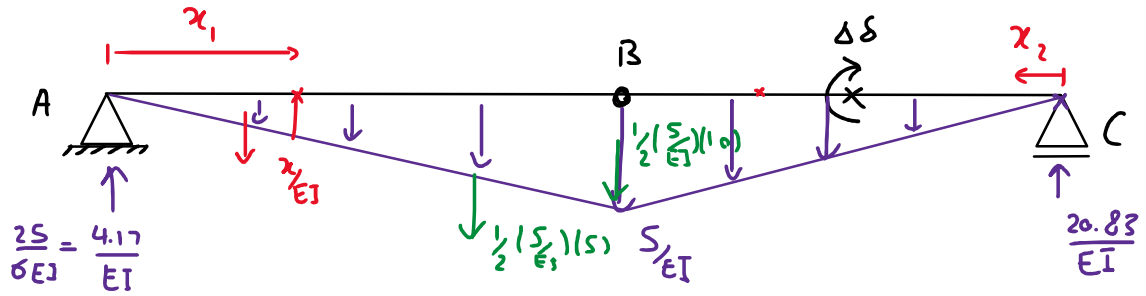
$\Delta\delta$



تیر اصلی δ

v

ΔM



تیر مزدوج M

$$\sum M_C = 0 \rightarrow \frac{4.17}{EI}(10) - \frac{25}{EI}(5) + \Delta\delta = 0 \rightarrow \boxed{\Delta\delta = \frac{83.33}{EI}}$$

$\frac{\delta(x)}{\Delta\delta}$ ↓

تا $\Delta\delta$ برابر واحد شود / $\Delta\delta$

$\delta(x_1)$ ↑

$$0 \le x_1 \le 5 \rightarrow M(x_1) = \frac{4.17}{EI}x_1 - \frac{1}{2}\left(\frac{x_1}{EI}\right)(x_1)\left(x_1/3\right) = \frac{1}{EI}\left(4.17x_1 - \frac{1}{6}x_1^3\right) / \Delta\delta$$

$\delta(x_2)$ ↑

$$5 \le x_2 \le 7 \rightarrow M(x_2) = \frac{20.83}{EI}x_2 - \frac{1}{2}\left(\frac{x_2}{EI}\right)(x_2)\left(x_2/3\right) = \frac{1}{EI}\left(20.83x_2 - \frac{1}{6}x_2^3\right) / \Delta\delta$$

$\delta(x_2)$ ↑

$$2 \le x_2 \le 5 \rightarrow M(x_2) = \frac{20.83}{EI}x_2 - \frac{1}{6}\frac{x_1^3}{EI} - \frac{83.33}{EI} = \frac{1}{EI}\left(-83.33 + 20.83x_2 - \frac{1}{6}x_2^3\right) / \Delta\delta$$

$$V_D = \frac{\delta(x)}{\Delta\delta} = \begin{cases} 0.05x_1 - 0.002x_1^3 & 0 \le x_1 \le 5 \\ 0.25x_2 - 0.002x_1^3 & 0 \le x_2 \le 2 \\ -1 + 0.25x_2 - 0.002x_2^3 & 2 \le x_2 \le 5 \end{cases}$$

$M_D = -2V_D$ در فیلم اشتباه شده $2(1-V_D)$

$M_D = \boxed{-2V_D} - 1(2-x_2)$

$\boxed{-0.5x_2 + 0.004x_1^3 + x_2 - 2}$

$-2V_D + x_2 = 0.5x_2 + 0.004x_1^3$

$$M_D = \begin{cases} -0.1x_1 + 0.004x_1^3 & 0 \le x_1 \le 3 \\ 0.5x_2 + 0.004x_2^3 - 2 & 0 \le x_2 \le 2 \\ 2 - 0.5x_2 + 0.004x_1^3 & 2 \le x_2 \le 5 \end{cases}$$

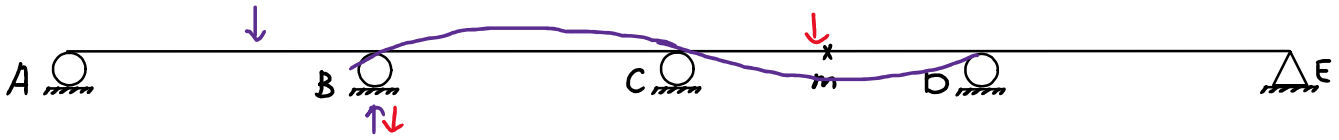
در فیلم اشتباه شده

Video 34

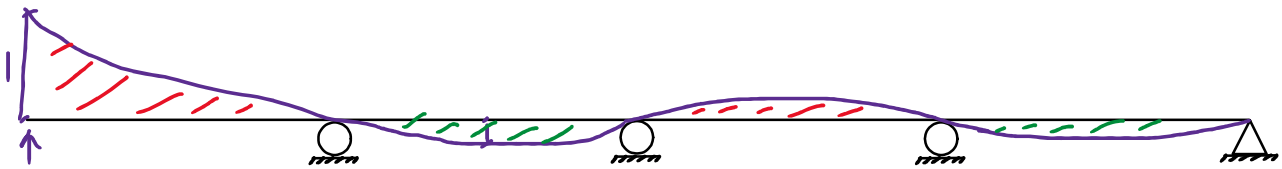
Influence Line 2

Thursday, June 13, 2024 18:56

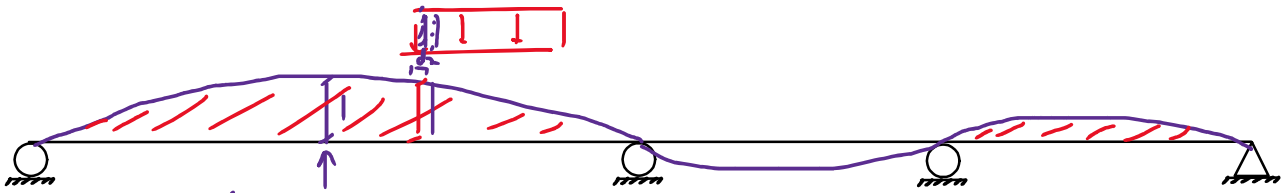
طرح ظاهری خط تأثیر تیرهای نامعین



(R_A)

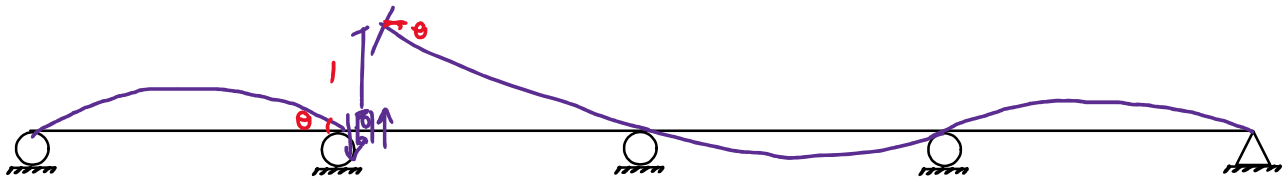


(R_B)

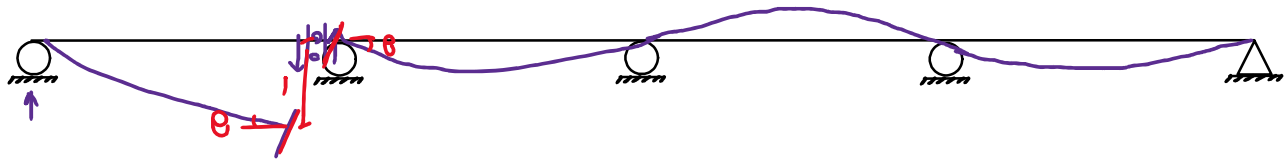


$\int (1 \cdot 1)\, w\, dx = w \int (1 \cdot 1)\, dx$

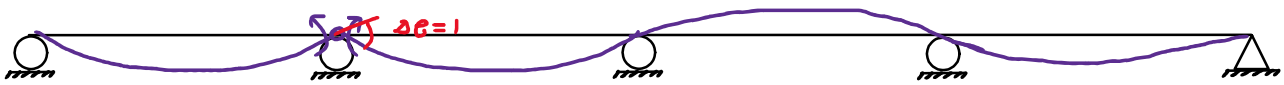
(V_{BR})



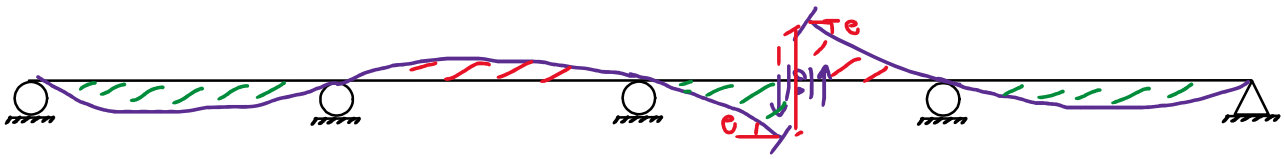
(V_{BL})



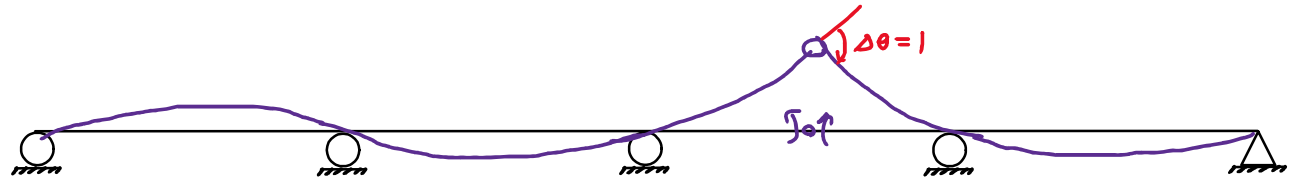
(M_B)



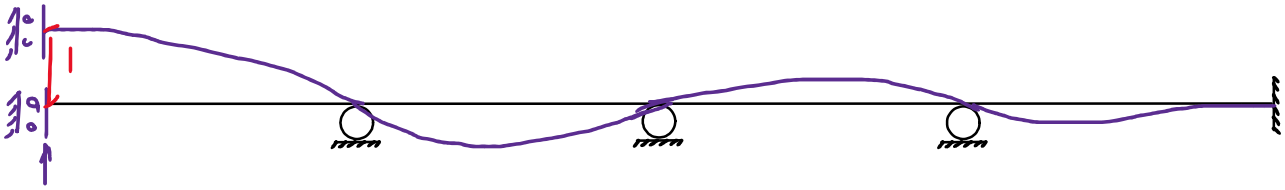
(V_m)



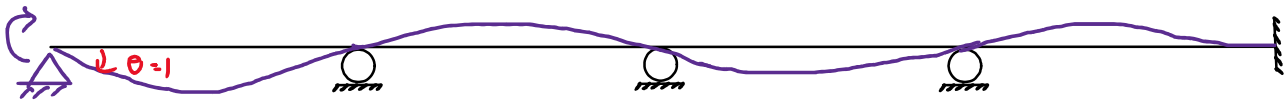
(M_m)



(V_A)

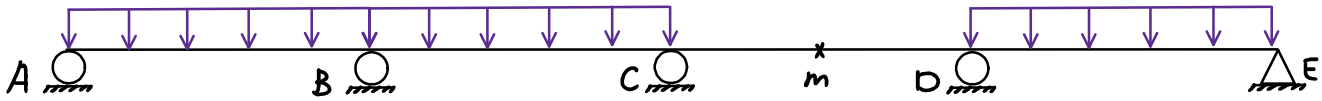


(M_A)

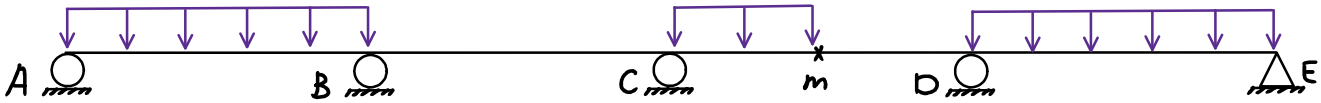


بحرانی ترین وضعیت بارگذاری

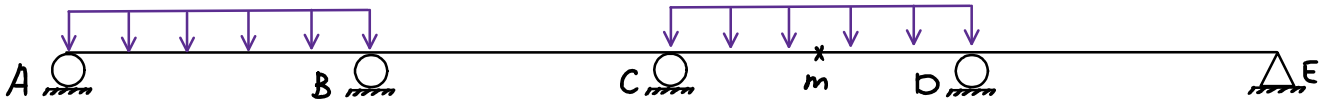
R_B , V_{BR} , V_{BL} , M_B^-



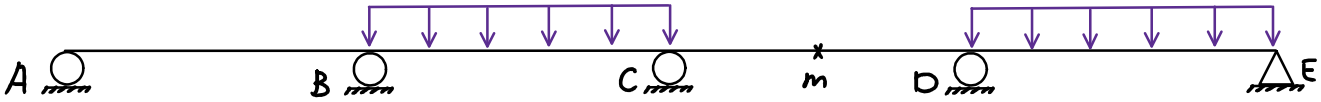
V_m



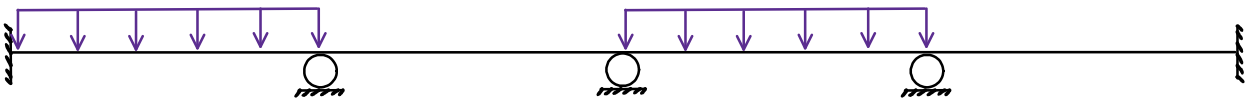
M_m^+



M_m^-



M_A^- , R_A



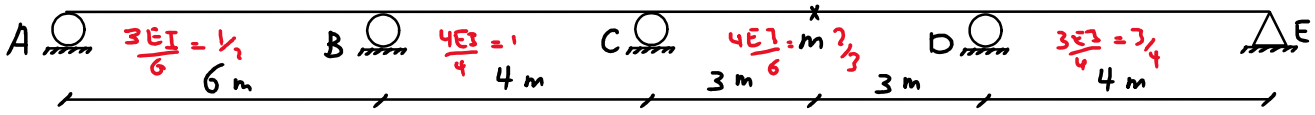
Video 35

Influence Line 3

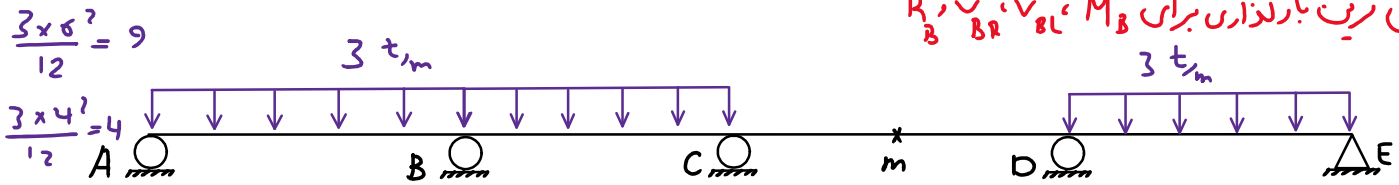
Thursday, June 13, 2024 18:56

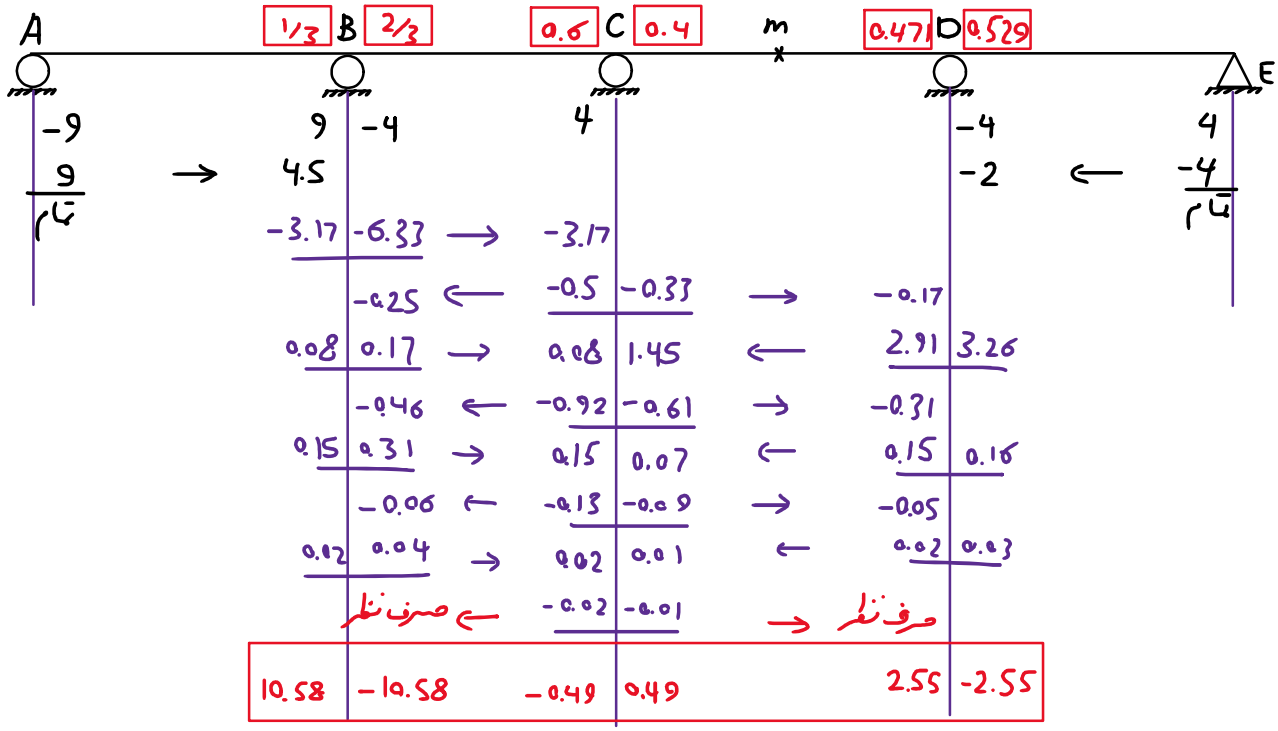
خط تأثیر سازه‌های نامعین

مثال: تیر سراسری شکل زیر تحت بار زنده 3 $t_{/m}$ قرار دارد. این بار می‌تواند برروی تمام یا قسمتی از هر دهانه قرار گیرد. بیشینه لنگر منفی در تکیه‌گاه B و برش سمت راست و چپ تکیه‌گاه B و برش نقطه m را به دست آورید.



بحران ترین بارگذاری برای R_B, V_{BR}, V_{BL}, M_B^-



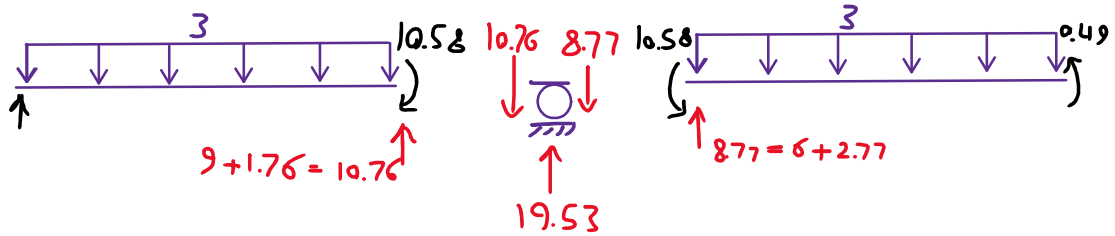


$M_B^- = 10.58$ t.m

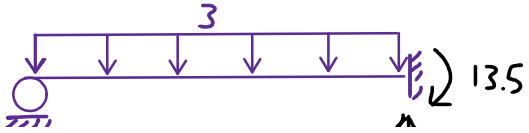
$V_{BL} = 10.76$ ton

$V_{BR} = 8.77$ ton

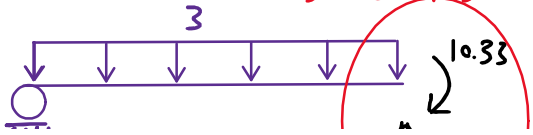
$R_B = 19.53$ ton

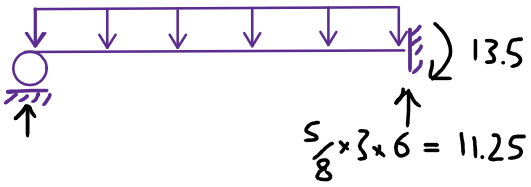


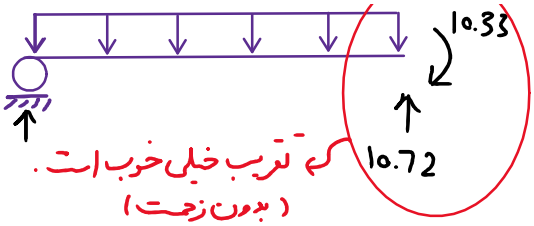
تقریب گیردار



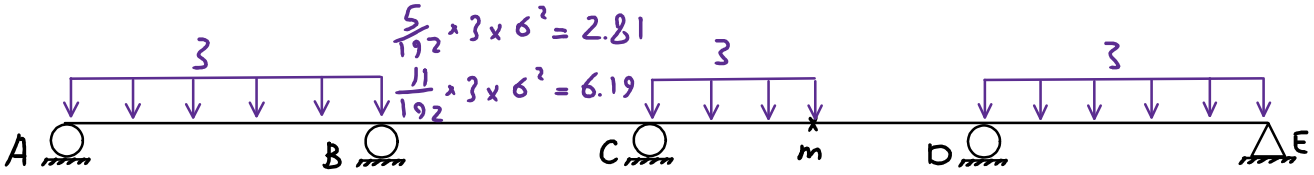
تقریب سیکل اول

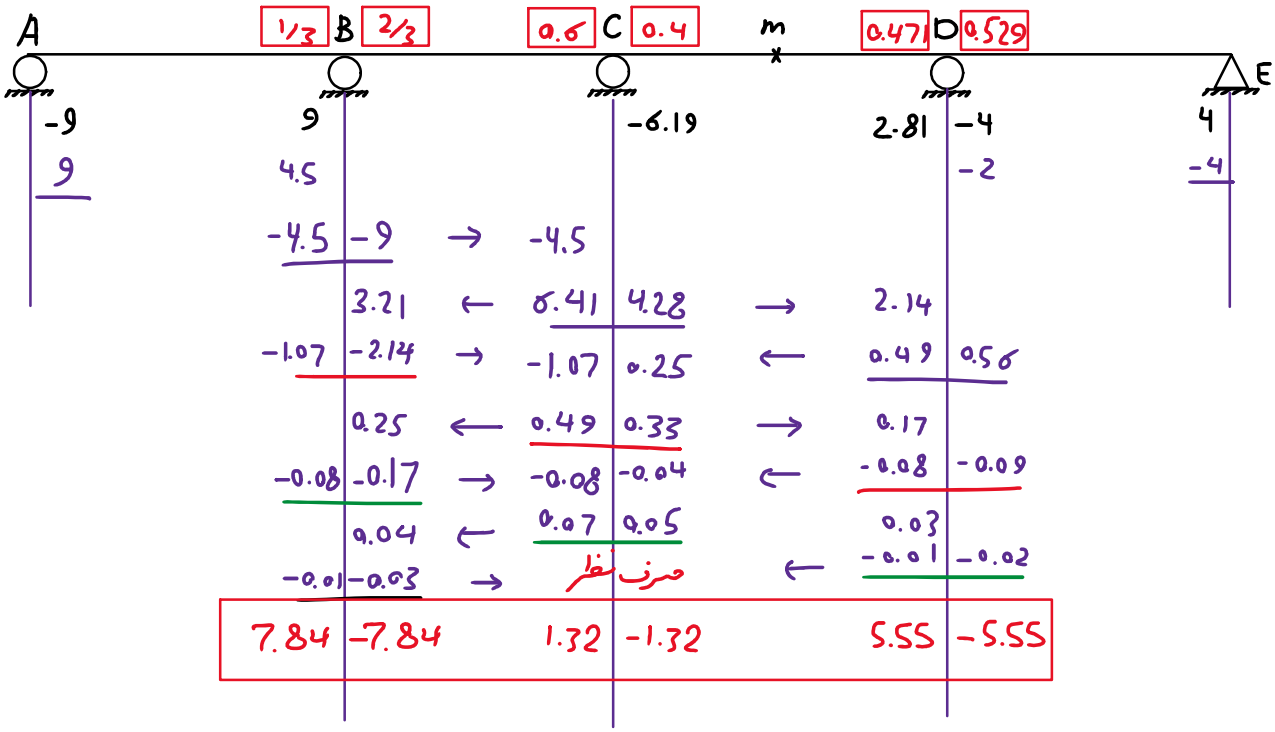






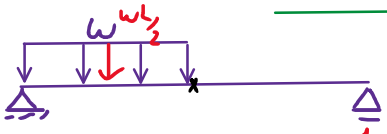
کم تقریب خیلی خوب است.
(بدون زحمت)

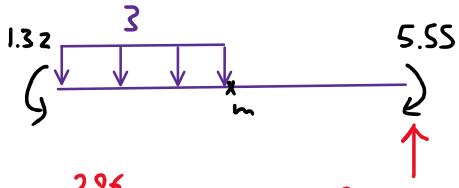




$V_m = 2.96$ ton

تقریب دوم سفل



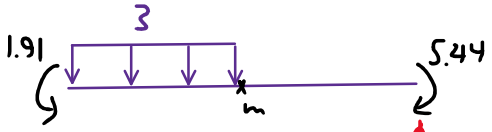


$2.25 \leq 2.96 \leq 3.23$

$\frac{wl}{8} = \frac{3\times 6}{8} = 2.25$

تقریب انتهای گیردار

تقریب سکل اول



$2.25 + 0.59 = 2.84 \leq 2.96$

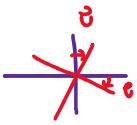
$2.25 + 0.98 = 3.23$

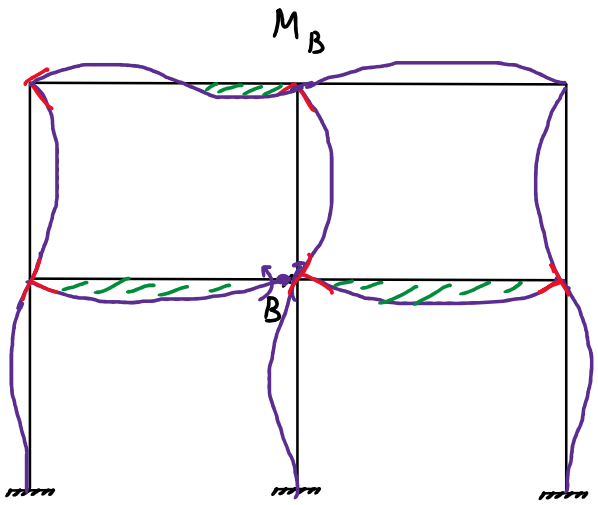
Video 36

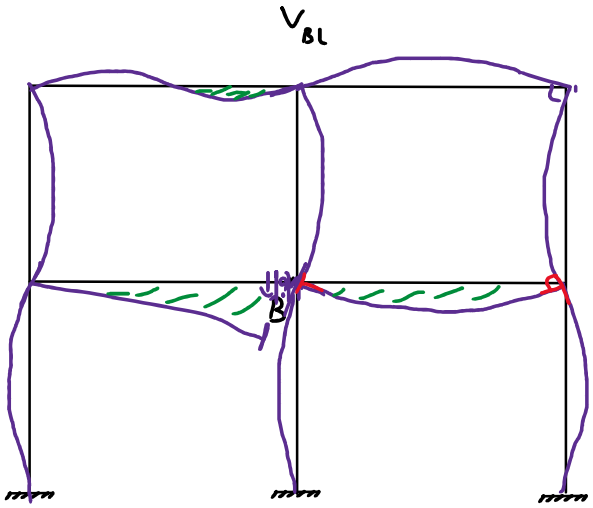
Influence Line 4

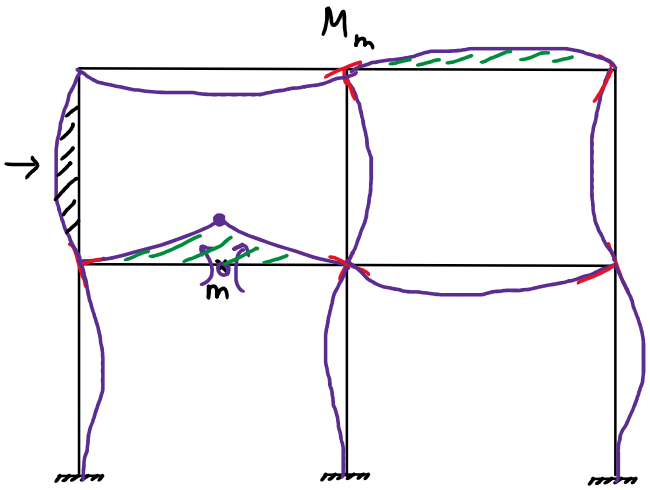
Thursday, June 13, 2024 19:03

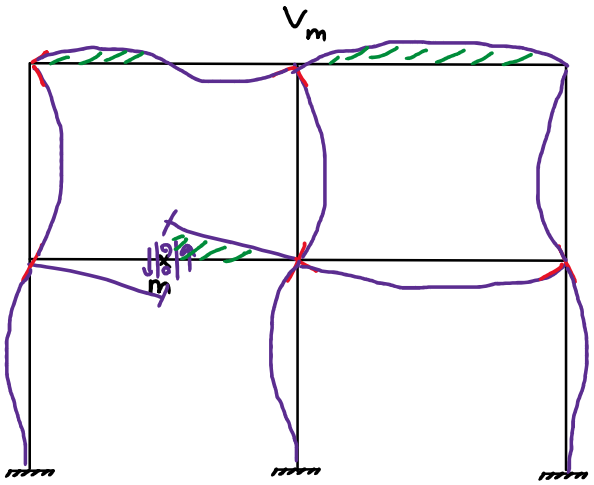
طرح ظاهری خط تأثیر قاب های نامعین

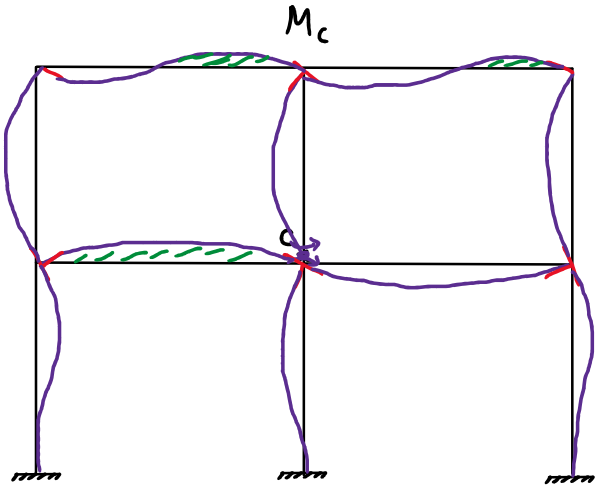


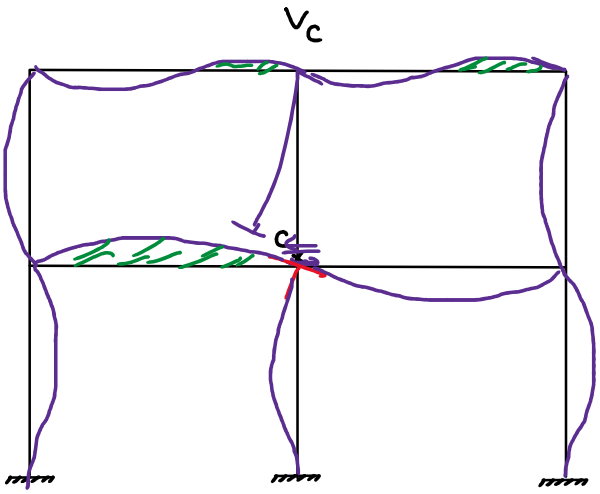


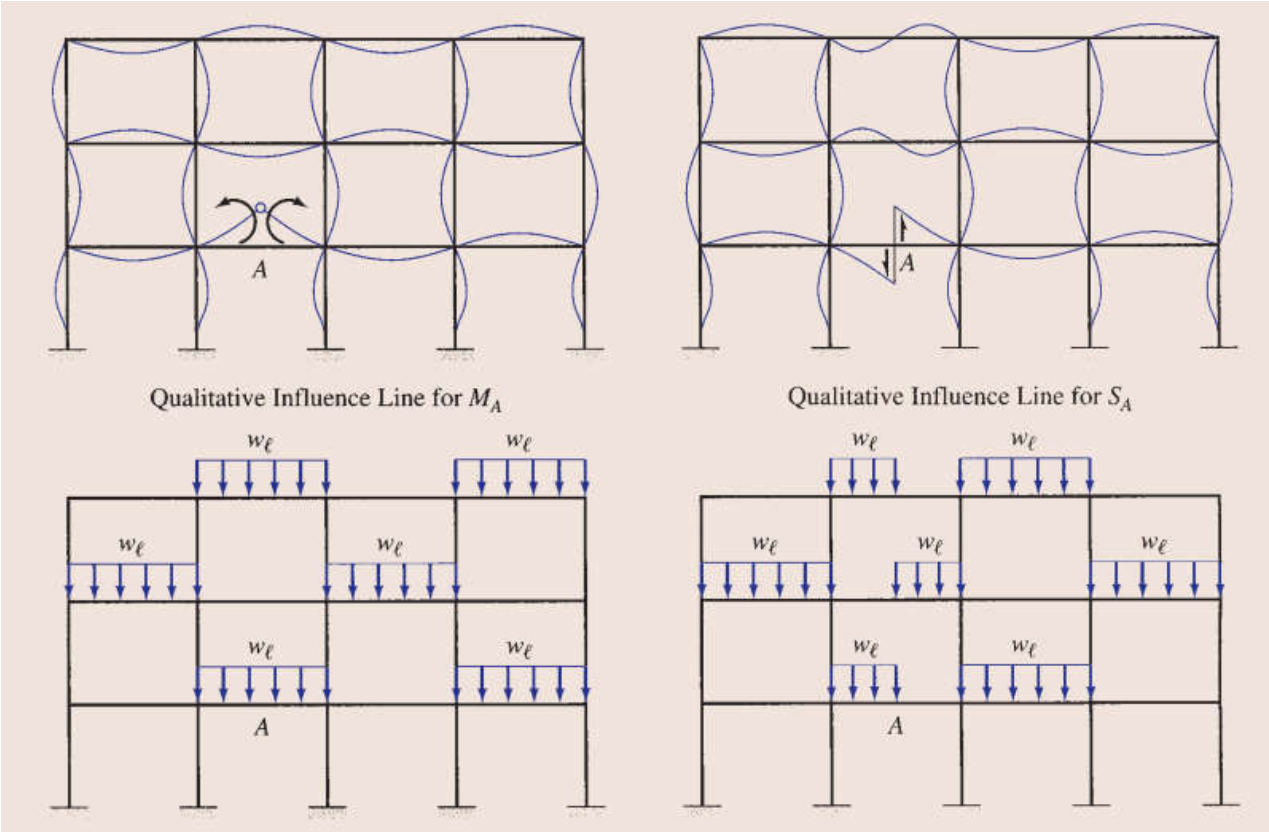




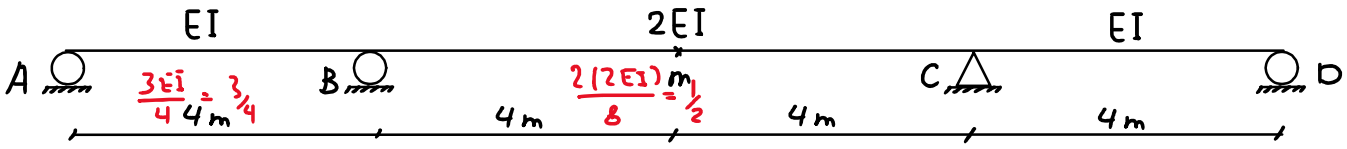


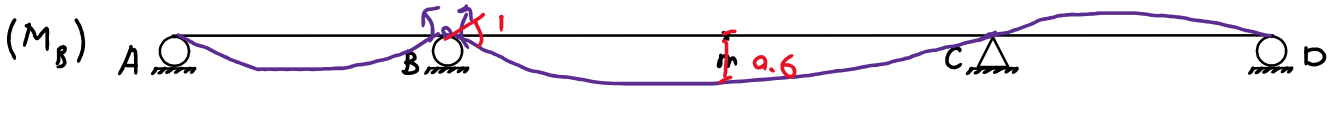


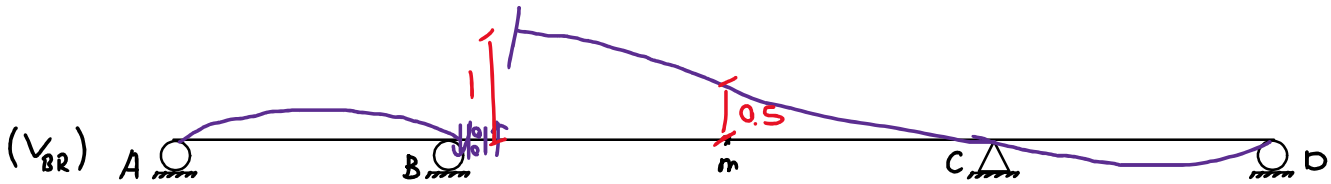


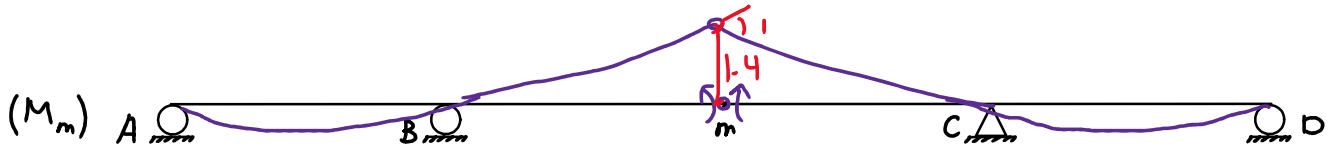


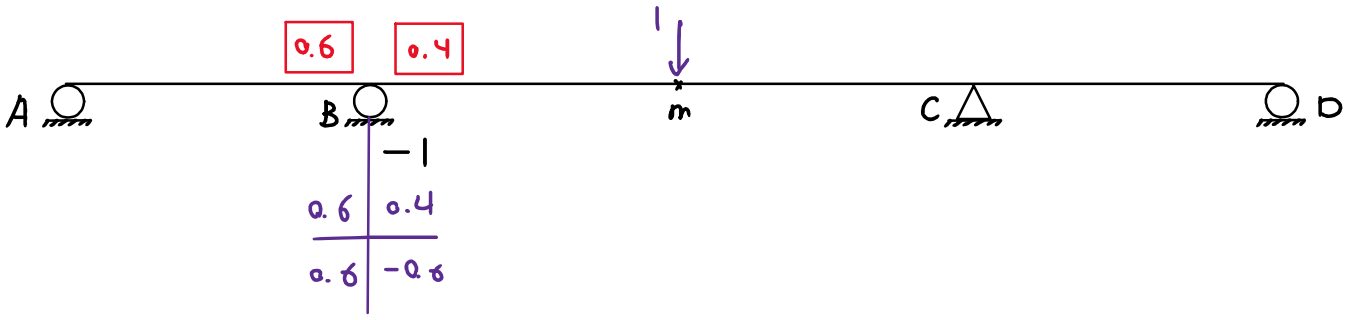
مثال: خط تأثیر M_B ، V_{BR} ، M_m را به صورت تقریبی رسم نموده و مقدار هر خط تأثیر در نقطه m را به دست آورید.

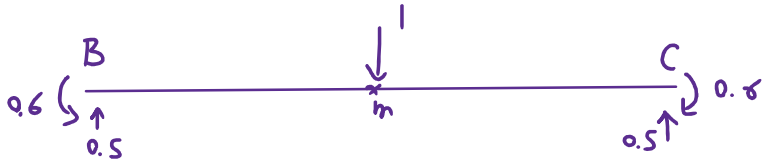




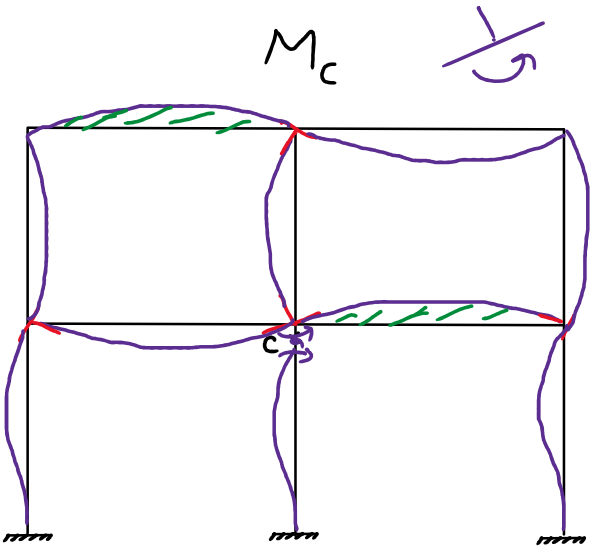


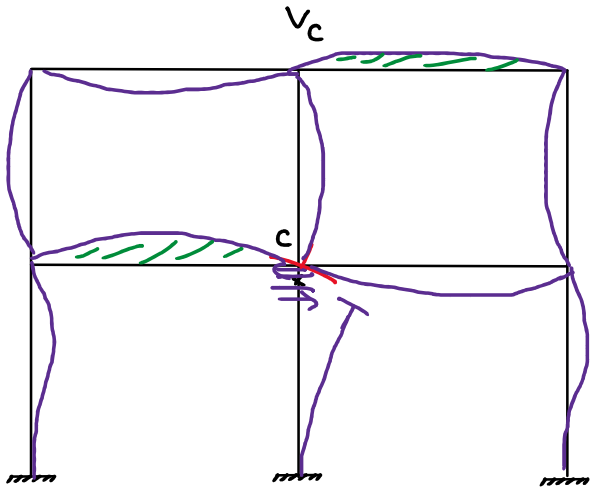






$$M_m = -0.6 + 0.5 \times 4 = 1.4$$





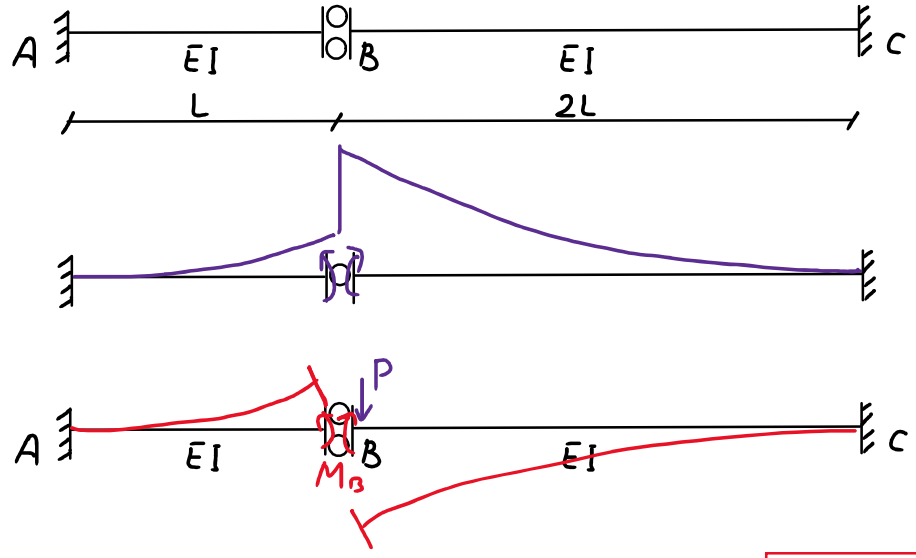
Video 37

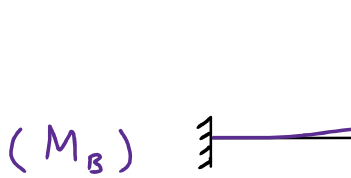
Influence Line 5

Thursday, June 13, 2024 18:56

خط تأثیر سازه‌های نامعین

مثال: حداکثر لنگر نقاط A، B و C در اثر بهره بار متمرکز P از روی تیر شکل زیر چقدر است؟

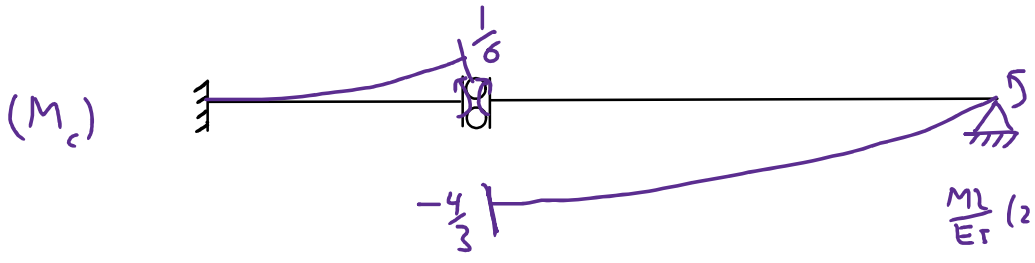




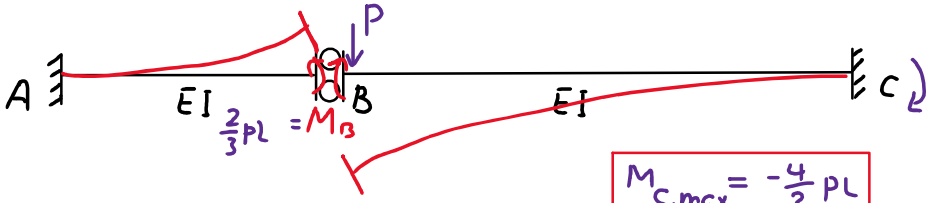
معادله $\theta_{BL} = \theta_{BR}$

مجهول M_B

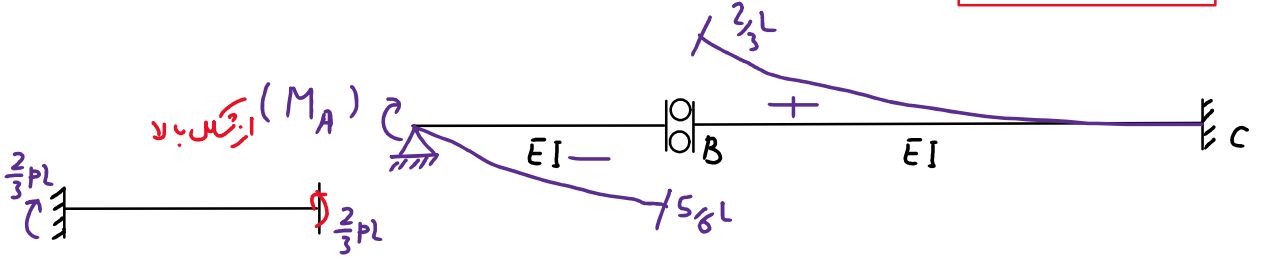
$$\frac{M_B L}{EI} = \frac{P(2L)^2}{2EI} - \frac{M_B(2L)}{EI} \rightarrow 3M_B = 2PL \rightarrow \boxed{M_{B,max} = \frac{2}{3}PL}$$

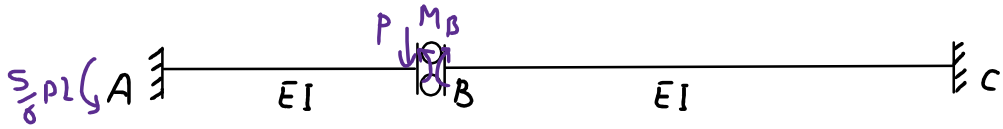


$$\frac{ML}{EI}(2L) + \frac{M(2L)^2}{2EI} = 4\frac{ML^2}{EI}$$



$$\boxed{M_{C,max} = -\frac{4}{3}PL}$$



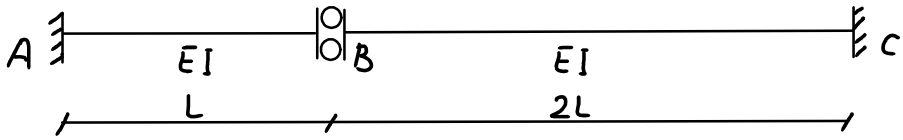


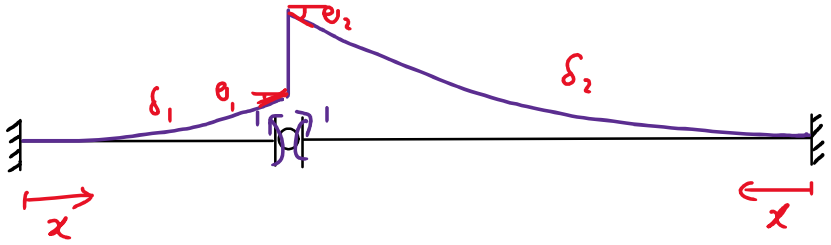
$$\frac{PL^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B(2L)}{EI} \rightarrow 3M_B = \frac{PL}{2} \rightarrow M_B = \frac{PL}{6}$$

$$\frac{PL^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{PL}{2} \rightarrow M_B = \frac{PL}{6}$$

$$\boxed{M_{A,max} = -\frac{5}{6}PL}$$

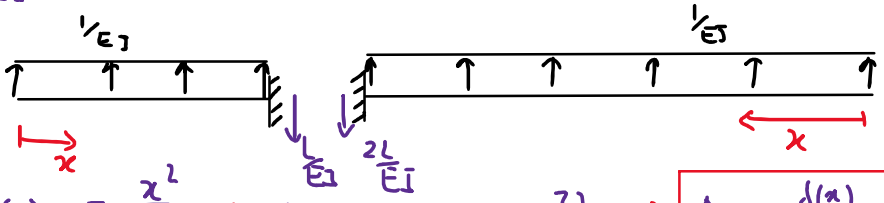
مثال: در مثال فوق، تابع خط تأثیر M_A و M_B را به دست آورید.





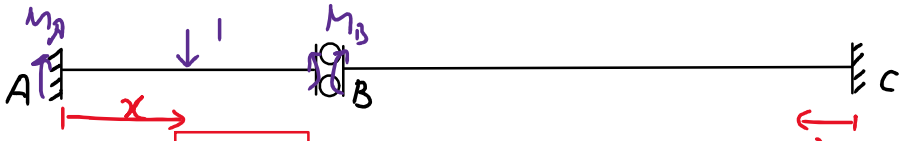
$$\begin{cases} \delta_1(x) = 1 \times \frac{x^2}{2EI} \\ \delta_2(x) = 1 \times \frac{x^2}{2EI} \end{cases} \rightarrow \Delta\theta = \frac{L}{EI} + \frac{(2L)}{EI} = \frac{3L}{EI} \rightarrow \boxed{M_B = \frac{\delta(x)}{\Delta\theta} = \frac{x^2}{6L}}$$

کنسر در تیر مزدوج



$$\delta(x)_{\text{اصلی}} = M(x)_{\text{تیر مزدوج}} = \frac{x^2}{2EI} \rightarrow \Delta\theta_{\text{اصلی}} = \Delta V_{\text{تیر مزدوج}} = \frac{3L}{EI} \rightarrow \boxed{M_B = \frac{\delta(x)}{\Delta\theta} = \frac{x^2}{6L}}$$

از معادله تعادل و M_B در بخش قبل

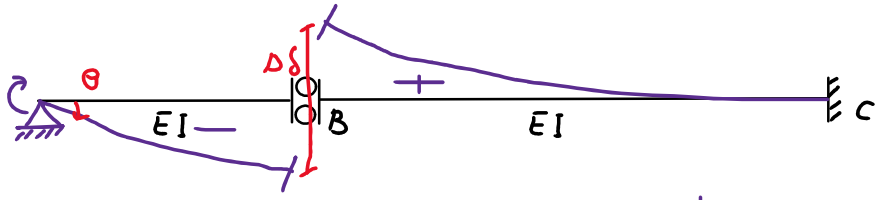


بار روی BC $\rightarrow M_A = M_B = \boxed{\frac{x^2}{6L} \quad BC}$

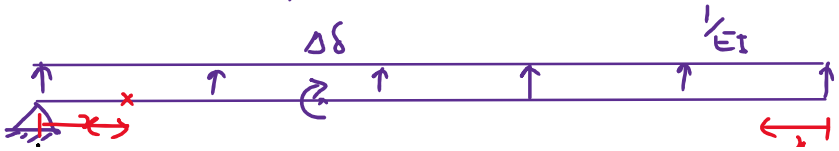
بار روی AB $\rightarrow M_A = M_B - 1 \times x = \boxed{\frac{x^2}{6L} - x \quad AB}$

$\Delta\delta \rightarrow \Delta M$

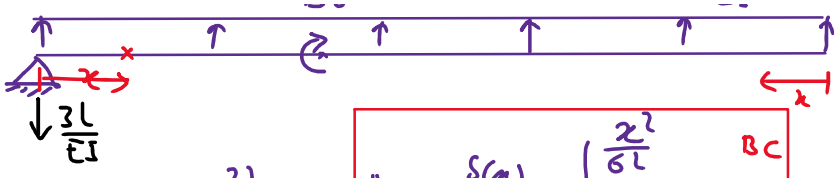
$\theta \rightarrow V$



تیر مزدوج



تیر مزدوج



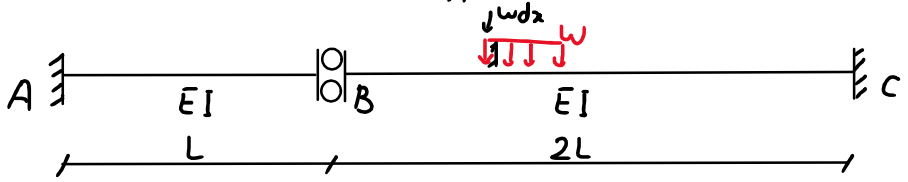
$\delta_2(x) = \frac{x^2}{2EI}$

$\delta_1(x) = \frac{-3L}{EI}x + \frac{x^2}{2EI}$

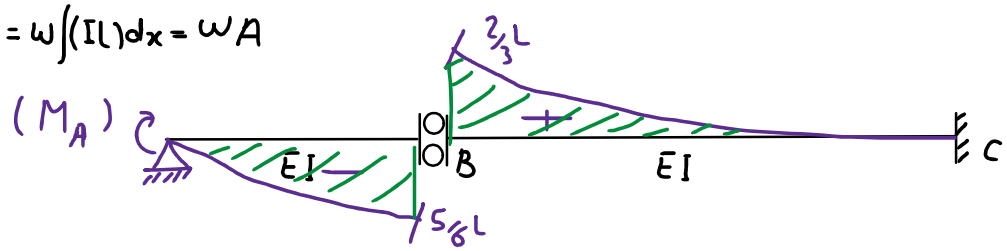
$\rightarrow \theta = \frac{2L}{EI} \rightarrow$

$$M_A = \frac{\delta(m)}{\theta} = \begin{cases} \frac{x^2}{6L} & BC \\ -x + \frac{x^2}{6L} & AB \end{cases}$$

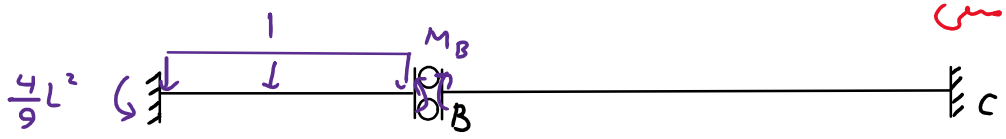
مثال: در مثال فوق، مساحت زیر منحنی خط تاثیر M_A را به دست آورید.



$$M_A = \int (IL)\, w\, dx = w \int (IL)\, dx = wA$$



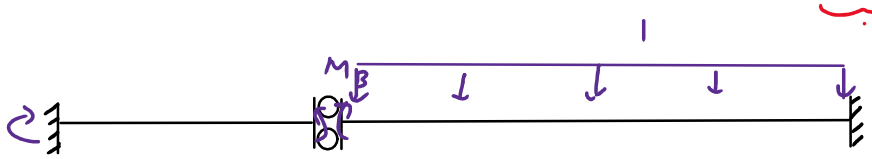
مساحت ناحیه منفی



$$\theta_{BL} = \theta_{BR} \rightarrow \frac{1 \times L^2}{2EI} - \frac{M_B L}{EI} = \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{L}{2} \rightarrow M_B = \frac{L}{18} \rightarrow M_A = \frac{-4}{9}L^2$$

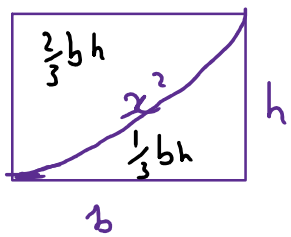
$$\boxed{A^- = \frac{4}{9}L^2}$$

مساحت ناحیه مثبت



$$\theta_{BL} = \theta_{BR} \rightarrow \frac{M_B L}{EI} = \frac{1 (2L)^3}{2EI} - \frac{M_B (2L)}{EI} \rightarrow 3M_B = \frac{4}{3}L^2 \rightarrow M_B = \frac{4}{9}L^2 \rightarrow M_A = M_B$$

$$\boxed{A^+ = \frac{4}{9}L^2}$$



$$A^+ = \frac{1}{3}\left(\frac{2}{3}L\right)(2L) = \frac{4}{9}L^2$$

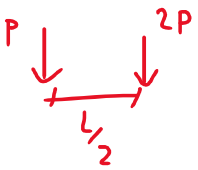
Video 38

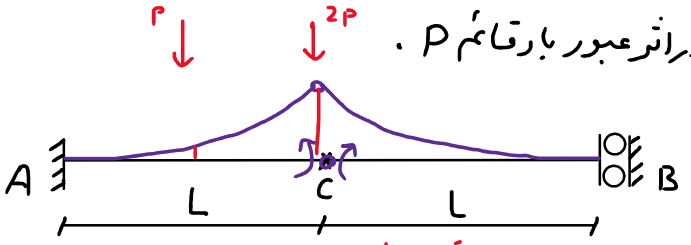
Influence Line 6

Thursday, June 13, 2024 19:03

خط تأثیر سازه‌های نامعین

مثال: حداکثر لنگر نقطه C در اثر عبور بار قائم P .

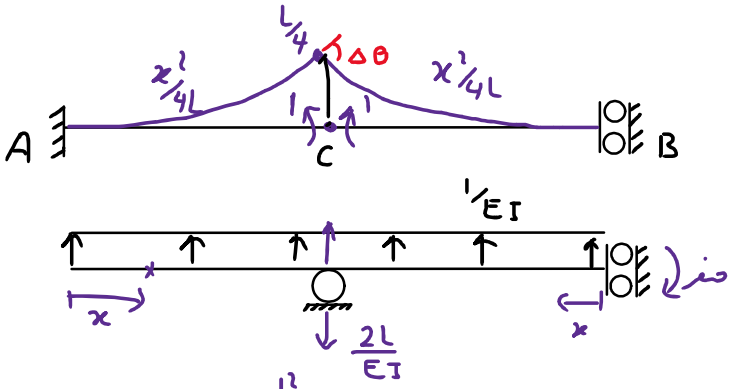




① تشکیل شماتیک خط تأثیر و تحلیل سازه تحت بار جزایی

$\theta_B = 0 \rightarrow \frac{PL^2}{2EI} = \frac{M_B(2L)}{EI} \rightarrow M_B = \frac{PL}{4} \rightarrow \boxed{M_{C,max} = \frac{PL}{4}}$

② اصل مولر- برسلاو



صفر

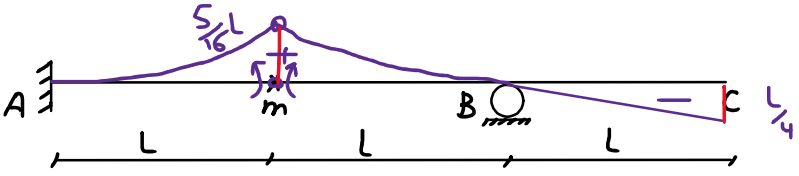
$\begin{cases} \delta_C = M_C = \frac{1}{EI}\frac{L^2}{2} \\ \text{مزدوج} \quad \text{اصلی} \end{cases}$

$\begin{cases} \Delta\theta = R_C = \frac{2L}{EI} \\ \text{مزدوج} \quad \text{اصلی} \end{cases}$

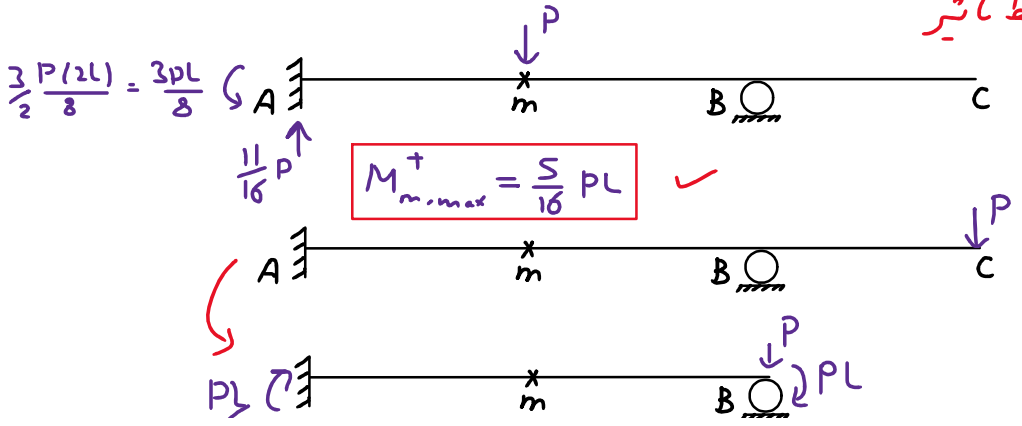
$\rightarrow M_C = \frac{\delta_C}{\Delta\theta} = \frac{L^2/2EI}{2L/EI} = \frac{L}{4} \rightarrow \boxed{M_{C,max} = \frac{PL}{4}}$

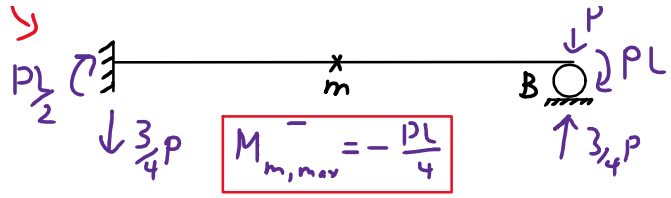
$M_C = \frac{\frac{1}{EI}\frac{x^2}{2}}{\frac{2L}{EI}} = \frac{x^2}{4L}$

مثال: حداکثر لنگر نقطه m در اثر عبور بار قائم P .

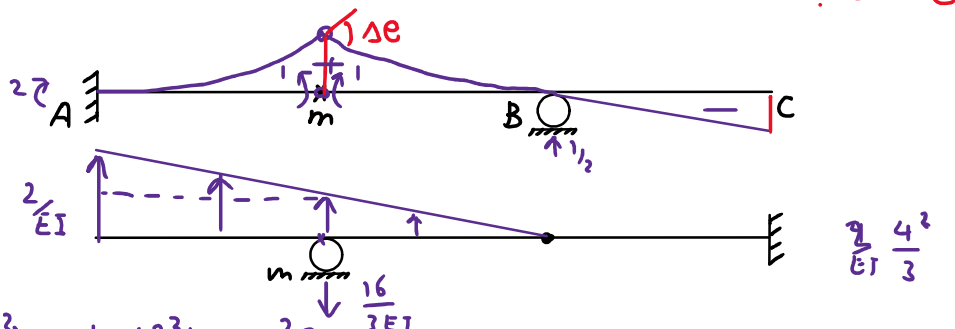


① تشکیل شماتیک خط تأثیر





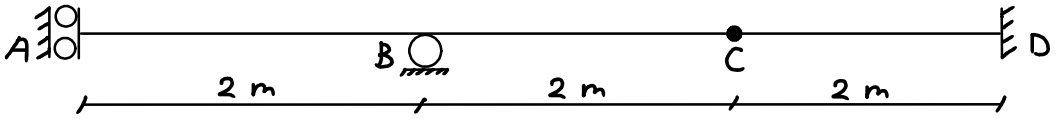
② اصل دولر - برسلاو



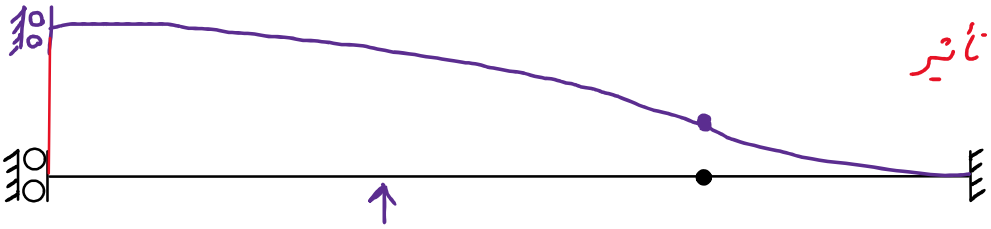
$$\begin{cases} \delta_m = M_m = \frac{1}{EI}\left(\frac{2}{2}\right)^2 + \frac{1}{EI}\left(\frac{2}{3}\right)^2 = \frac{20}{6EI} \\ \text{اصل} \quad \text{مزدوج} \\ \Delta\theta = R_m = \frac{16}{3EI} \\ \text{اصل} \quad \text{مزدوج} \end{cases}$$

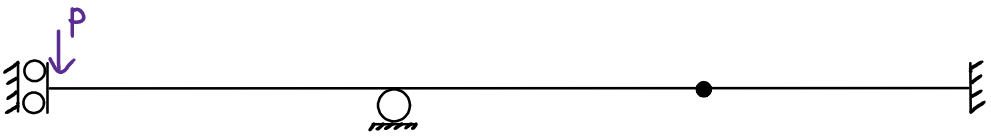
$$\rightarrow M_m = \frac{\delta_m}{\Delta\theta} = \frac{\frac{10}{3}}{\frac{16}{3}} = \frac{5}{8} \searrow \frac{5}{16}L$$

مثال: حداکثر عکس العمل تکیه‌گاه B در اثر عبور بار قائم P .

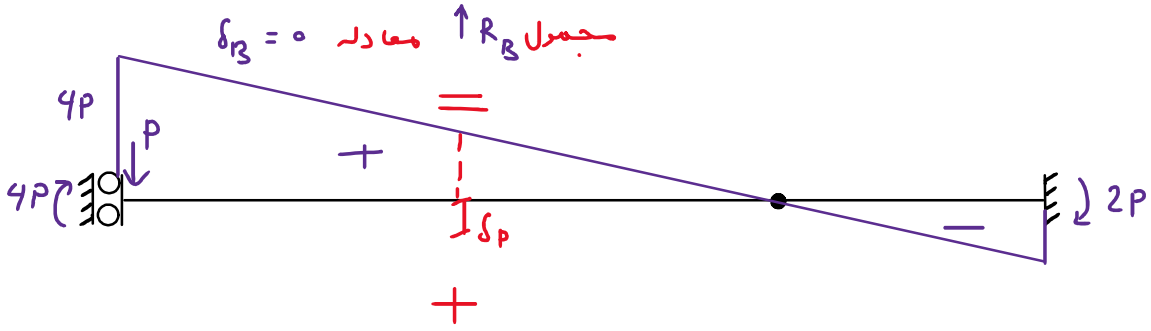


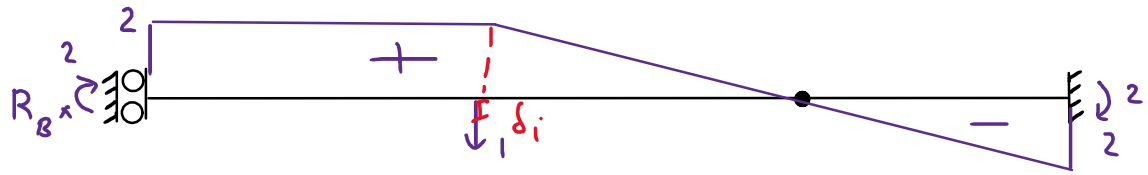
① شکل شماتیک خط تاثیر





$\delta_B = 0$ معادله مجهول



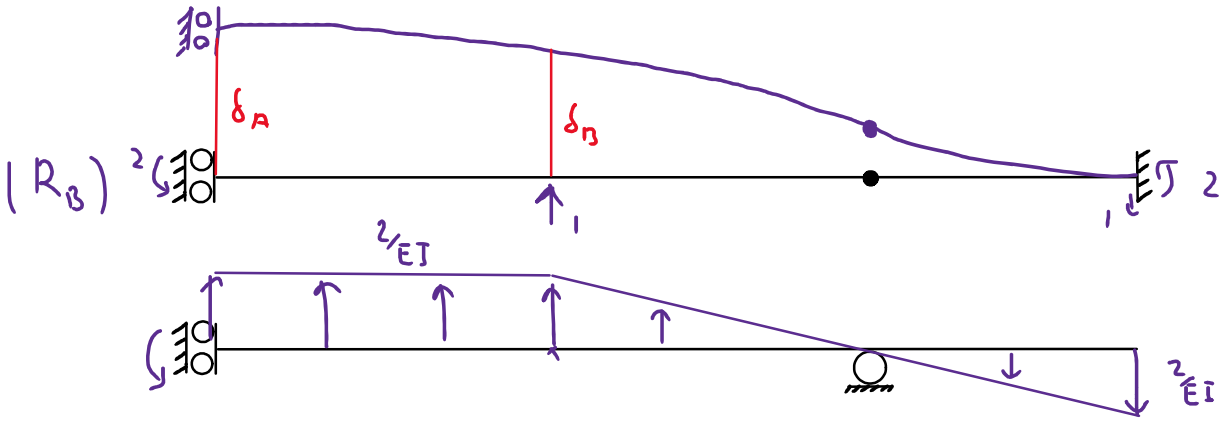


$$1 \times \delta_P = \int \frac{m M}{EI} dx = \frac{1}{EI}\left[(2)(3p)(2) + \frac{2}{3} \times (2p)(2) \times 2\right] = \frac{52}{3}\frac{P}{EI}$$

$$1 \times \delta_i = \int \frac{m^2}{EI} dx = \frac{1}{EI}\left[(2^2)(2) + \frac{2}{3} \times (2^2) \times 2\right] = \frac{40}{3EI}$$

$$\delta_B = 0 \rightarrow \delta_P + R_B \delta_i = 0 \rightarrow \frac{52}{3}\frac{P}{EI} + R_B \frac{40}{3EI} = 0 \rightarrow \boxed{R_B = -\frac{52}{40}P}$$

② اصل دوم- برسلاو



$$\begin{cases} \delta_A = M_A = \left(\frac{2}{EI}\right)(2)(3) + \left(\frac{2}{EI}\right)\left(\frac{2^2}{3}\right) \times 2 = \frac{52}{3EI} \\ \text{اصل} \quad \text{مزدوج} \\ \delta_B = M_B = -\frac{52}{3EI} + \frac{2}{EI}\left(\frac{2^2}{2}\right) = -\frac{40}{3EI} \\ \text{اصل} \quad \text{مزدوج} \end{cases} \rightarrow R_B = \frac{\delta_A}{\delta_B} = \frac{\frac{52}{3}}{\frac{40}{3}} = \boxed{\frac{52}{40}}$$

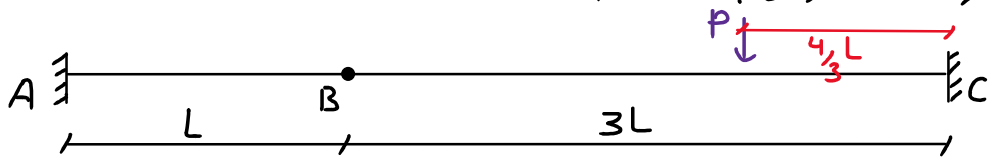
Video 39

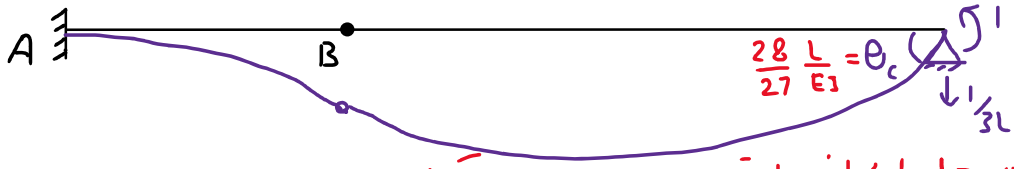
Influence Line 7

Thursday, June 13, 2024 19:03

خط تأثیر سازه‌های نامعین

مثال: حداکثر لنگر نقطه C در اثر عبور بار قائم P .

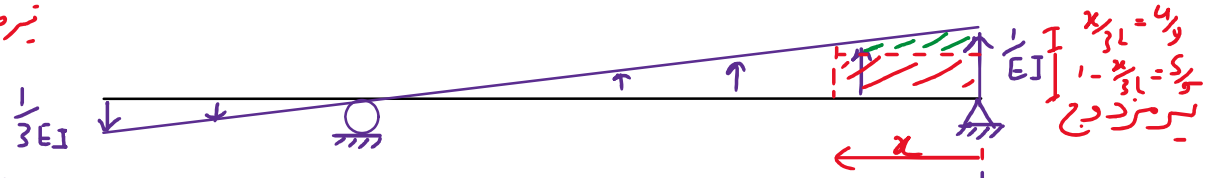




تیر اصلی

برای پیدا کردن محل بار بحرانی، از تیر مزدوج استفاده می‌کنیم.

δ_{max} تیر اصلی = M_{max} تیر مزدوج



$$\frac{1}{EI}\frac{(3L)^2}{3}+\frac{1}{3EI}\frac{L^2}{3}-R_c(3L)=0 \rightarrow R_c=\frac{28}{27}\frac{L}{EI}$$

$$M_{max} \rightarrow V=0 \rightarrow -\frac{28}{27}\frac{L}{EI}+\frac{1}{2EI}\left[1+1-\frac{x}{3L}\right]x=0 \rightarrow x-\frac{x^2}{6L}=\frac{28}{27}L \rightarrow x^2-(6L)x+\frac{56}{9}L^2=0$$

$$x=\frac{4}{3}L$$

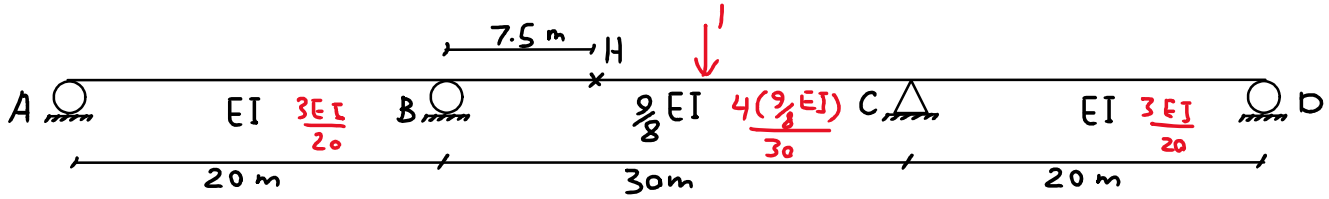
$$M_{max}=-\frac{28}{27}\frac{L}{EI}\left(\frac{4}{3}L\right)+\frac{5}{9EI}\frac{\left(\frac{4}{3}L\right)^2}{2}+\frac{4}{9EI}\frac{\left(\frac{4}{3}L\right)^3}{3}=-0.626\frac{L^2}{EI}$$

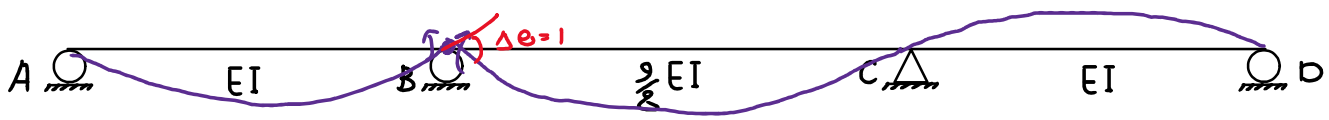
δ_{max} تیر اصلی = M_{max} تیر مزدوج

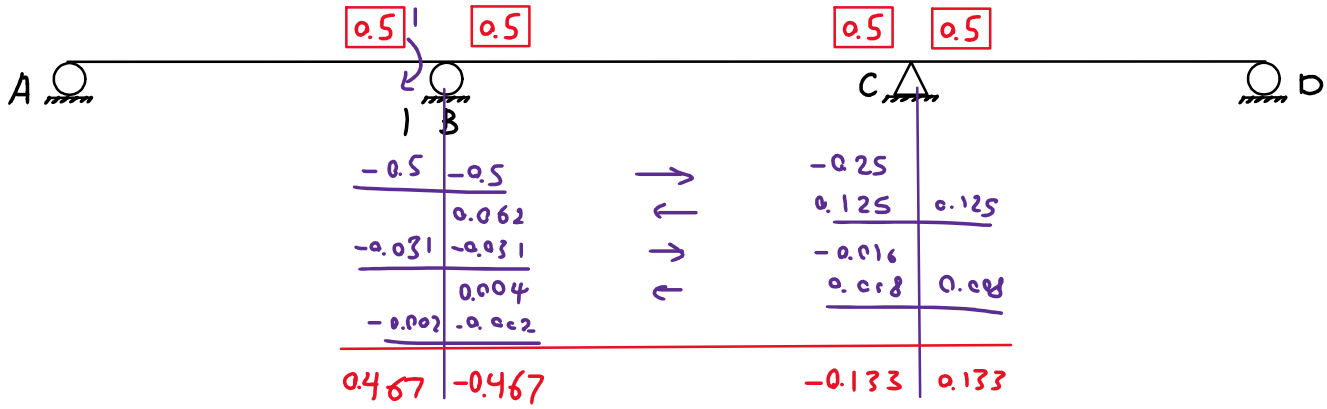
$$M_c=\frac{\delta_{max}}{\theta_c}=\frac{-0.626\frac{L^2}{EI}}{\frac{28}{27}\frac{L}{EI}}=-0.603L$$

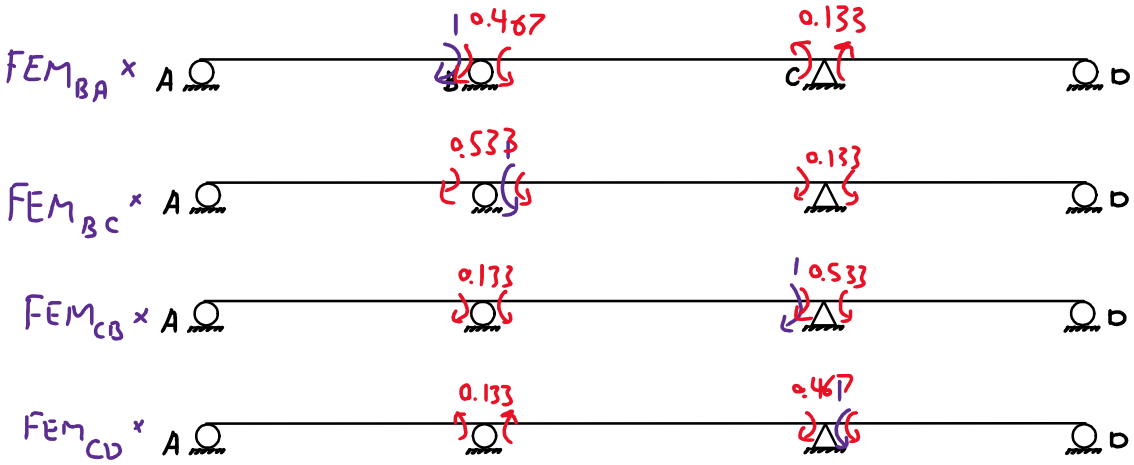
$$M_{c,max}=-0.603PL$$

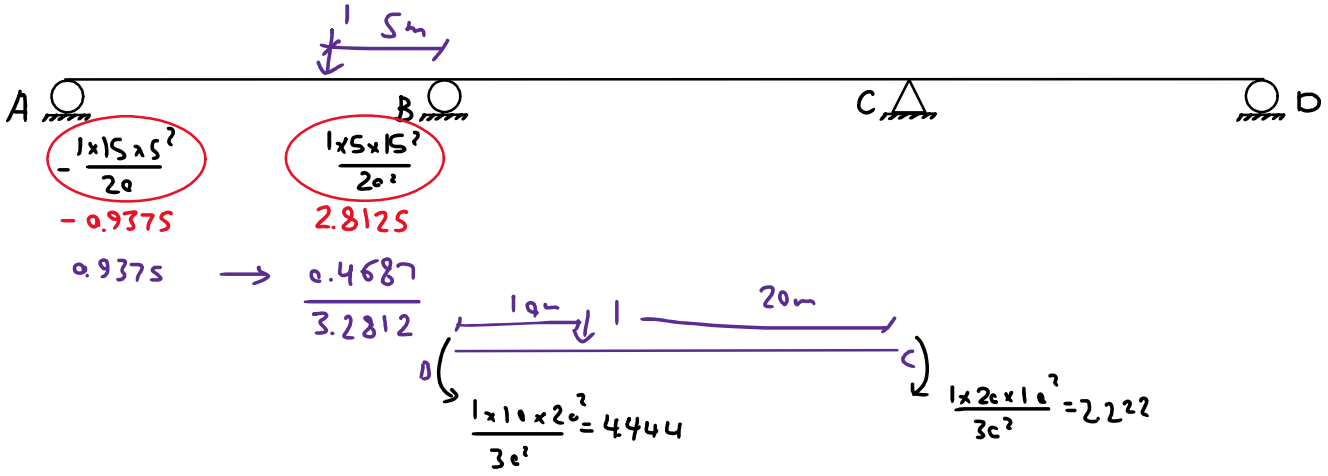
مثال: مقدار عددی خط تأثیر تیر پیوسته شکل زیر را برای M_B، M_c، V_H و V_{BR} به صورت کامل به دست آورید.



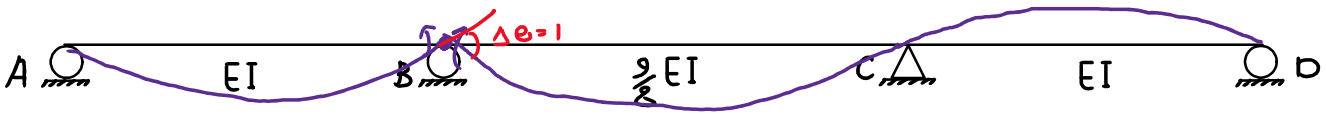


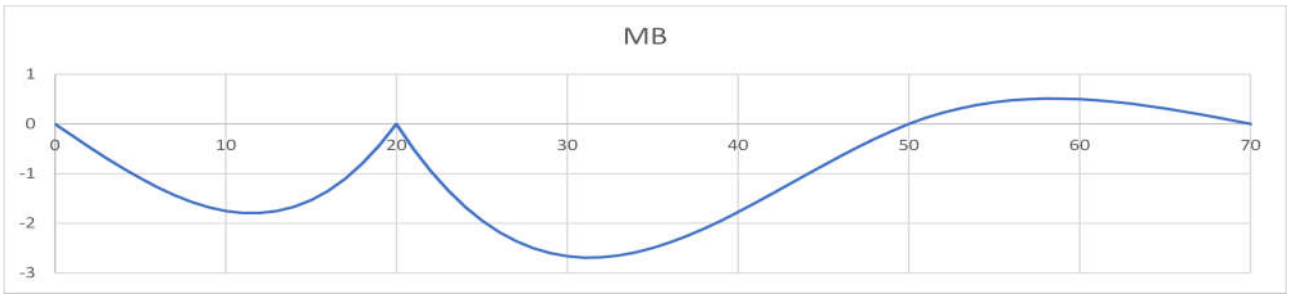


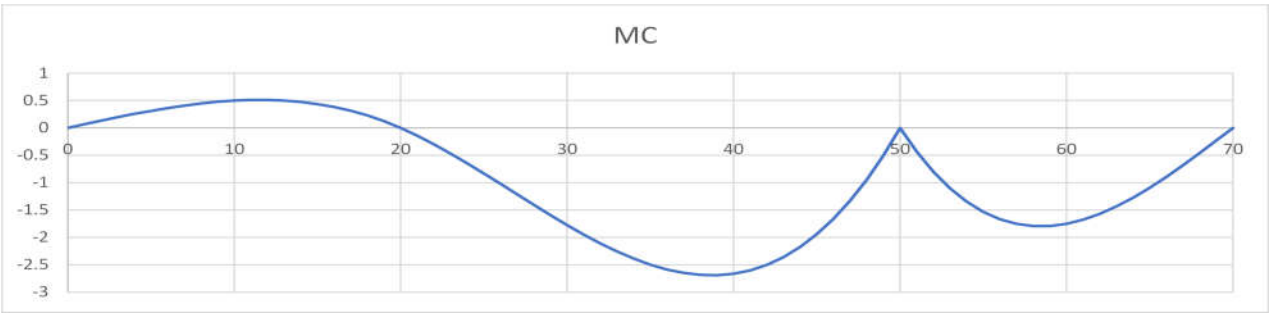


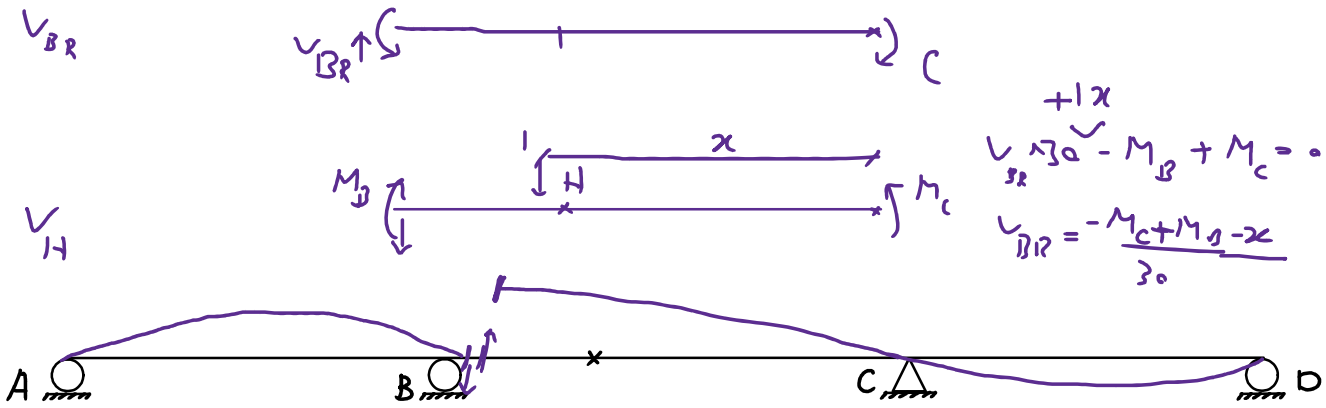


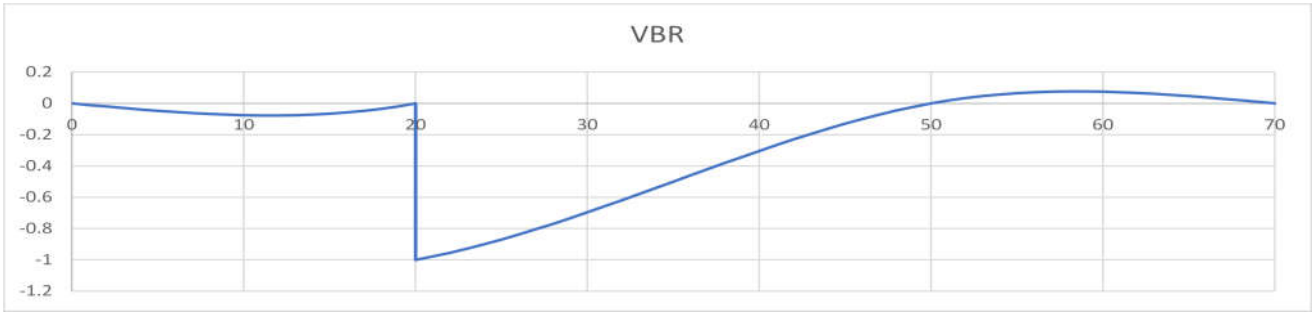
$$M_a = 4.444 \times 0.533 + 2.222 \times 0.133$$











Loc (m)	M_B	M_C	V_BR
0	**0**	0	0
1	-0.23292	0.066334	-0.00998
2	-0.46233	0.13167	-0.0198
3	-0.68474	0.195011	-0.02933
4	-0.89664	0.25536	-0.0384
5	-1.09453	0.311719	-0.04688
6	-1.27491	0.36309	-0.0546
7	-1.43427	0.408476	-0.06143

4	-0.89664	0.25536	-0.0384
5	-1.09453	0.311719	-0.04688
6	-1.27491	0.36309	-0.0546
7	-1.43427	0.408476	-0.06143
8	-1.56912	0.44688	-0.0672
9	-1.67595	0.477304	-0.07178
10	-1.75125	0.49875	-0.075
11	-1.79153	0.510221	-0.07673
12	-1.79328	0.51072	-0.0768
13	-1.753	0.499249	-0.07508
14	-1.66719	0.47481	-0.0714
15	-1.53234	0.436406	-0.06563
16	-1.34496	0.38304	-0.0576
17	-1.10154	0.313714	-0.04718
18	-0.79857	0.22743	-0.0342
19	-0.43256	0.123191	-0.01853
20	0	0	0
20	0	0	-1
21	-0.50234	-0.14146	-0.9787
22	-0.94516	-0.29804	-0.9549
23	-1.3311	-0.4671	-0.9288
24	-1.66284	-0.64596	-0.90056
25	-1.94306	-0.83194	-0.87037
26	-2.1744	-1.0224	-0.8384
27	-2.35954	-1.21466	-0.80483
28	-2.50116	-1.40604	-0.76984
29	-2.6019	-1.5939	-0.7336
30	-2.66444	-1.77556	-0.6963
31	-2.69146	-1.94834	-0.6581
32	-2.6856	-2.1096	-0.6192
33	-2.64954	-2.25666	-0.57976
34	-2.58596	-2.38684	-0.53997
35	-2.4975	-2.4975	-0.5
36	-2.38684	-2.58596	-0.46003
37	-2.25666	-2.64954	-0.42024
38	-2.1096	-2.6856	-0.3808
39	-1.94834	-2.69146	-0.3419
40	-1.77556	-2.66444	-0.3037
41	-1.5939	-2.6019	-0.2664
42	-1.40604	-2.50116	-0.23016
43	-1.21466	-2.35954	-0.19517
44	-1.0224	-2.1744	-0.1616
45	-0.83194	-1.94306	-0.12963
46	-0.64596	-1.66284	-0.09944
47	-0.4671	-1.3311	-0.0712
48	-0.29804	-0.94516	-0.0451
49	-0.14146	-0.50234	-0.0213
50	0	0	0
51	0.123191	-0.43256	0.018525
52	0.22743	-0.79857	0.0342
53	0.313714	-1.10154	0.047175
54	0.38304	-1.34496	0.0576
55	0.436406	-1.53234	0.065625
56	0.47481	-1.66719	0.0714
57	0.499249	-1.753	0.075075
58	0.51072	-1.79328	0.0768
59	0.510221	-1.79153	0.076725
60	0.49875	-1.75125	0.075
61	0.477304	-1.67595	0.071775
62	0.44688	-1.56912	0.0672
63	0.408476	-1.43427	0.061425
64	0.36309	-1.27491	0.0546
65	0.311719	-1.09453	0.046875
66	0.25536	-0.89664	0.0384
67	0.195011	-0.68474	0.029325
68	0.13167	-0.46233	0.0198
69	0.066334	-0.23292	0.009975
70	0	0	0

Video 40

Matrix Analysis 1

Wednesday, July 17, 2024 11:46

تحلیل ماتریسی سازه ها

① تاریخچه

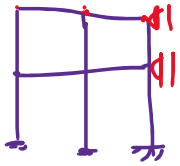
مبانی تئوری:
① روش سازگاری تغییر شکل ها : ماکسول 1864
② روش شیب-انت : مان 1915

مجهول	معادله
F	$\theta, \delta = 0$
θ, δ	$\Sigma M = 0$
	$\Sigma F_h = 0$

← حل هم‌زمان دستگاه معادلات جبری

انتشار دهه 1960 ← اختراع کامپیوتر ← شکل ماتریسی معادلات

② مقایسه روش های کلاسیک و تحلیل ماتریسی



① معادلات تعادل
② معادلات سازگاری
③ سختی اعضا
← فرم ماتریسی قابل برنامه نویسی

چرا تحلیل کلاسیک؟
① فهم رفتار سازه
② تحلیل سازه های کوچک
③ چک کردن تحلیل های کامپیوتری
④ تحلیل تقریبی و طراحی اولیه
⑤ تعیین رابطه نیرو-تغییرمکان

① سیستماتیک ← الگوریتمی، قابل برنامه نویسی
② عمومی ← برای انواع سازه ها، الگوریتم مشابهی دارد.

③ مقایسه تحلیل ماتریسی و اجزای محدود

کاربرد:
تحلیل ماتریسی ← فقط برای سازه های خطی
اجزای محدود ← برای هر شکل سازه

تئوری:
تحلیل ماتریسی ← رابطه دقیق نیرو-تغییرمکان براساس حل دقیق معادلات دیفرانسیل
اجزای محدود ← فرض توابع تغییرمکان یا تنش و استفاده از روابط کار و انرژی

④ روش های سخت و نرم

روش سخت ✓
روش نرم

روش سخت: more systematic
پیاده سازی کامپیوتری راحت‌تر

اکثر نرم افزارهای تجاری ← روش سخت (شیب-انت)

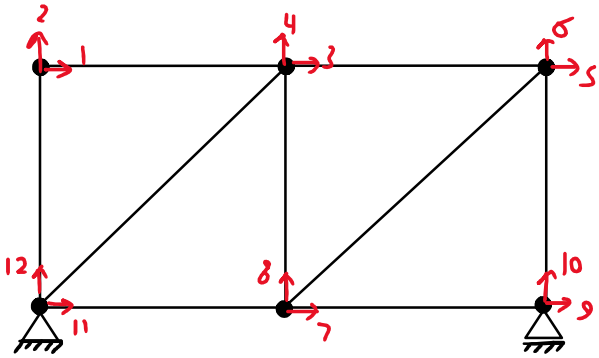
درجات آزادی

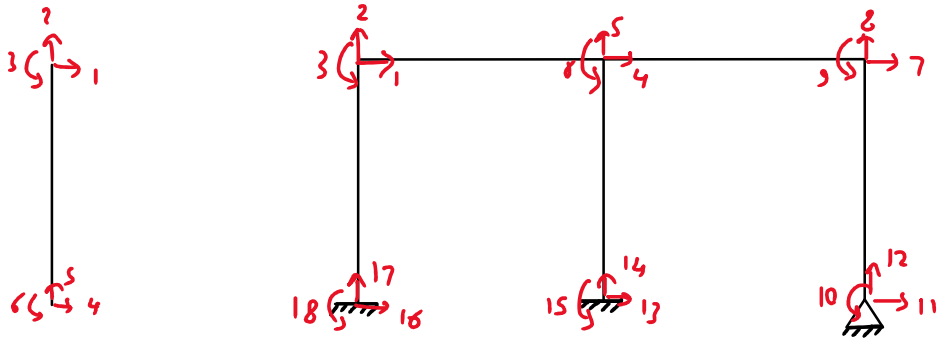
درجات آزادی

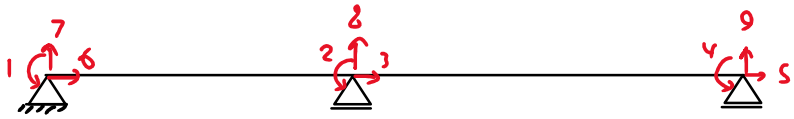
مجهولات: δ , θ

معادلات: $\sum M = 0$ ، $\sum F = 0$

درجه آزادی = آزاد / مقید



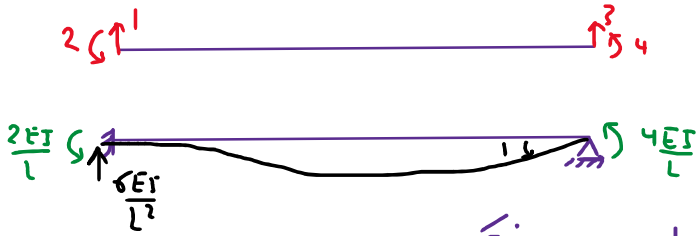




ماتریس سختی اعضا

ماتریس سختی، نیروهای گرهی را به تغییرمکان‌های گرهی ربط می‌دهد.

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix}_{4\times1} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & & & \\ k_{31} & & & \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}_{4\times4} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}_{4\times1}$$



* هر درایه ماتریس سختی که با k_{ij} نشان داده می‌شود، عبارت است از نیروی لازم در درجه آزادی i برای ایجاد تغییرمکان واحد در درجه آزادی j، هنگامی که از تغییرمکان سایر درجات آزادی جلوگیری شود.

* بنابراین برای پیدا کردن هر ستون ماتریس سختی کافی است که به درجه آزادی مربوطه تغییرمکان واحد اعمال شود و نیروی کلیه درجات آزادی به دست آید؛ در حالی که تغییرمکان سایر درجات آزادی صفر است.

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & & & \\ k_{31} & & & \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$P_1 = k_{11} \times 1$

$P_2 = k_{21} \times 1$

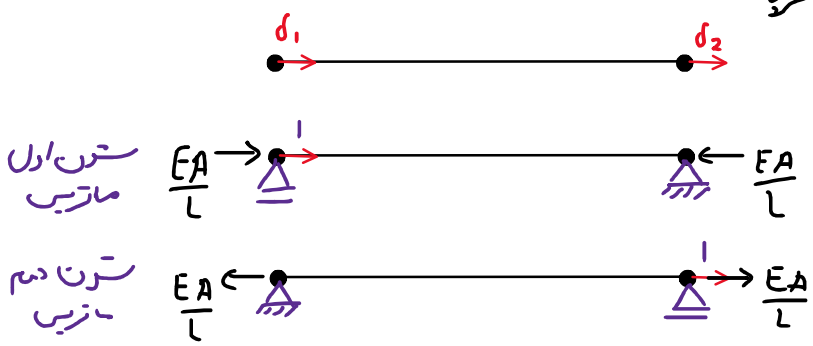
$\vdots$

* مطابق قانون ماکسول، $k_{ij} = k_{ji}$ بوده و بنابراین ماتریس سختی متقارن است.

$k_{12} = k_{21}$

مثال: ماتریس سختی هر عضو را برای درجات آزادی نشان داده شده به دست آورید.

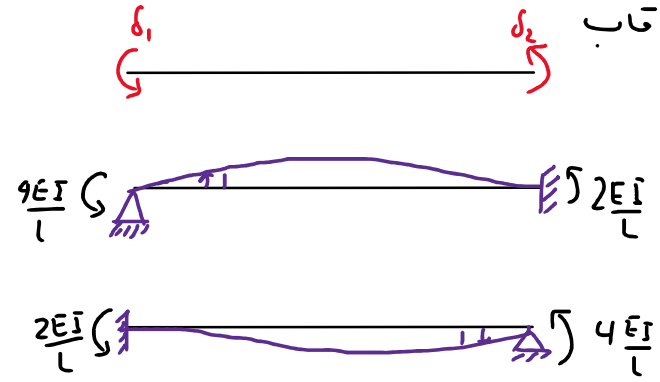
خرپا



$$k = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}_{2\times 2}$$

$$\boxed{k = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

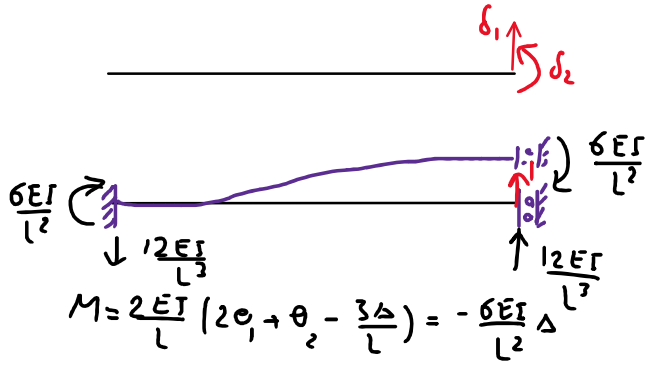
تاب



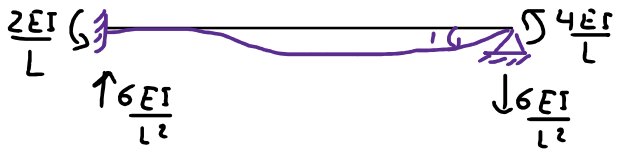
$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix}$$

$$k = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$$

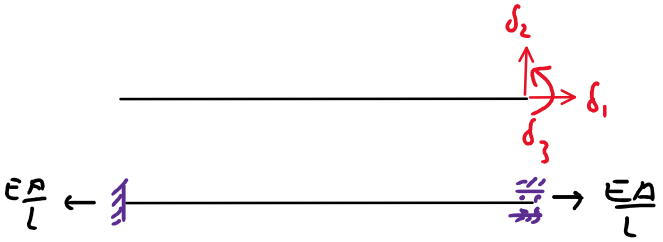
$$\boxed{k = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}$$



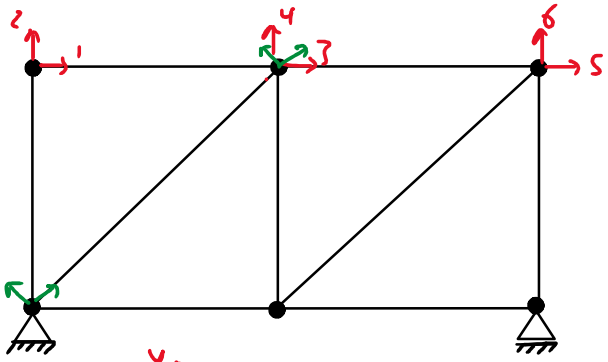
$$\boxed{k = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}}$$



$$K=\begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}_{3\times 3}$$



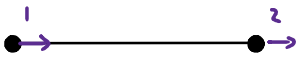
مختصات محلی و کلی

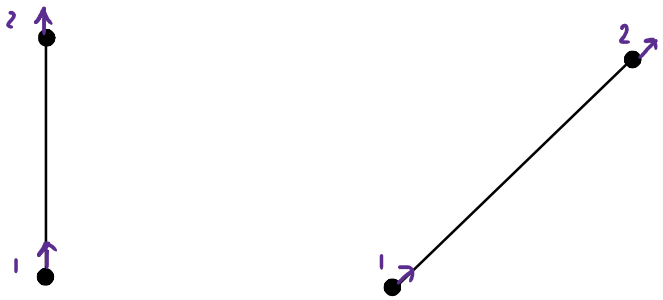


Local

Global

$$P_G = k_G \delta_G$$





$$K=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Video 41

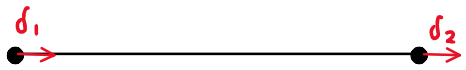
Matrix Analysis 2

Wednesday, July 17, 2024 11:47

تحلیل ماتریسی خرپاها

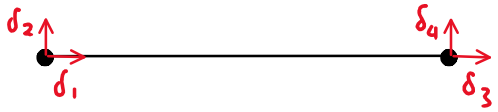
ماتریس سختی عضو خرپایی با ۲ درجه آزادی در مختصات محلی (Local)

$$k_l = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



ماتریس سختی عضو خرپایی با ۴ درجه آزادی در مختصات محلی (Local)

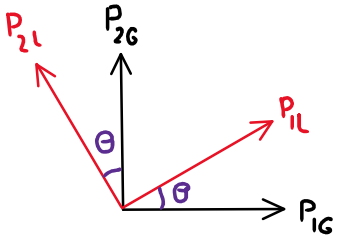
$$k_l = \frac{EA}{L}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



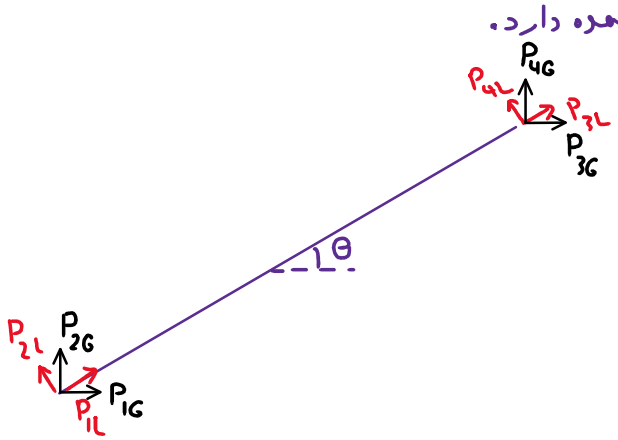
ماتریس مبدل یا ماتریس انتقال (R)

مختصات { کلی Global / محلی Local }

نقش انتقال کمیت ها را بین دستگاه مختصات محلی و کلی به عهده دارد.



$$\begin{cases} P_{1L} = P_{1G}\cos\theta + P_{2G}\sin\theta \\ P_{2L} = -P_{1G}\sin\theta + P_{2G}\cos\theta \\ P_{3L} = P_{3G}\cos\theta + P_{4G}\sin\theta \\ P_{4L} = -P_{3G}\sin\theta + P_{4G}\cos\theta \end{cases}$$



$$\begin{Bmatrix} P_{1L} \\ P_{2L} \\ P_{3L} \\ P_{4L} \end{Bmatrix} = \left[\begin{array}{cc|cc} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ \hline 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{array}\right] \begin{Bmatrix} P_{1G} \\ P_{2G} \\ P_{3G} \\ P_{4G} \end{Bmatrix}$$

$$P_L = R\,P_G$$

$$\delta_L = R\,\delta_G$$

$P_L =$ R (ماتریس انتقال یا) P_G

$P_L = R \; P_G$

ماتریس انتقال یا ماتریس دوران (Rotation)

* برای سادگی، می توان $\cos\theta$ و $\sin\theta$ را با حروف C و S نشان داد. بنابراین ماتریس مدل عضو خرپایی را می توان به شکل زیر نوشت:

$$R = \begin{bmatrix} R_J & 0 \\ 0 & R_J \end{bmatrix} \qquad R_J = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$$

* ماتریس مدل دارای خاصیت تعامد است.

$$R R^T = R^T R = I$$

$$R^{-1} = R^T$$

$$\begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} \begin{bmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A A^{-1} = I \qquad AX = b \rightarrow X = A^{-1} b$$

خلاصه

$$R = \begin{bmatrix} R_J & 0 \\ 0 & R_J \end{bmatrix} \qquad R_J = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$$

$$P_L = R \, P_G \qquad \delta_L = R \, \delta_G$$

$$P_G = R^T P_L \qquad \delta_G = R^T \delta_L$$

ماتریس سختی عضو در مختصات کلی (Global)

برای تشکیل ماتریس سختی کل سازه لازم است که ماتریس سختی کلیه اعضا در یک دستگاه مختصات مشترک نوشته شود تا امکان ترکیب آنها وجود داشته باشد.

به عبارت دیگر با تبدیل ماتریس سختی اعضا از مختصات محلی (local) به مختصات کلی (Global)، امکان ترکیب ماتریس سختی اعضا و تشکیل ماتریس سختی کل سازه فراهم می شود.

$$P_L = k_L \delta_L \rightarrow R^T P_L = R^T k_L R \, \delta_G \rightarrow P_G = \underbrace{R^T k_L R}_{k_G} \delta_G$$

$$* \; k_G = R^T k_L R \; *$$

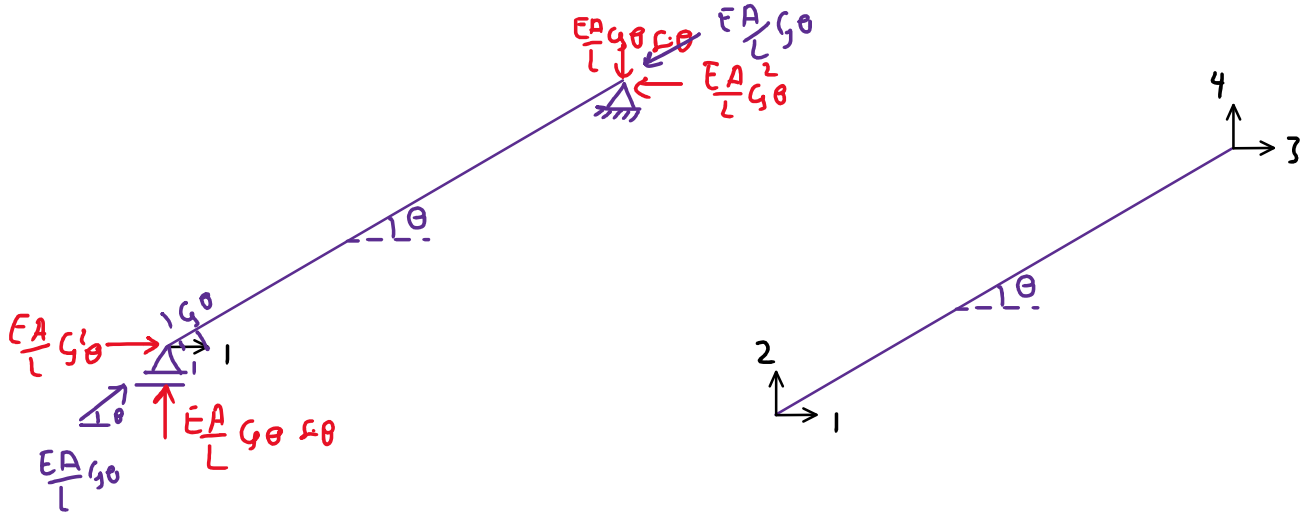
ماتریس سختی عضو خرپایی در مختصات کلی (Global)

ماتریس سختی عضو خرپایی در مختصات کلی (Global)

$$K_G = \left[\begin{array}{cc|cc} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ \hline 0 & 0 & C & -S \\ 0 & 0 & S & C \end{array}\right] \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left[\begin{array}{cc|cc} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ \hline 0 & 0 & C & S \\ 0 & 0 & -S & C \end{array}\right]$$

$$K_G = \left[\begin{array}{cc|cc} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ \hline 0 & 0 & C & -S \\ 0 & 0 & S & C \end{array}\right] \frac{EA}{L} \begin{bmatrix} C & S & -C & -S \\ 0 & 0 & 0 & 0 \\ -C & -S & C & S \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{L} \left[\begin{array}{cc|cc} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ \hline -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{array}\right]$$

$$k_G = \left[\begin{array}{c:c} K_J & -K_J \\ \hdashline -K_J & K_J \end{array}\right] \qquad k_J = \frac{EA}{L} \begin{bmatrix} C^2 & CS \\ CS & S^2 \end{bmatrix}$$



Video 42

Matrix Analysis 3

Wednesday, July 17, 2024 11:47

ماتریس سختی کل سازه

در بخش های قبلی، ابتدا ماتریس سختی عضو در مختصات محلی (k_L) به دست آمد. سپس با استفاده از ماتریس مبدل (R)، ماتریس سختی به مختصات کلی (k_G) انتقال داده شد.

حال در این قسمت، با ترکیب یا سرهم بندی (Assembly) ماتریس های سختی اعضا (k_G)، ماتریس سختی کل سازه را به دست می آید.

$$[k_L]_{mem} \xrightarrow{[R]} [k_G]_{mem} \xrightarrow{Assembly} [k_G]_{str}$$

ماتریس سختی عضو در مختصات محلی → (ماتریس انتقال) → ماتریس سختی عضو در مختصات کلی → (سرهم بندی) → ماتریس سختی کل سازه

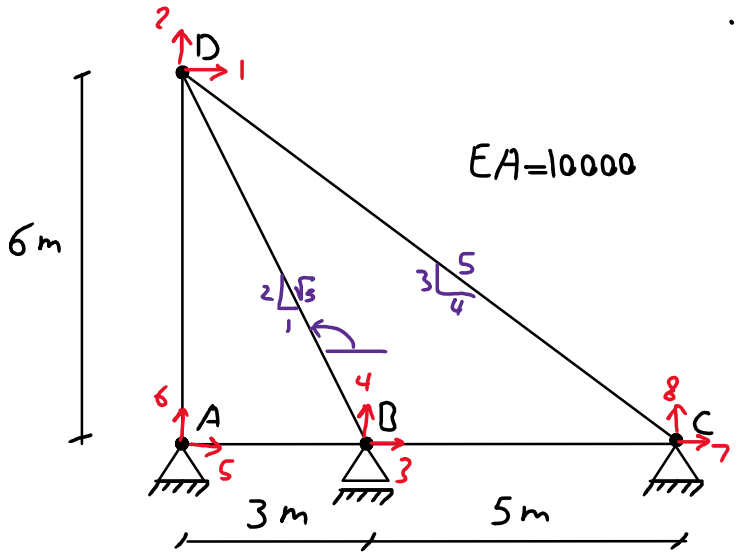
* از آنجایی که ماتریس سختی سازه بدون توجه به شرایط مرزی محاسبه می شود، ناپایدار بوده و $|k| = 0$ است. در ادامه با اعمال شرایط مرزی، سازه پایدار شده و قابل تحلیل می شود.

سرهم بندی ماتریس سختی اعضا

ماتریس سختی کل سازه از سرهم بندی ماتریس های سختی کلیه اعضا (k_G) به دست می آید. به این ترتیب که هر درایه ماتریس سختی سازه با جمع کردن درایه های همنام ماتریس سختی اعضا تعیین می شود.

* توجه کنید که سختی عضو بر نقطه روی درایه های سختی سازه وابسته به درجات آزادی آن تأثیر دارد.

مثال: ماتریس سختی کل سازه را تشکیل دهید.



$$k_L = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_G = \frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

ماتریس کل کلیه اعضا در مختصات Global ①

AB: $\frac{EA}{L} = \frac{10000}{3} = 3333$

$$k_{AB} = 3333 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 3 \\ 4 \end{matrix} \quad \begin{matrix} 5 & 6 & 3 & 4 \end{matrix}$$

BC: $\frac{EA}{L} = \frac{10000}{5} = 2000$

$$k_{BC} = 2000 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix} \quad \begin{matrix} 3 & 4 & 7 & 8 \end{matrix}$$

AD: $\frac{EA}{L} = \frac{10000}{6} = 1667$

$$k_{AD} = 1667 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 1 \\ 2 \end{matrix} \quad \begin{matrix} 5 & 6 & 1 & 2 \end{matrix}$$

BD: $\frac{EA}{L} = \frac{10000}{3\sqrt{5}} = 1491$

$\theta = 116.565$, $C = \frac{-1}{\sqrt{5}}$, $S = \frac{2}{\sqrt{5}}$

$$k_{BD} = 1491 \begin{bmatrix} 0.2 & -0.4 & -0.2 & 0.4 \\ -0.4 & 0.8 & 0.4 & -0.8 \\ -0.2 & 0.4 & 0.2 & -0.4 \\ 0.4 & -0.8 & -0.4 & 0.8 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 1 \\ 2 \end{matrix} \quad \begin{matrix} 3 & 4 & 1 & 2 \end{matrix}$$

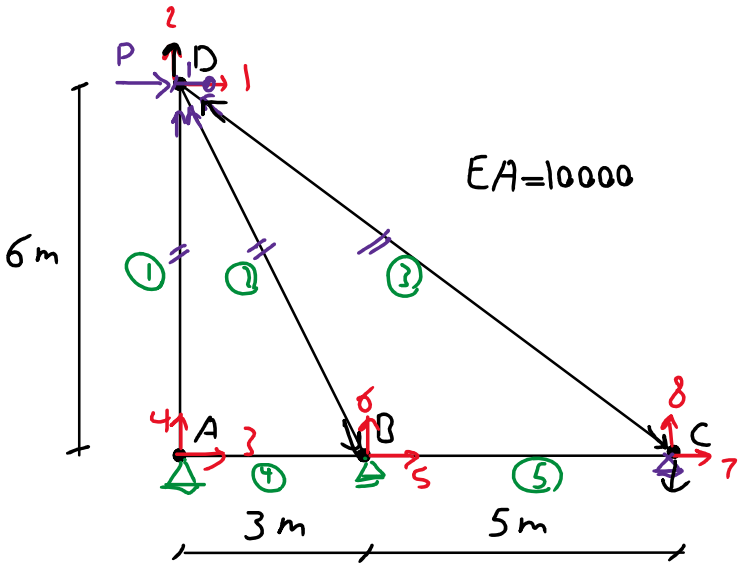
CD: $\frac{EA}{L} = \frac{10000}{10} = 1000$

$\theta = 143.13^\circ$, $C = -0.8$, $S = 0.6$

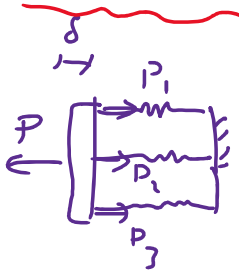
$$k_{CD} = 1000 \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 1 \\ 2 \end{matrix} \quad \begin{matrix} 7 & 8 & 1 & 2 \end{matrix}$$

۴) ماتریس سختی سازه با سرهم بندی

$$k_{str} = \begin{bmatrix} 938 & -1076 & -298 & 596 & 0 & 0 & -640 & 480 \\ -1076 & 3220 & 596 & -1193 & 0 & -1667 & 480 & -360 \\ -298 & 596 & 5631 & -596 & -3333 & 0 & -2000 & 0 \\ 596 & -1193 & -596 & 1193 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3333 & 0 & 3333 & 0 & 0 & 0 \\ 0 & -1667 & 0 & 0 & 0 & 1667 & 0 & 0 \\ -640 & 480 & -2000 & 0 & 0 & 0 & 2640 & -480 \\ 480 & -360 & 0 & 0 & 0 & 0 & -480 & 360 \end{bmatrix}_{8\times 8}$$



مفهوم سری هم بند



$$P = k_1\delta + k_2\delta + k_3\delta$$

$$P = \Sigma k_i \delta$$

$$P = k\,\delta \quad k_{11} + k_{11} + k_{11}$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_8 \end{Bmatrix}_{8\times 1} = \begin{bmatrix} k_{11} & \cdots & k_{18} \\ k_{21} & & \\ \vdots & & \\ k_{81} & & k_{88} \end{bmatrix}_{8\times 8} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_8 \end{Bmatrix}_{8\times 1}$$

(1)

$$k_G = \frac{EA}{L}\begin{array}{c} \\ 3 \\ 4 \\ 1 \\ 2 \end{array}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}\begin{array}{c} 3 \\ 4 \\ 1 \\ 2 \end{array}$$

(2)

$$k_G = \frac{EA}{L}\begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}\begin{array}{c} 5 \\ 6 \\ 1 \\ 2 \end{array}$$

(3)

$$k_G = \frac{EA}{L}\begin{array}{c} \\ 7 \\ 8 \\ 1 \\ 2 \end{array}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}\begin{array}{c} 7 \\ 8 \\ 1 \\ 2 \end{array}$$

(4)

$$k_G = \frac{EA}{L}\begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}\begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$\begin{bmatrix} -cs & -s^2 & cs & s^2 \end{bmatrix} \; 2$$

$$\begin{bmatrix} -cs & -s^2 & cs & s^2 \end{bmatrix} \; 6$$

(5)

$$K_G = \frac{EA}{L} \begin{array}{c} \begin{matrix} 5_2 & 6 & 7_2 & 8 \end{matrix} \\ \left[\begin{array}{cc|cc} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ \hline -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{array}\right] \end{array} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

$$= \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Video 43

Matrix Analysis 4

Wednesday, July 17, 2024 11:47

تحلیل سازه با اعمال شرایط مرزی

از آنجایی که شرایط مرزی (تکیه‌گاهی) در ماتریس سختی کل سازه اعمال نشده است، سازه ناپایدار است و $|k|=0$.

یعنی
① از نظر تغییر مکان، هیچ درجات آزادی باز نبود، و سازه دارای حرکت به شکل جسم صلب است.
② از نظر نیرویی، معادلات $P=k\delta$ مستقل نیستند و با معادلات تعادل به یکدیگر وابسته‌اند.

* با ایجاد تکیه‌گاه‌های کافی برای سازه، سازه پایدار و قابل تحلیل می‌شود.

یعنی حداقل ۳ مجهول تکیه‌گاهی برای ارضاء ۳ معادله تعادل

اعمال شرایط مرزی با جداسازی درجات آزادی آزاد (δ_f) و مقید (δ_s) در ماتریس سختی و یا به عبارت دیگر با پارتیشن (Partition) بندی ماتریس سختی انجام می‌پذیرد.

	مجهول	معلوم
آزاد f	δ_f	P_f
مقید s	P_s	δ_s

$$\begin{Bmatrix} P_f \\ P_s \end{Bmatrix} = \left[\begin{array}{c|c} k_{ff} & k_{fs} \\ \hline k_{sf} & k_{ss} \end{array}\right] \begin{Bmatrix} \delta_f \\ \delta_s \end{Bmatrix}$$

$$P_f = k_{ff}\,\delta_f + k_{fs}\,\delta_s \quad (\delta_s = 0)$$

$$\boxed{P_f = k_{ff}\,\delta_f}$$

* محاسبه مجهولات تغییر مکان (δ_f)

با اعمال شرایط مرزی یعنی $\delta_s = 0$، سازه پایدار شده و این معادله قابل حل است. $|k_{ff}| \neq 0$

$$P_s = k_{sf}\,\delta_f + k_{ss}\,\delta_s \quad (\delta_s = 0)$$

* $P_s = k_{sf}\,\delta_f$

* محاسبه نیروهای تکیه‌گاهی (P_s)

نیروی داخلی اعضا نیز به صورت زیر به دست می‌آید:

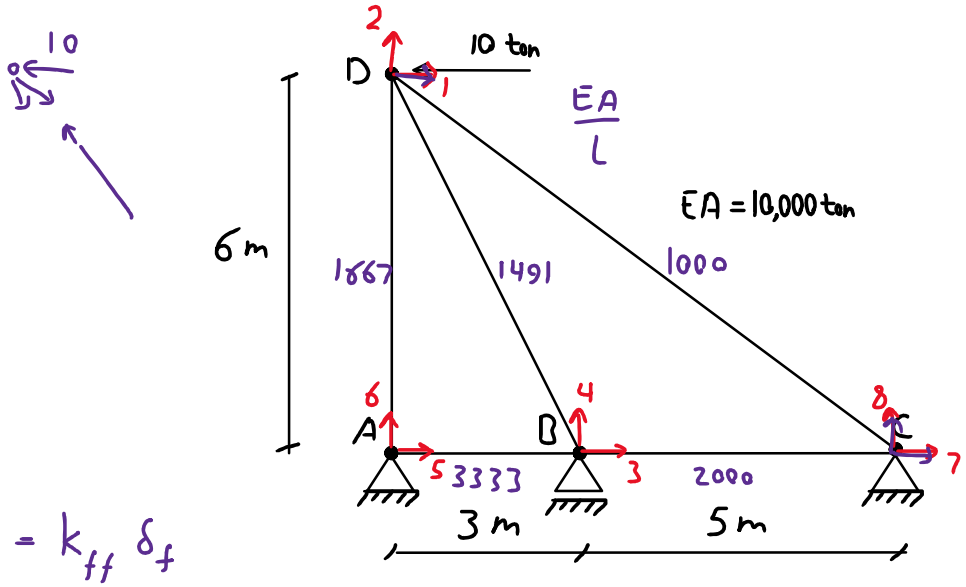
$$P_L = k_L \delta_L = k_L R \delta_G$$

$$\boxed{P_L = k_L R \delta_G}$$

* محاسبه نیروی اعضا

معمولاً برای محاسبه نیروهای عکس‌العمل تکیه‌گاهی از فرمول بالا استفاده نمی‌شود؛ بلکه پس از به دست آوردن نیروی کلیه اعضا، عکس‌العمل‌ها با نوشتن معادلات تعادل در تکیه‌گاه‌ها به دست می‌آید.

مثال: تغییرمکان های گره، نیروی عضو CD و عکس العمل های تکیه گاهی را به دست آورید.



$$P_f = k_{ff}\,\delta_f$$

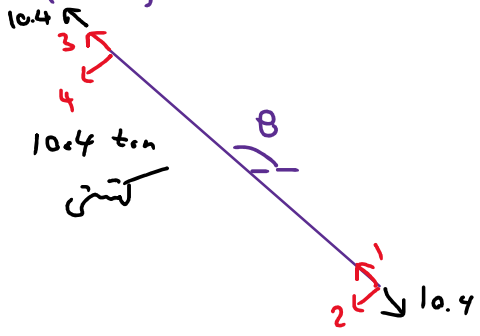
$$\begin{bmatrix} 938 & -1076 & -298 \\ -1076 & 3220 & 596 \\ -298 & 596 & 5631 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 0 \\ 0 \end{Bmatrix} \longrightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -17.34\times10^{-3} \\ -5.74\times10^{-3} \\ -0.31\times10^{-3} \end{Bmatrix}}$$

$C = -0.8$, $S = 0.6$

$$P_{L,CD} = k_L R \delta_G = 1000 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -0.8 & 0.6 & 0 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & -0.8 & 0.6 \\ 0 & 0 & -0.6 & -0.8 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -17.34\times10^{-3} \\ -5.74\times10^{-3} \end{Bmatrix}$$

$$P_{L,CD} = 1000 \begin{bmatrix} -0.8 & 0.6 & 0.8 & -0.6 \\ 0 & 0 & 0 & 0 \\ 0.8 & -0.6 & -0.8 & 0.6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -17.34\times10^{-3} \\ -5.74\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} -10.4 \\ 0 \\ 10.4 \\ 0 \end{Bmatrix}$$

$$\boxed{P_{L,CD} = \begin{Bmatrix} -10.4 \\ 0 \\ 10.4 \\ 0 \end{Bmatrix}}$$



$$P_s = k_{sf}\,\delta_f$$

$$P_f = \begin{bmatrix} 596 & -1193 & -596 \\ 0 & 0 & -3333 \\ 0 & -1667 & 0 \\ -640 & 480 & -2000 \\ 480 & -360 & 0 \end{bmatrix} \begin{Bmatrix} -17.34\times10^{-3} \\ -5.74\times10^{-3} \\ -0.31\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} -3.3 \\ 1 \\ 9.6 \\ 9 \\ -6.3 \end{Bmatrix}$$

$$\boxed{P_f = \begin{Bmatrix} -3.3 \\ 1 \\ 9.6 \\ 9 \\ -6.3 \end{Bmatrix}}$$

محاسبه k_{ff} به صورت مستقیم بدون سرهم بندی

$$k_{11} = 1000\times0.64 + 1491\times0.2 = 938$$

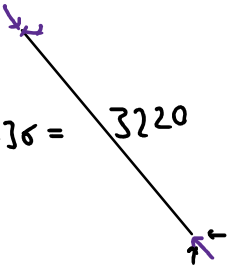
$$k_{21} = -1000\times0.48 - 1491\times0.4 = -1076$$

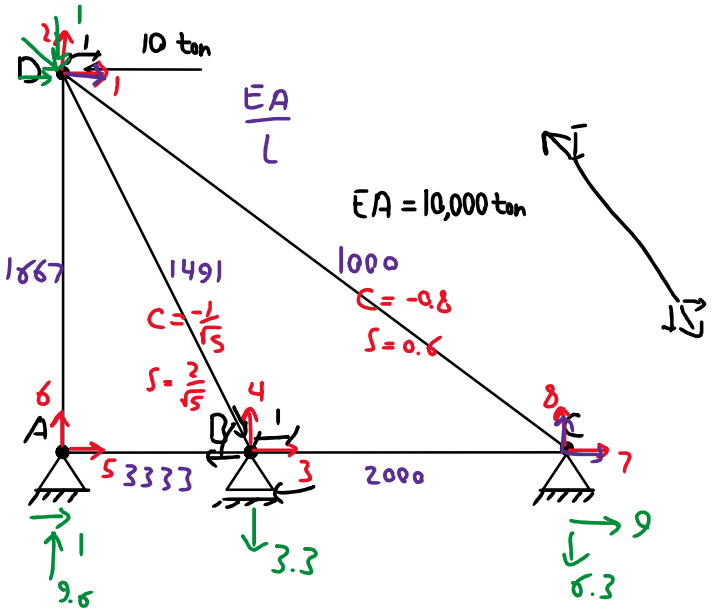
$$k_{31} = -1491\times0.2 = -298$$

$$k_{22} = 1667 + 1491\times0.8 + 1000\times0.36 = 3220$$

$$k_{32} = 1491\times0.4 = 596$$

$$k_{33} = 2000 + 3333 + 1491\times0.2 = 5631$$





$$k_{ff} = \begin{bmatrix} 938 & -1076 & -298 \\ -1076 & 3220 & 596 \\ -298 & 596 & 5631 \end{bmatrix}$$

حل معادلات به روش حذف گوس :

$$\begin{bmatrix} 938 & -1076 & -298 \\ -1076 & 3220 & 596 \\ -298 & 596 & 5631 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ 0 \\ 0 \end{Bmatrix}$$

$\frac{1076}{938}\times$

$$\begin{bmatrix} 938 & -1076 & -298 & | & -10 \\ -1076 & 3220 & 596 & | & 0 \\ -298 & 596 & 5631 & | & 0 \end{bmatrix}$$

$\frac{298}{938}\times$

$$\begin{bmatrix} 938 & -1076 & -298 & | & -10 \\ 0 & 1984.7 & 254 & | & -11.47 \\ -298 & 596 & 5631 & | & 0 \end{bmatrix} \longrightarrow$$

$\frac{-254}{1984.7}\times$

$$\begin{bmatrix} 938 & -1076 & -298 & | & -10 \\ 0 & 1984.7 & 254 & | & -11.47 \\ 0 & 254 & 5536 & | & -3.18 \end{bmatrix} \longrightarrow \begin{bmatrix} 938 & -1076 & -298 & | & -10 \\ 0 & 1984.7 & 254 & | & -11.47 \\ 0 & 0 & 5503 & | & -1.71 \end{bmatrix}$$

بنابراین معادلات به شکل زیر در می آید:

$$\begin{bmatrix} 938 & -1076 & -298 \\ 0 & 1984.7 & 254 \\ 0 & 0 & 5503 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -10 \\ -11.47 \\ -1.71 \end{Bmatrix} \longrightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -17.3\times10^{-3} \\ -5.7\times10^{-3} \\ -0.3\times10^{-3} \end{Bmatrix}}$$

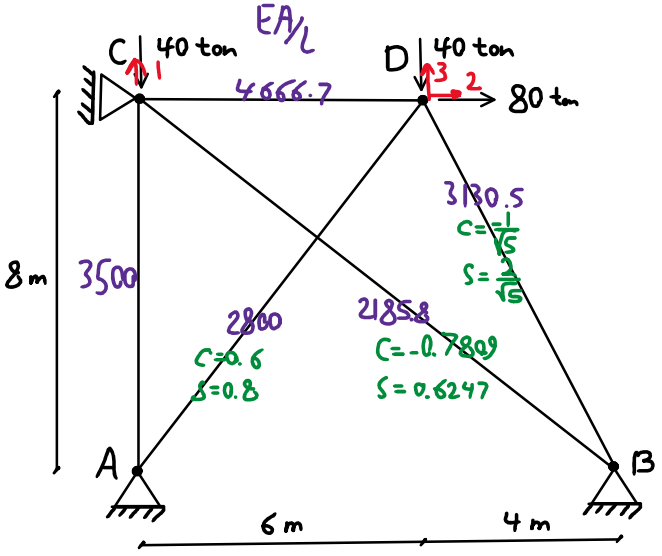
Video 44

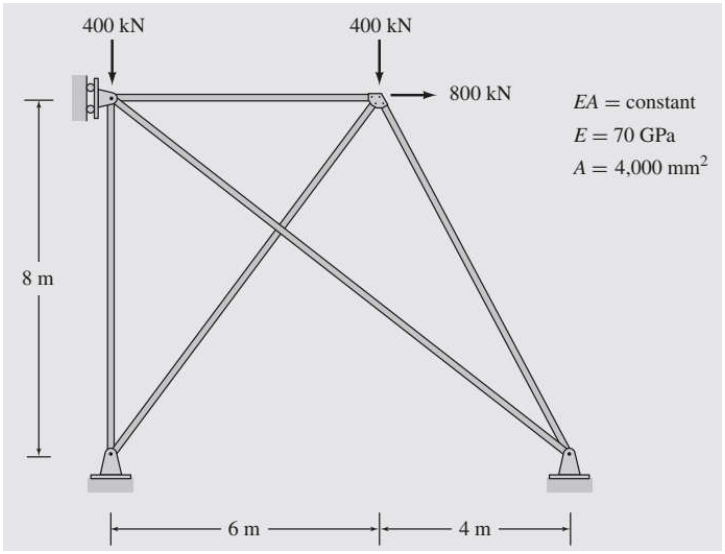
Matrix Analysis 5

Wednesday, July 17, 2024 11:47

تحلیل ماتریسی سازه‌ها

مثال: خرپای شکل زیر را با روش ماتریسی تحلیل کنید و جابجایی گره‌های C و D و نیروی اعضای AC و BD را به دست آورید.





$EA = 70\times10^3 \times 4{,}000 = 280\times10^6\ N$

$EA = 280\times10^3\ kN = 28000\ ton$

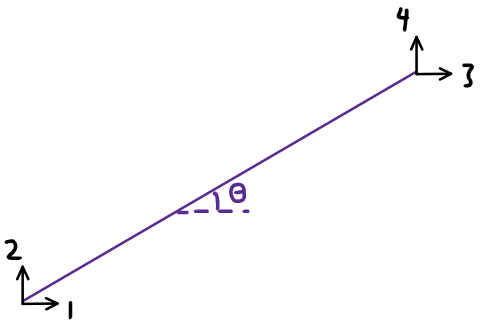
ماتریس k_{ff} به روش مستقیم

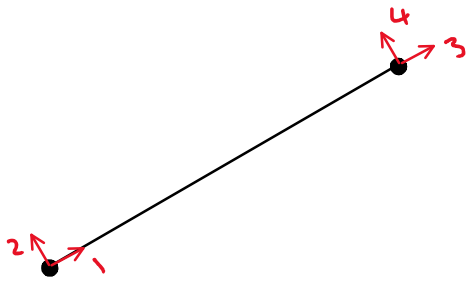
$$k_{ff} = \begin{bmatrix} 4353 & 0 & 0 \\ 0 & 6300.8 & 91.8 \\ 0 & 91.8 & 4296.4 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$$k_{ff}\,\delta_f = P_f$$

$$\begin{bmatrix} 4353 & 0 & 0 \\ 0 & 6300.8 & 91.8 \\ 0 & 91.8 & 4296.4 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -40 \\ 80 \\ -40 \end{Bmatrix} \longrightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} -9.18\times10^{-3} \\ 12.83\times10^{-3} \\ -9.58\times10^{-3} \end{Bmatrix}$$

ماتریس k_{ff} با سرفهم بندی ماتریس کلی اعضا





$$K_L = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}①\\②\\③\\④\end{matrix} \qquad \begin{matrix}① & ② & ③ & ④\end{matrix}$$

$$K_G = \frac{EA}{L}\left[\begin{array}{cc|cc} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ \hline -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{array}\right]\begin{matrix}①\\②\\③\\④\end{matrix} \qquad \begin{matrix}① & ② & ③ & ④\end{matrix}$$

① ماتریس k_{ff} مربوط به سرعت بندی

$$[k_{ff}]_{AC} = 3500\,[1]\ ① \qquad \begin{matrix}①\end{matrix}$$

$$[k_{ff}]_{AD} = 2800\begin{bmatrix} 0.36 & 0.48 \\ 0.48 & 0.64 \end{bmatrix}\begin{matrix}②\\③\end{matrix} \qquad \begin{matrix}② & ③\end{matrix}$$

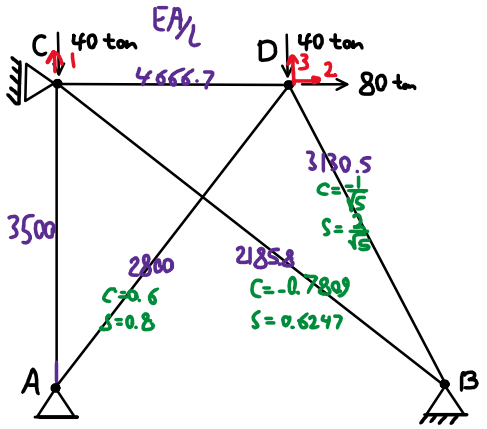
$$[k_{ff}]_{BC} = 2185.8\,[0.39]\ ① \qquad \begin{matrix}①\end{matrix}$$

$$[k_{ff}]_{BD} = 3130.5\begin{bmatrix} 0.2 & -0.4 \\ -0.4 & 0.8 \end{bmatrix}\begin{matrix}②\\③\end{matrix} \qquad \begin{matrix}② & ③\end{matrix}$$

$$[k_{ff}]_{CD} = 46667\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{matrix}①\\②\\③\end{matrix} \qquad \begin{matrix}① & ② & ③\end{matrix}$$

① ماتریس k_{ff} سازه با سرعم بندی

$$k_{ff} = \begin{bmatrix} 4353 & 0 & 0 \\ 0 & 6300.8 & 91.8 \\ 0 & 91.8 & 4296.4 \end{bmatrix}\begin{matrix}①\\②\\③\end{matrix} \qquad \begin{matrix}① & ② & ③\end{matrix}$$



---

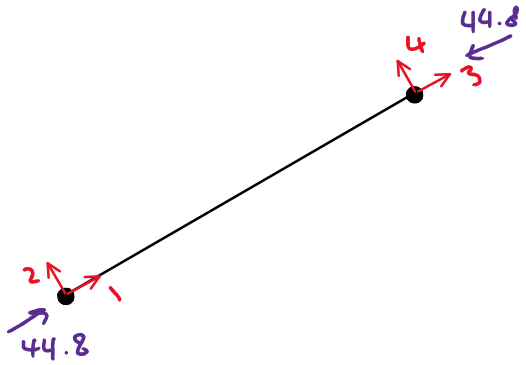
② محاسبه نیروی اعضا

$$P_L = k_L \delta_L = k_L R \delta_G$$

$$(P_L)_{BD} = 3130.5 \underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{k_L} \underbrace{\begin{bmatrix} -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 & 0 \\ -\frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 0 & 0 & -\frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{bmatrix}}_{R} \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 12.83\times10^{-3} \\ -9.58\times10^{-3} \end{Bmatrix}}_{\delta_G}$$

$$R = \left[\begin{array}{cc:cc} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ \hdashline 0 & 0 & C & S \\ 0 & 0 & -S & C \end{array}\right]$$

$$(P_L)_{BD} = 3130.5 \underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{k_L} \underbrace{\begin{Bmatrix} 0 \\ 0 \\ -14.3\times10^{-3} \\ -7.2\times10^{-3} \end{Bmatrix}}_{\delta_L} = \begin{Bmatrix} 44.8 \\ 0 \\ -44.8 \\ 0 \end{Bmatrix}$$



$$(P_L)_{BD} = \begin{Bmatrix} 44.8 \\ 0 \\ -44.8 \\ 0 \end{Bmatrix}$$

اگر فقط P_L را نیاز داریم و δ_L را نمی‌خواهیم:

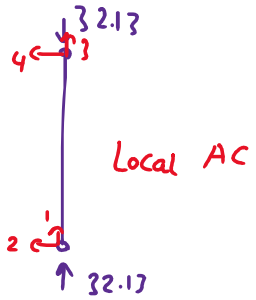
$$P_L = k_L \delta_L = k_L R \delta_G$$

$$*k_L R = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} = \begin{bmatrix} C & S & -C & -S \\ 0 & 0 & 0 & 0 \\ -C & -S & C & S \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

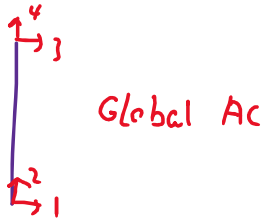
$$(P_L)_{BD} = 3130.5 \begin{bmatrix} \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 12.83\times10^{-3} \\ -9.58\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} 44.8 \\ 0 \\ -44.8 \\ 0 \end{Bmatrix}$$

دستی

$$(P_L)_{Ac} = 3500 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -9.18\times10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 32.13 \\ 0 \\ -32.13 \\ 0 \end{Bmatrix}$$



$$(P_L)_{Ac} = \begin{Bmatrix} 32.13 \\ 0 \\ -32.13 \\ 0 \end{Bmatrix}$$



کنوبیس

$$(P_L)_{Ac} = k_L R \delta_G = 3500 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -9.18\times10^{-3} \end{Bmatrix}$$

$R\delta_G = \delta_L$

$$= 3500 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -9.18 \times 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 32.13 \\ 0 \\ -32.13 \\ 0 \end{Bmatrix}$$

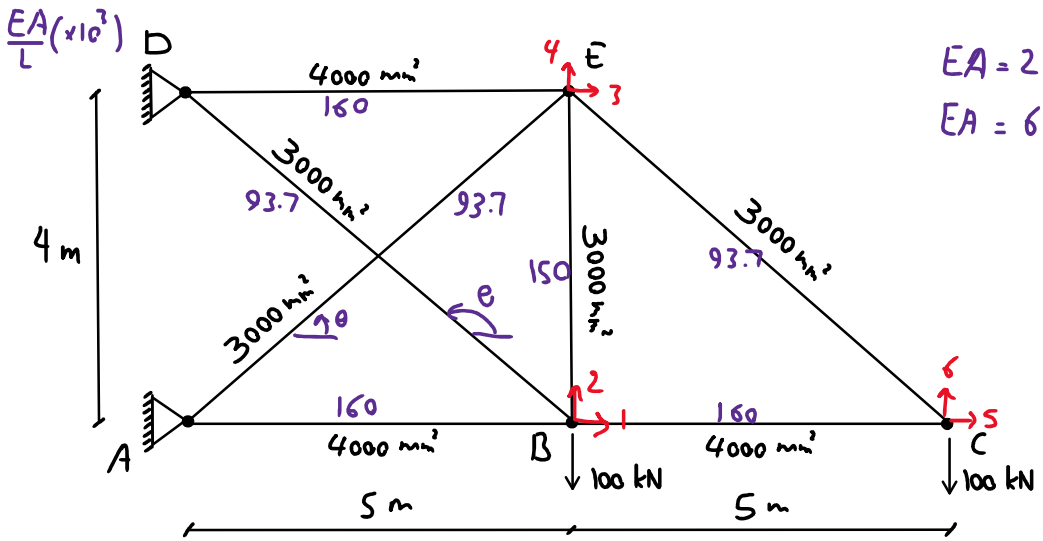
Video 45

Matrix Analysis 6

Wednesday, July 17, 2024 11:47

تحلیل ماتریسی سازه‌ها

مثال: خرپای شکل زیر را به روش ماتریسی تحلیل کنید و جابجایی گره‌های آزاد و نیروی اعضای AE و BE را به دست آورید. ($E = 200\ GPa$)



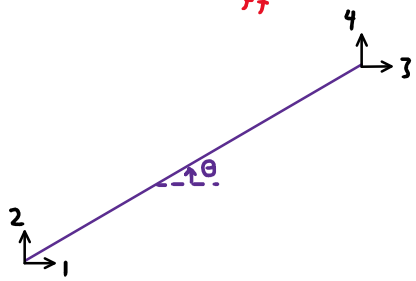
$EA = 200 \times 10^3 \times 4000 = 800 \times 10^3\ kN$

$EA = 600 \times 10^3\ kN$

$\begin{cases} C = 0.7809 \\ S = 0.6247 \end{cases}$

$[k_{ff}]_{6\times 6}$

① ماتریس سختی k_{ff} هر عضو



$$K_G = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$$[k_{ff}]_{AB} = 160\times 10^3 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[k_{ff}]_{BC} = 160\times 10^3 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[k_{ff}]_{DE} = 160\times 10^3 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[k_{ff}]_{BE} = 150\times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$[k_{ff}]_{AE} = 93.7\times 10^3 \begin{bmatrix} 0.6098 & 0.4878 \\ 0.4878 & 0.3903 \end{bmatrix}$$

$$[k_{ff}]_{BD} = 93.7\times 10^3 \begin{bmatrix} 0.6098 & -0.4878 \\ -0.4878 & 0.3903 \end{bmatrix}$$

$$[k_{ff}]_{CE} = 93.7\times 10^3 \begin{bmatrix} 0.6098 & -0.4878 & -0.6098 & 0.4878 \\ -0.4878 & 0.3903 & 0.4878 & -0.3903 \\ -0.6098 & 0.4878 & 0.6098 & -0.4878 \\ 0.4878 & -0.3903 & -0.4878 & 0.3903 \end{bmatrix}$$

② ماتریس سختی k_{ff} سازه با سر هم بندی

② ماتریس سختی k_{ff} سازه با شماره‌بندی

$$k_{ff} = 10^{3} \times \begin{bmatrix} 377.14 & -45.71 & 0 & 0 & -160 & 0 \\ -45.71 & 186.57 & 0 & -150 & 0 & 0 \\ 0 & 0 & 274.28 & 0 & -57.14 & 45.71 \\ 0 & -150 & 0 & 223.14 & 45.71 & -36.57 \\ -160 & 0 & -57.14 & 45.71 & 217.14 & -45.71 \\ 0 & 0 & 45.71 & -36.57 & -45.71 & 36.57 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{matrix}$$

$$P_f = \begin{Bmatrix} 0 \\ -100 \\ 0 \\ 0 \\ 0 \\ -100 \end{Bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{matrix}$$

③ حل دستگاه معادلات $k_{ff}\delta_f = P_f$

MATLAB از ←

$$\delta_f = 10^{-3} \times \begin{Bmatrix} -1.6 \\ -4.7 \\ 1.6 \\ -4.7 \\ -2.3 \\ -12.3 \end{Bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{matrix}$$

① محاسبه نیروی اعضا

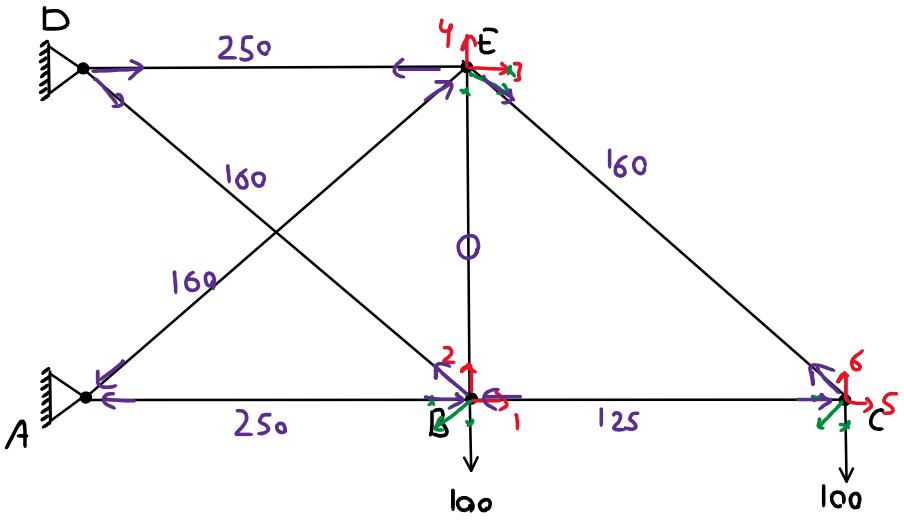
$P_L = k_L R \delta_G$

$$(P_L)_{BE} = 150 \times 10^{3} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} 10^{-3} \times \begin{Bmatrix} -1.6 \\ -4.7 \\ 1.6 \\ -4.7 \end{Bmatrix} =$$

$$150 \times 10^{3} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-3} \begin{Bmatrix} -4.7 \\ 1.6 \\ -4.7 \\ -1.6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$(P_L)_{AE} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left[\begin{array}{cc:cc} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ \hdashline 0 & 0 & C & S \\ 0 & 0 & -S & C \end{array}\right] \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = \begin{bmatrix} C & S & -C & -S \\ 0 & 0 & 0 & 0 \\ -C & -S & C & S \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

$$(P_L)_{AE} = 93.7\times10^{-3}\begin{bmatrix} 0.7809 & 0.6247 & -0.7809 & -0.6247 \\ 0 & 0 & 0 & 0 \\ -0.7809 & -0.6247 & 0.7809 & 0.6247 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-3}\times\begin{Bmatrix} 0 \\ 0 \\ 1.6 \\ -4.7 \end{Bmatrix} = \begin{Bmatrix} 160 \\ 0 \\ -160 \\ 0 \end{Bmatrix}$$



Video 46

Matrix Analysis 7

Wednesday, July 17, 2024 11:48

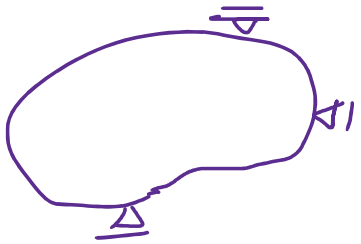
مقدار ویژه و بردار ویژه ماتریس سختی

① توضیح در خلال مثال عضو خرپایی

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$\Sigma F_x = 0 \rightarrow P_1 + P_2 = 0 \rightarrow |k| = 0$$

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$



$$Ax = bx$$

بردار ویژه ← x
مقدار ویژه ← b

* مقادیر ویژه و بردارهای ویژه به ترتیب نمایانگر ازرهای ذخیره شده و تغییرمکان عضو در یک حالت تغییرمکان می باشد.

$$\delta^T (k\delta = \lambda\delta) \rightarrow \delta^T k \delta = \lambda \delta^T \delta$$

$$\boxed{W = \delta^T P = \lambda a}$$

$$k\delta = \lambda\delta \rightarrow (k - \lambda I)\delta = 0 \rightarrow \begin{cases} ① \ \delta = 0 \\ ② \ |K - \lambda I| = 0 \end{cases}$$ جواب بدیهی

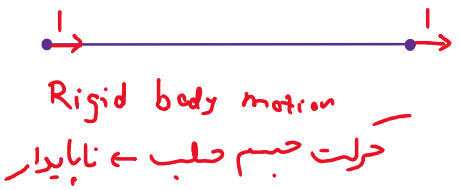
$$\begin{bmatrix} 60 & -6 \\ -6 & 60 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$\begin{vmatrix} \frac{EA}{L}-\lambda & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L}-\lambda \end{vmatrix} = 0 \rightarrow \left(\frac{EA}{L}-\lambda\right)^2 - \left(-\frac{EA}{L}\right)^2 = 0$$

$$\rightarrow \lambda^2 - \frac{2EA}{L}\lambda = 0 \rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = \frac{2EA}{L} \end{cases}$$

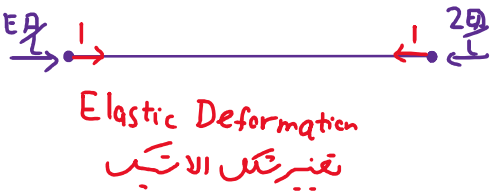
$\lambda = 0$

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = 0 \rightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

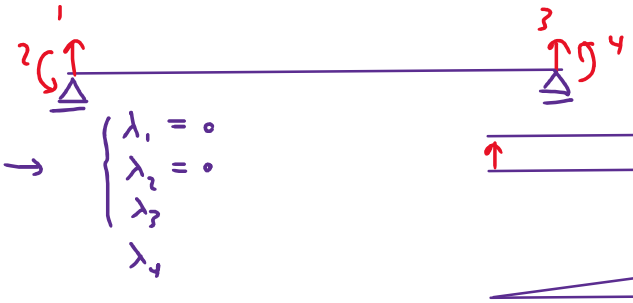


$\lambda = \frac{2EA}{L}$

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{2EA}{L} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \rightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

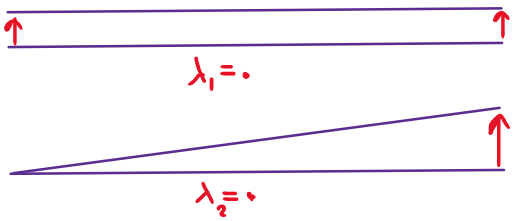


$$W = \lambda \delta^T \delta \rightarrow W = \frac{2EA}{L} \times 2 = \frac{4EA}{L}$$



$$K\delta = \lambda\delta \rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 \\ \lambda_4 \end{cases}$$

مثال نی ②



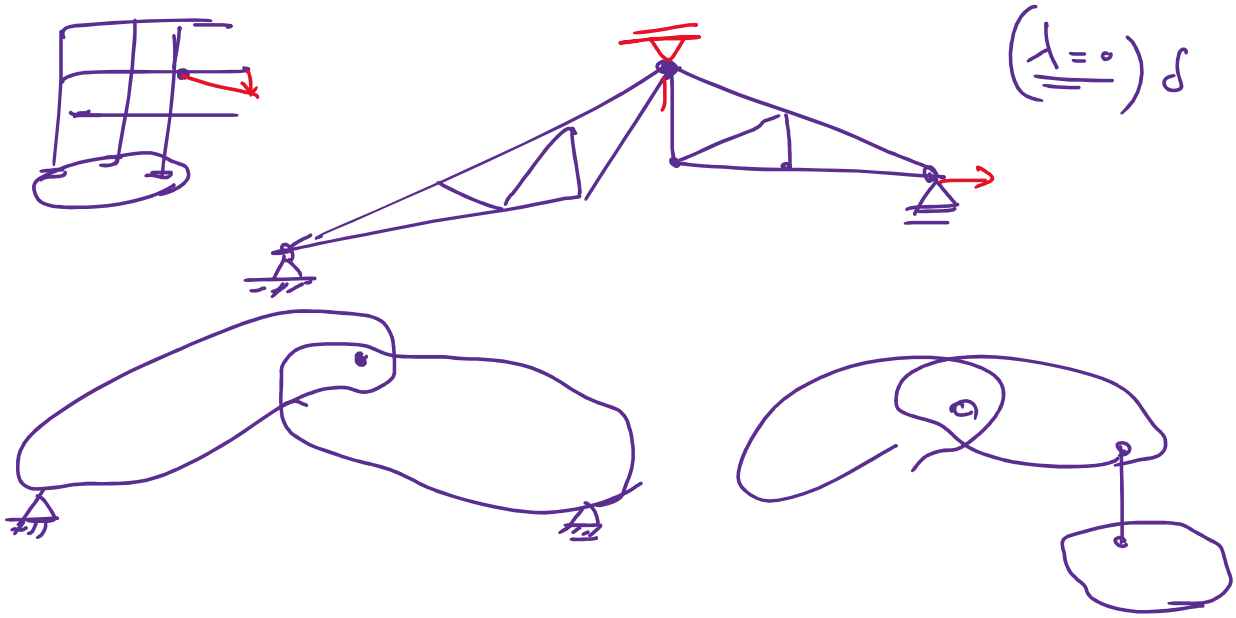
$$K\delta = \lambda\delta$$

$$|K - \lambda I| = 0$$

نتیجه در مورد سازه ③

① به تعداد $\lambda = 0$ حرکت صلب داریم و برای پایدار شدن سازه به همین تعداد قید نیاز هستیم

② برای هر بردار ویژه مربوط به $\lambda = 0$، می‌توان حرکت صلب سازه و قید مورد نیاز برای پایداری را پیدا کرد



Video 47

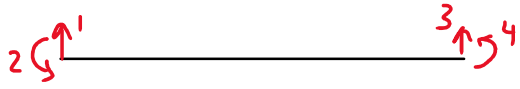
Matrix Analysis 8

Wednesday, July 17, 2024 11:48

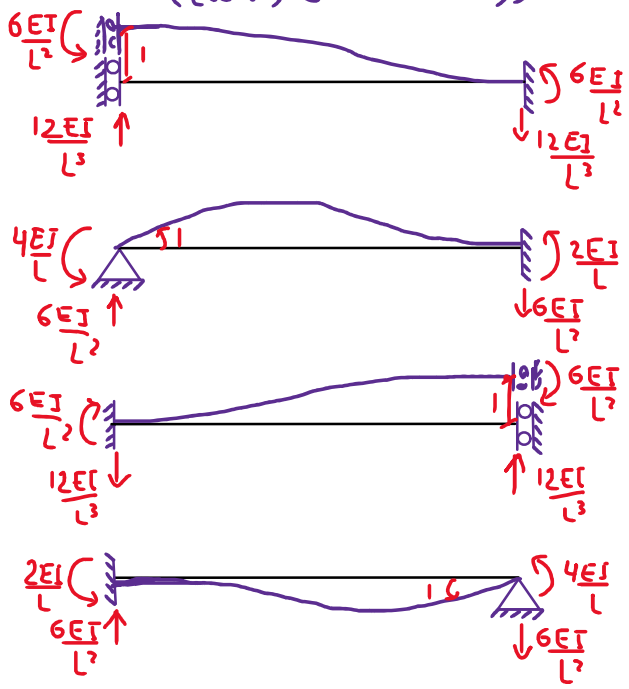
تحلیل ماتریسی تیر و قاب ها

① ماتریس سختی عضو در مختصات محلی (Local)

ماتریس سختی عضو خمشی با ۴ درجه آزادی (در مختصات محلی (Local))



$$k_l = \begin{array}{c} \begin{matrix} ① & ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$



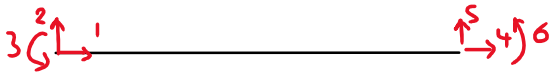
k_{ss} $\frac{12EI}{L^3}$ 1,3

$k_{s\theta}$ $\frac{6EI}{L^2}$

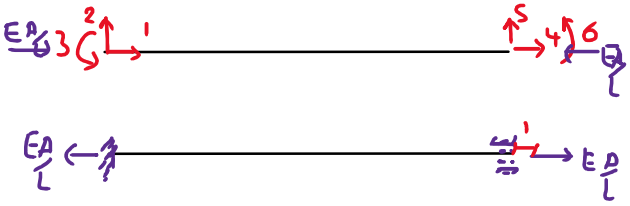
$k_{\theta\theta}$ $\frac{4EI}{L}, \frac{2EI}{L}$

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

ماتریس سختی عضو خمشی با ۶ درجه آزادی (در مختصات محلی (Local))

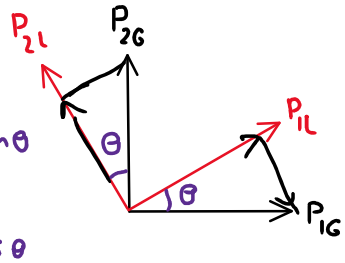


$$k_l = \begin{array}{c} \begin{matrix} ① & ② & ③ & ④ & ⑤ & ⑥ \end{matrix} \\ \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{matrix}$$



$$k_l = \begin{array}{c} \begin{matrix} 2 & 3 & 5 & 6 \\ ① & ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} 1,2 \\ 2,3 \\ 3,5 \\ 4,6 \end{matrix}$$

② ماتریس مبدل یا ماتریس انتقال (R)



$$P_{1L} = P_{1G}\cos\theta + P_{2G}\sin\theta$$

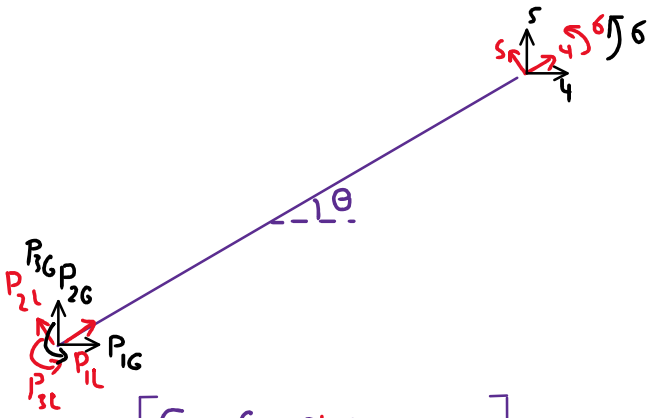
$$P_{2L} = -P_{1G}\sin\theta + P_{2G}\cos\theta$$

$$P_{3L} = P_{3G}$$

$$P_{4L} = P_{4G}\cos\theta + P_{5G}\sin\theta$$

$$P_{5L} = -P_{4G}\sin\theta + P_{5G}\cos\theta$$

$$P_{6L} = P_{6G}$$



$$R = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

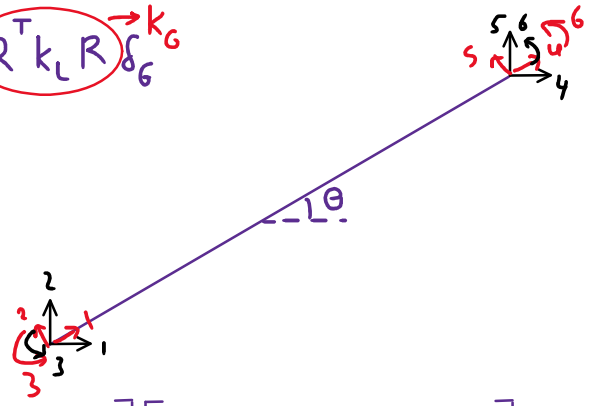
$$P_L = R\,P_G \qquad \delta_L = R\,\delta_G$$

$$R = \begin{bmatrix} R_J & 0 \\ 0 & R_J \end{bmatrix} \qquad R_J = \begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ماتریس سختی عضو در مختصات کلی (Global) ③

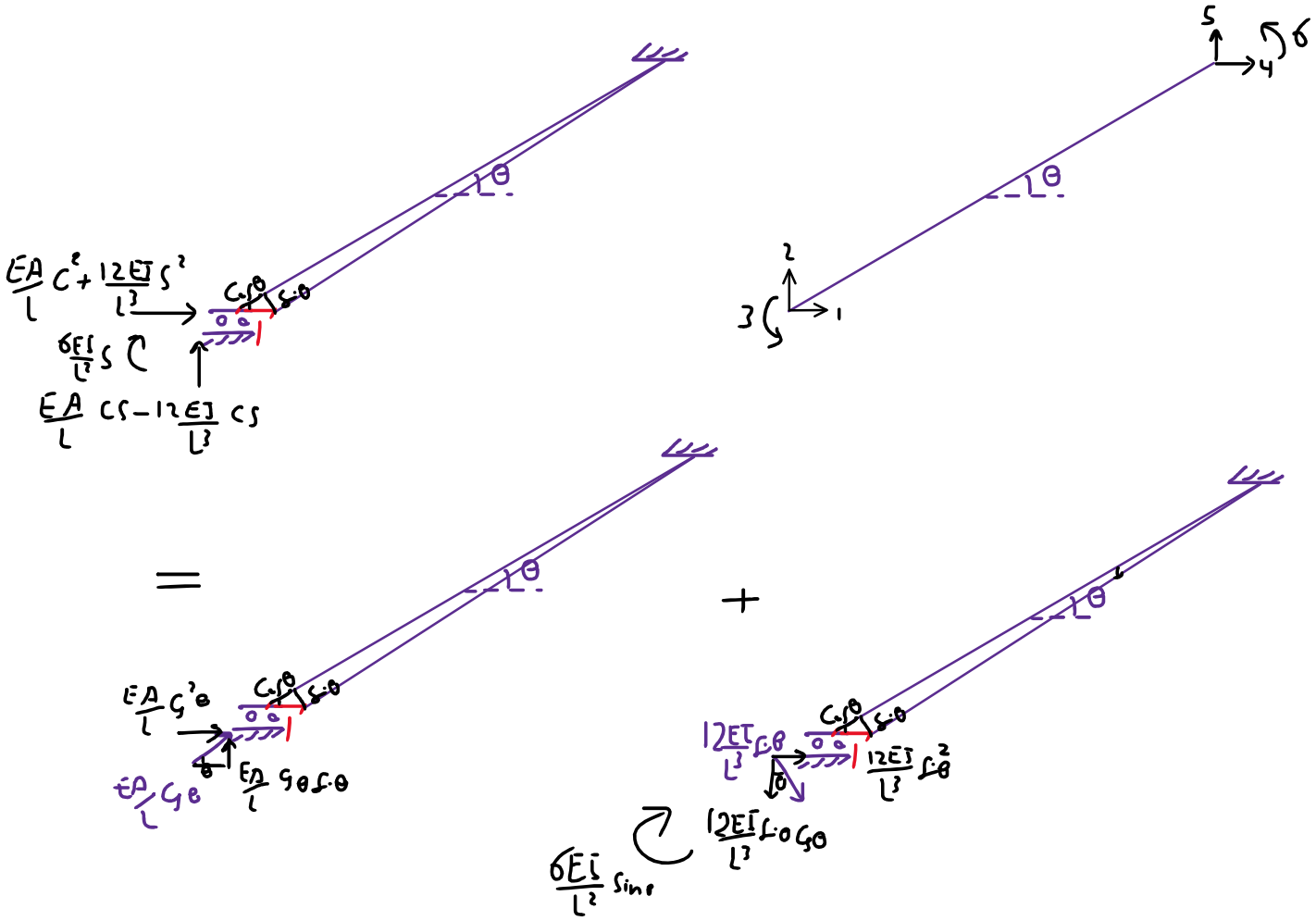
$$P_L = k_L\,\delta_L \longrightarrow R\,P_G = k_L\,R\,\delta_G \longrightarrow P_G = \overbrace{R^T k_L R}^{k_G}\,\delta_G$$

$$k_G = R^T k_L R$$



$$k_G = \begin{bmatrix} C & -S & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & -S & 0 \\ 0 & 0 & 0 & S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_G = \left[\begin{array}{ccc|ccc} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} & -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} & c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} \\ -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$



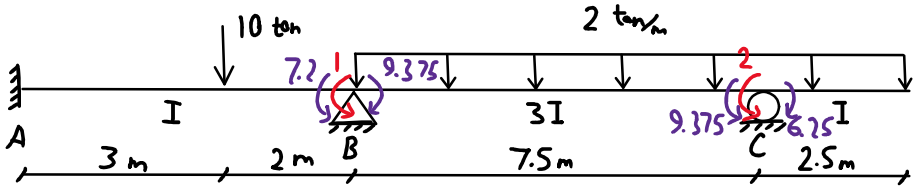
Video 48

Matrix Analysis 9

Wednesday, July 17, 2024 11:48

تحلیل ماتریسی سازه ها

مثال: تیر شکل زیر را به روش ماتریسی تحلیل کنید.

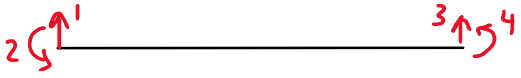


$$k_{ff}\,\delta_f = P_f \qquad P_L = k_L \delta_L + FER$$

$$[k_{ff}]_{AB} = \left[\frac{4EI}{5}\right] = EI\,[0.8] \quad ①$$

* ماتریس سختی k_{ff} اعضا

$$[k_{ff}]_{BC} = \begin{bmatrix} \frac{4(3EI)}{7.5} & \frac{2(3EI)}{7.5} \\ \frac{2(3EI)}{7.5} & \frac{4(3EI)}{7.5} \end{bmatrix} = EI \begin{bmatrix} 1.6 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$



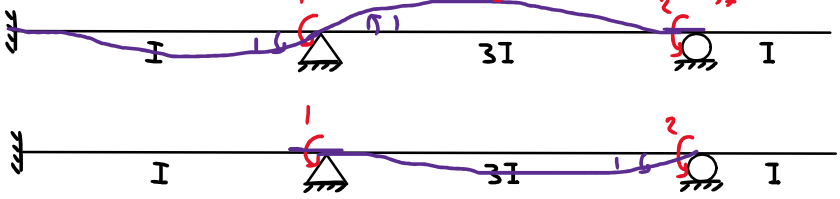
$$k_G = k_L = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$$\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$$

* ماتریس سختی k_{ff} سازه با سوهم بندی

$$k_{ff} = EI \begin{bmatrix} 2.4 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

ماتریس k_{ff} به صورت مستقیم



* محاسبه بارگذاری P_f

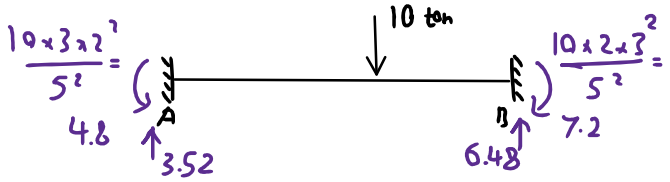
$$P_f = \begin{Bmatrix} 7.2 - 9.375 \\ 9.375 - 6.25 \end{Bmatrix} = \begin{Bmatrix} -2.175 \\ 3.125 \end{Bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

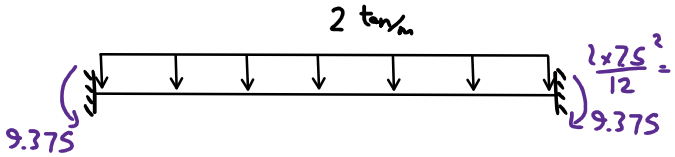
* محاسبه تغییرمکان های گره ای δ_f

$$EI \begin{bmatrix} 2.4 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -2.175 \\ 3.125 \end{Bmatrix}$$

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -1.87 \\ 2.89 \end{Bmatrix}$$

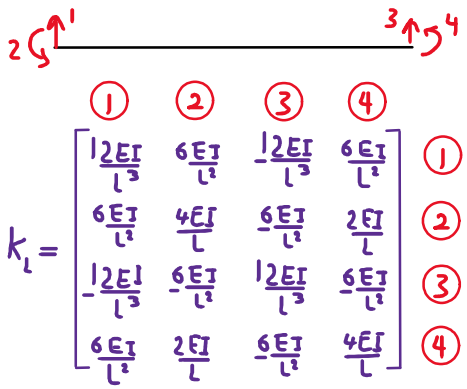
$$k_{ff} = \begin{bmatrix} \frac{4EI}{5} + \frac{4(3EI)}{7.5} & \frac{2(3EI)}{7.5} \\ \frac{2(3EI)}{7.5} & \frac{4(3EI)}{7.5} \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix} \qquad \begin{matrix} ① & ② \end{matrix}$$



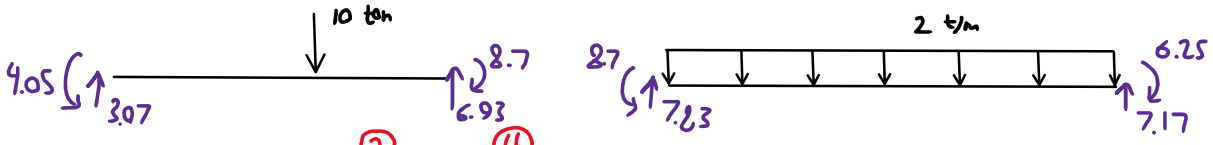


$$(P_L)_{AB} = k_L\delta_L + FER = \begin{bmatrix} \frac{6EI}{5^2} & 0 \\ \frac{2EI}{5} & 0 \\ \frac{6EI}{5^2} & -1.87/EI \\ \frac{4EI}{5} & \end{bmatrix} \begin{matrix} \\ \\ \\ \leftarrow 4 \end{matrix} + \begin{Bmatrix} 3.52 \\ 4.8 \\ 6.48 \\ -7.2 \end{Bmatrix}$$

$$(P_L)_{AB} = \begin{Bmatrix} 3.07 \\ 4.05 \\ 6.93 \\ -8.7 \end{Bmatrix}$$



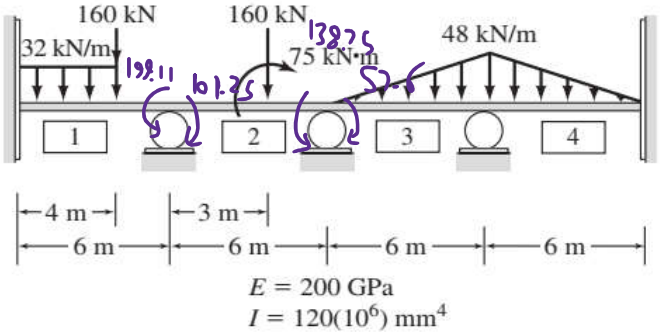
$$k_L = \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix}$$
$$\begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix}$$



$$(P_L)_{BC} = k_L\delta_L + FER = \begin{bmatrix} \textcircled{2} & \textcircled{4} & \\ \frac{6(3EI)}{7.5^2} & \frac{6(3EI)}{7.5^2} & 0 \\ \frac{4(3EI)}{7.5} & \frac{2(3EI)}{7.5} & -1.87/EI \\ -\frac{6(3EI)}{7.5^2} & -\frac{6(3EI)}{7.5^2} & 0 \\ \frac{4(3EI)}{7.5} & \frac{4(3EI)}{7.5} & 2.89/EI \end{bmatrix} + \begin{Bmatrix} 7.5 \\ 9.375 \\ 7.5 \\ -9.375 \end{Bmatrix} = \begin{Bmatrix} 7.83 \\ 8.7 \\ 7.17 \\ -6.25 \end{Bmatrix}$$

$$(P_L)_{BC} = \begin{Bmatrix} 7.83 \\ 8.7 \\ 7.17 \\ -6.25 \end{Bmatrix}$$

تیر با بارگذاری های متنوع



$E = 200$ GPa
$I = 120(10^6)$ mm^4

$$P_f = \begin{Bmatrix} 199.11 - 101.25 \\ 138.75 - 576 \\ \\ \end{Bmatrix}$$

ازجزوه انتشارات کتاب کاسمعلی (دردانشیت daneshesaze.com)

Member 1: (Loading types 1 and 3)

$$FS_b = \frac{160(2)^2(14)}{(6)^3} + \frac{32(6)}{2}\left[1 - \frac{(2)^3(10)}{(6)^4}\right] = 131.56 \text{ kN}$$

$$FM_b = \frac{160(4)(2)^2}{(6)^2} + \frac{32(6)^2}{12}\left[1 - \frac{(2)^3(18)}{(6)^4}\right] = 156.44 \text{ kN}\cdot\text{m}$$

$$FS_e = \frac{160(4)^2(10)}{(6)^3} + \frac{32(6)}{2}\left[1 - \frac{2(392)}{(6)^4}\right] = 156.44 \text{ kN}$$

$$FM_e = -\frac{160(4)^2(2)}{(6)^2} - \frac{32(6)^2}{12}\left[1 - \frac{(2)^2(132)}{(6)^4}\right] = -199.11 \text{ kN}\cdot\text{m}$$

Member 2: (Loading types 1 and 2)

$$FS_b = 80 - \frac{6(75)(3)^2}{(6)^3} = 61.25 \text{ kN}$$

$$FM_b = \frac{160(3)^3}{(6)^2} + \frac{75(3)(-3)}{(6)^2} = 101.25 \text{ kN}\cdot\text{m}$$

$$FS_e = 80 + \frac{6(75)(3)^2}{(6)^3} = 98.75 \text{ kN}$$

$$FM_e = -\frac{160(3)^3}{(6)^2} + \frac{75(3)(-3)}{(6)^2} = -138.75 \text{ kN}\cdot\text{m}$$

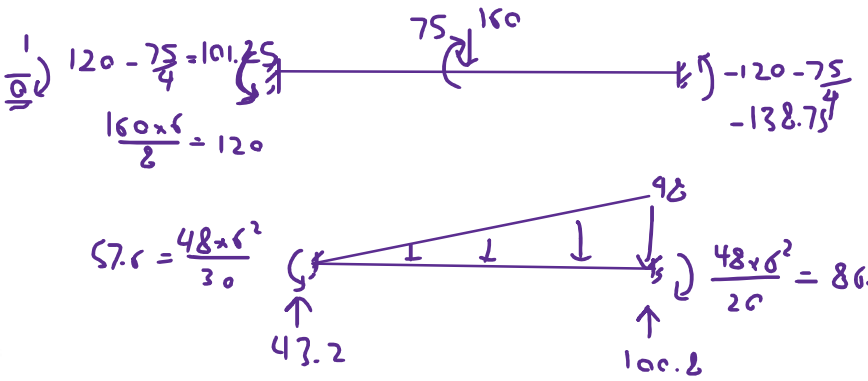
Member 3: (Loading type 4)

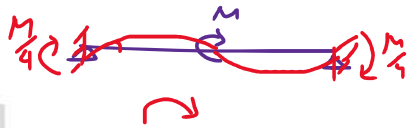
$$FS_b = \frac{48(6)^3(3)(6)}{20(6)^3} = 43.2\text{kN}$$

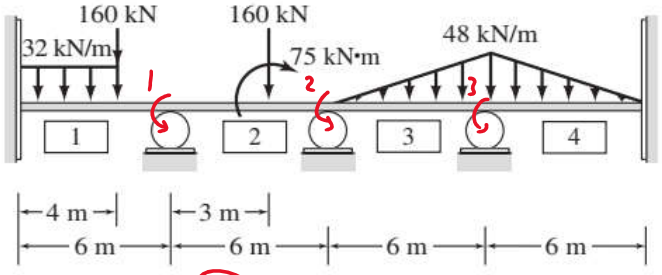
$$FM_b = \frac{48(6)^3(2)(6)}{60(6)^2} = 57.6 \text{ kN}\cdot\text{m}$$

$$FS_e = \left(\frac{48}{2}\right)(6) - 43.2 = 100.8 \text{ kN}$$

$$FM_e = (-48)(6) + (43.2)(6) - 57.6 = -86.4 \text{ kN}\cdot\text{m}$$







$E = 200$ GPa

$I = 120(10^6)$ mm^4

$$k_{ff} = \begin{bmatrix} \frac{4EI_1}{L_1} + \frac{4EI_2}{L_2} & \frac{2EI_2}{L_2} & 0 \\ \frac{2EI_2}{L_2} & \frac{4EI_2}{L_2} + \frac{4EI_3}{L_3} & \frac{2EI_3}{L_3} \\ 0 & \frac{2EI_3}{L_3} & \frac{4EI_3}{L_3} + \frac{4EI_4}{L_4} \end{bmatrix} \quad P_f = \begin{Bmatrix} \\ \\ \end{Bmatrix}$$

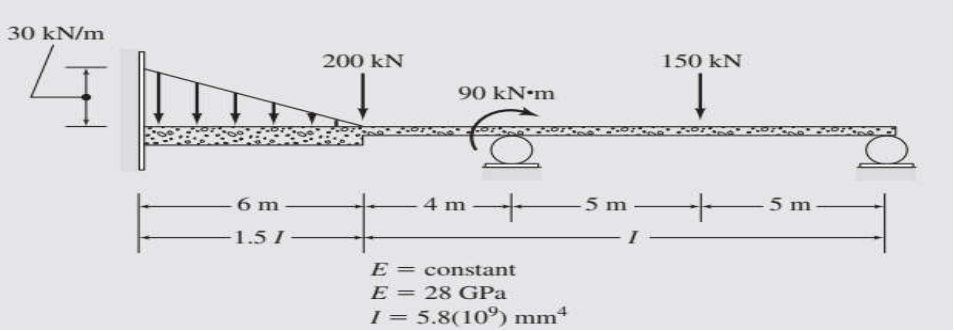
Video 49

Matrix Analysis 10

Wednesday, July 17, 2024 11:48

تحلیل ماتریسی سازه‌ها

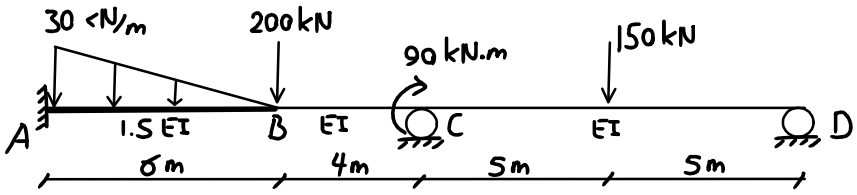
مثال: تیر شکل زیر را به روش ماتریسی تحلیل کنید.

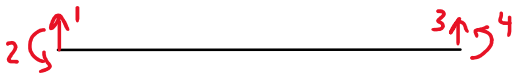


$EI = 28\times10^3 \frac{N}{mm^2}\times 5.8\times10^9 mm^4$

$= 162.4\times10^{12}\ N.mm^2$

$= 162400\ kN.m^2$





① k_{ff} مرحله

$$[k_{ff}]_{AB} = \begin{bmatrix} \frac{12(1.5EI)}{L^3} & \frac{6(1.5EI)}{L^2} \\ \frac{6(1.5EI)}{L^2} & \frac{4(1.5EI)}{L} \end{bmatrix} = EI \begin{bmatrix} 0.08333 & -0.25 \\ -0.25 & 1 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

$$k_L = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$$[k_{ff}]_{BC} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = EI \begin{bmatrix} 0.1875 & 0.375 & 0.375 \\ 0.375 & 1 & 0.5 \\ 0.375 & 0.5 & 1 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$$[k_{ff}]_{CD} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = EI \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{matrix} ③ \\ ④ \end{matrix}$$

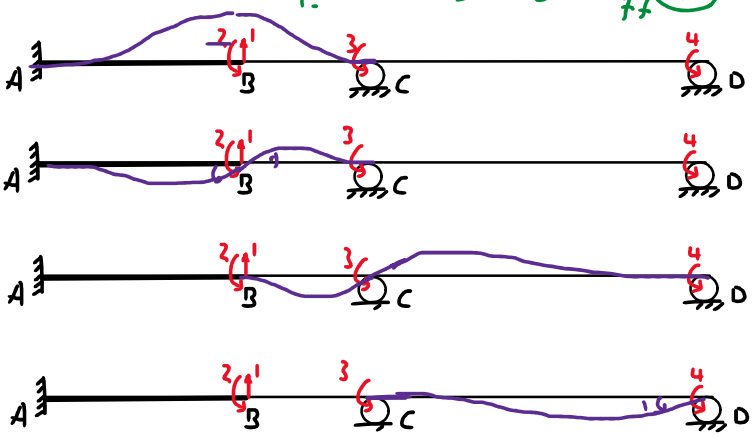
② k_{ff} سازه با سر هم بندی

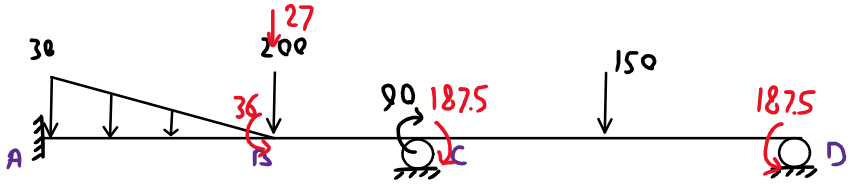
$$k_{ff} = EI \begin{bmatrix} 0.27083 & 0.125 & 0.375 & 0 \\ 0.125 & 2 & 0.5 & 0 \\ 0.375 & 0.5 & 1.4 & 0.2 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$$K_{ff} = EI \begin{bmatrix} & & & \\ 0.375 & 0.5 & 1.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix} \begin{matrix} \\ ③ \\ ④ \end{matrix}$$

(۲) K_{ff} را به صورت مستقیم

$$K_{ff} = EI \begin{bmatrix} ① & ② & ③ & ④ \\ 0.27083 & 0.125 & 0.375 & 0 \\ 0.125 & 2 & 0.5 & 0 \\ 0.375 & 0.5 & 1.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$





(۳) محاسبه P_f و δ_f

$$K_{ff}\,\delta_f = P_f$$

$$EI \begin{bmatrix} 0.27083 & 0.125 & 0.375 & 0 \\ 0.125 & 2 & 0.5 & 0 \\ 0.375 & 0.5 & 1.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = \begin{Bmatrix} -227 \\ 36 \\ -277.5 \\ 187.5 \end{Bmatrix}$$

$$P_f = \begin{Bmatrix} -227 \\ 36 \\ -277.5 \\ 187.5 \end{Bmatrix}$$

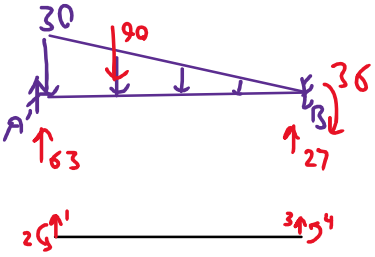
$$\delta_f = 10^{-3} \times \begin{Bmatrix} -4.4729 \\ 0.56143 \\ -0.68416 \\ 3.2285 \end{Bmatrix}$$

(۴) محاسبه نیروی اعضا

$$P_L = k_L \delta_L + FER$$

$$EI \to 1.5EI \qquad L \to 6m$$

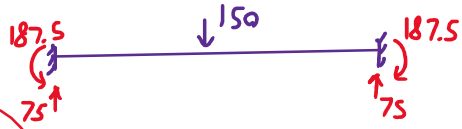
$$(P_L)_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} 10^{-3} \times \begin{Bmatrix} 0 \\ 0 \\ -4.4729 \\ 0.56143 \end{Bmatrix} + \begin{Bmatrix} 63 \\ 54 \\ 27 \\ -36 \end{Bmatrix} = \begin{Bmatrix} 146.33 \\ 281.19 \\ -56.33 \\ 236.78 \end{Bmatrix}$$



$EI \rightarrow EI$

$L \rightarrow 4$

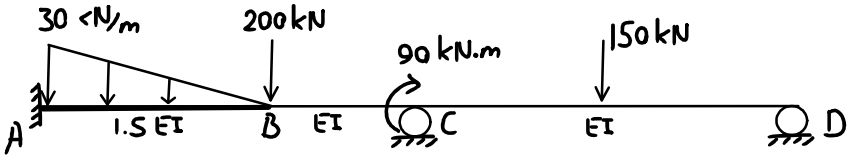
$$(P_L)_{BC} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} 10^{-3} \times \begin{Bmatrix} -4.4729 \\ 0.56143 \\ 0 \\ -0.68416 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -143.67 \\ -236.78 \\ 143.67 \\ -337.92 \end{Bmatrix}$$

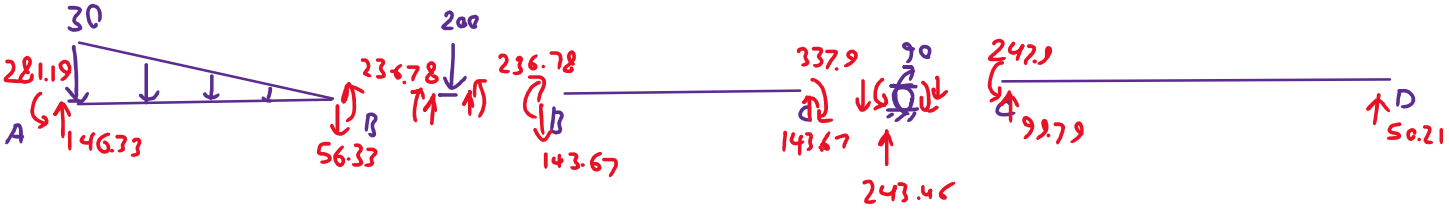


$EI \rightarrow EI$

$L \rightarrow 10$

$$(P_L)_{CD} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} 10^{-3} \times \begin{Bmatrix} 0 \\ -0.68416 \\ 0 \\ 3.2285 \end{Bmatrix} + \begin{Bmatrix} 75 \\ 187.5 \\ 75 \\ -187.5 \end{Bmatrix} = \begin{Bmatrix} 99.79 \\ 247.92 \\ 50.21 \\ 0 \end{Bmatrix}$$





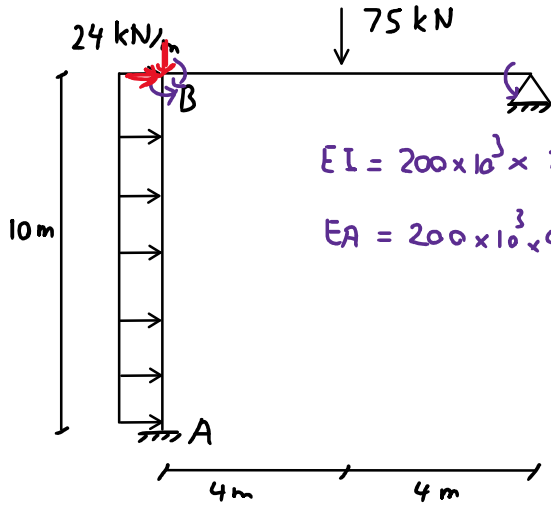
Video 50

Matrix Analysis 11

Tuesday, August 6, 2024 10:29

تحلیل ماتریسی سازه‌ها

مثال: قاب شکل زیر را به روش ماتریسی تحلیل کنید.



$E = 200\ GPa$

$A = 4740\ mm^2$

$I = 22.2 \times 10^6\ mm^4$

تقریبا IPB160

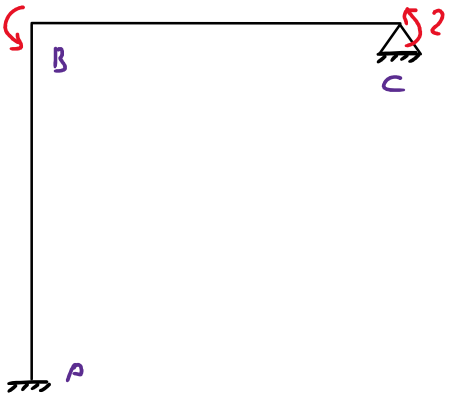
$10^{-3}\ kN.(10^{-1}m)^2 = 10^{-9}\ kN.m^2$

$EI = 200\times10^3 \times 22.2\times10^6\ N.mm^2 = 4440\ kN.m^2$

$EA = 200\times10^3\times4740\ N = 948\times10^3\ kN$

(الف) با صرف نظر از تغییر شکل‌های محوری اعضا

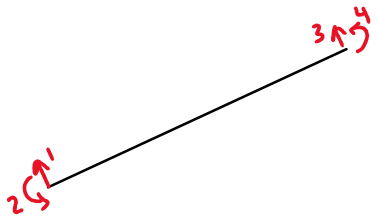
$$k_{ff} = \begin{bmatrix} \frac{4EI}{10}+\frac{4EI}{8} & \frac{2EI}{8} \\ \frac{2EI}{8} & \frac{4EI}{8} \end{bmatrix} = EI \begin{bmatrix} 0.9 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$



$$P_f = \begin{Bmatrix} \frac{24\times10^2}{12} - \frac{75\times8}{8} \\ \frac{75\times8}{8} \end{Bmatrix} = \begin{Bmatrix} 125 \\ 75 \end{Bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

$$\boxed{k_{ff}\,\delta_f = P_f}$$

$$EI \begin{bmatrix} 0.9 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 125 \\ 75 \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} \frac{112.91}{EI} \\ \frac{93.54}{EI} \end{Bmatrix} \begin{matrix} \rightarrow 0.0254 \\ \rightarrow 0.02107 \end{matrix}}$$



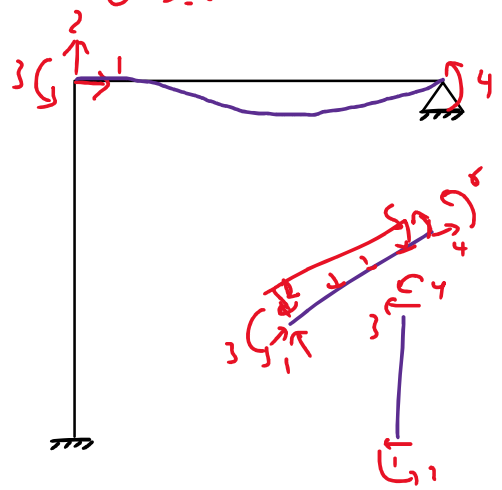
$$k_L = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

$$(P_L)_{AB} = k_L \delta_L + FER = \begin{bmatrix} \frac{6EI}{10^2} \\ \frac{2EI}{10} \\ -\frac{6EI}{10^2} \\ \frac{4EI}{10} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{112.91}{EI} \end{Bmatrix} + \begin{Bmatrix} 120 \\ 200 \\ 120 \\ -200 \end{Bmatrix}$$

$$(P_L)_{AB} = \begin{Bmatrix} 6.775 \\ 22.58 \\ -6.775 \\ 45.16 \end{Bmatrix} + \begin{Bmatrix} 120 \\ 200 \\ 120 \\ -200 \end{Bmatrix} = \begin{Bmatrix} 126.775 \\ 222.58 \\ 113.225 \\ -154.84 \end{Bmatrix}$$

$$\boxed{(P_L)_{AB} = \begin{Bmatrix} 126.775 \\ 222.58 \\ 113.225 \\ -154.84 \end{Bmatrix}}$$

(ب) با در نظر گرفتن تغییر شکل های محوری اعضا



$$k_{ff} = \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix}$$

$$\begin{bmatrix} \frac{12EI}{10^3} + \frac{EA}{8} & 0 & \frac{6EI}{10^2} & 0 \\ 0 & \frac{12EI}{8^3} + \frac{EA}{10} & \frac{6EI}{8^2} & \frac{6EI}{8^2} \\ \frac{6EI}{10^2} & \frac{6EI}{8^2} & \frac{4EI}{10} + \frac{4EI}{8} & \frac{2EI}{8} \\ 0 & \frac{6EI}{8^2} & \frac{2EI}{8} & \frac{4EI}{8} \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix}$$

$$\boxed{k_{ff}\,\delta_f = P_f}$$

$$\begin{bmatrix} 118553 & 0 & 266.4 & 0 \\ 0 & 94904 & 416.25 & 416.25 \\ 266.4 & 416.25 & 3996 & 1110 \\ 0 & 416.25 & 1110 & 2220 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = \begin{Bmatrix} 120 \\ -37.5 \\ 125 \\ 75 \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = \begin{Bmatrix} 0.001 \\ -0.0006 \\ 0.0254 \\ 0.0212 \end{Bmatrix}}$$

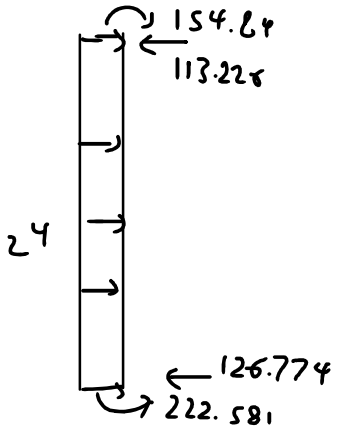
$$(P_L)_{AB} = k_L \delta_L + FER = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0.0006 \\ -0.001 \\ 0.0254 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 120 \\ 200 \\ 0 \\ 120 \\ -200 \end{Bmatrix} = \begin{Bmatrix} -56.88 \\ 126.815 \\ 222.802 \\ 56.88 \\ 113.185 \\ -154.651 \end{Bmatrix}$$

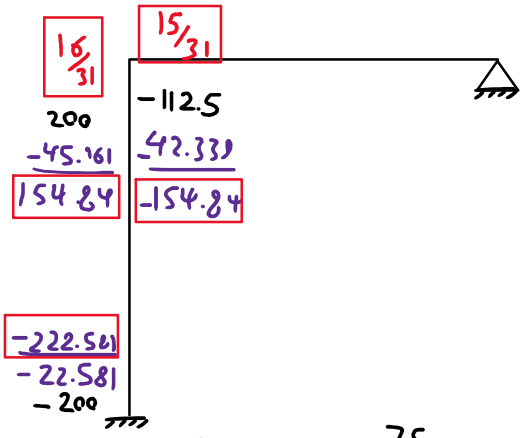
$$(P_L)_{AB} = \begin{Bmatrix} -56.88 \\ 126.815 \\ 222.862 \\ 56.82 \\ 113.185 \\ -154.651 \end{Bmatrix}$$

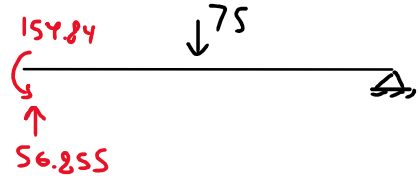
مقایسه با روش کلاس

$\frac{3EI}{8}$ ، $\frac{4EI}{10}$

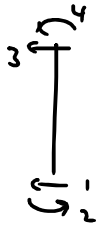
$\frac{\frac{3}{8}}{\frac{3}{8}+\frac{4}{10}} - \frac{\frac{15}{40}}{\frac{15+16}{40}}$







$$(P_L)_{AB} = \begin{Bmatrix} 6.775 \\ 22.58 \\ -6.775 \\ 45.16 \end{Bmatrix} + \begin{Bmatrix} 120 \\ 200 \\ 120 \\ -200 \end{Bmatrix} = \begin{Bmatrix} 126.775 \\ 222.58 \\ 113.225 \\ -154.84 \end{Bmatrix}$$



Video 51

Matrix Analysis 12

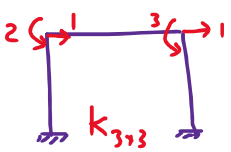
Tuesday, August 6, 2024 10:29

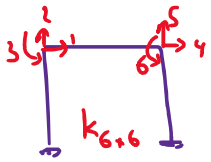
تحلیل ماتریسی سازه ها

پیرامون ماتریس سختی المان تاب

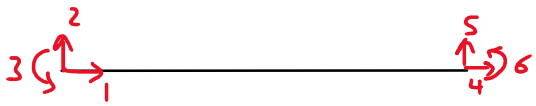
* در حل دستی به روش تحلیل ماتریسی، معمولا از تغییر طول محوری اعضا صرف نظر می شود که راه حل را کوتاه می کند ولی در حل با ماشین تر، تغییر طول محوری اعضا لحاظ می شود.

* بنابراین در ادامه از ماتریس سختی اعضا فقط بر اساس تغییر شکل های خمشی استفاده می شود و از تغییر شکل های برشی و محوری صرف نظر می کنیم.





① عضو افقی



$$k_G = k_l = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ② \\ ③ \\ ⑤ \\ ⑥ \end{matrix}$$

(② ③ ⑤ ⑥)

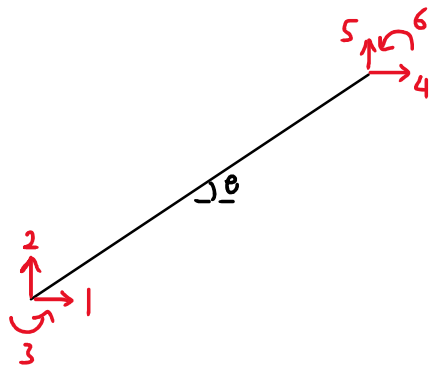
② عضو قائم

$$k_G = \begin{bmatrix} \frac{12EI}{l^3} & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ -\frac{6EI}{l^2} & \frac{4EI}{l} & \frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ -\frac{6EI}{l^2} & \frac{2EI}{l} & \frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ① \ \delta \\ ③ \ \theta \\ ④ \ \delta \\ ⑥ \ \theta \end{matrix}$$

(δ θ δ θ ; ① ③ ④ ⑥)

③ عضو مایل

$$k_G = \begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} & \textcircled{6} \end{matrix} \\ \left[\begin{array}{cccccc} s^2\frac{12EI}{L^3} & -cs\frac{12EI}{L^3} & -s\frac{6EI}{L^2} & -s^2\frac{12EI}{L^3} & cs\frac{12EI}{L^3} & -s\frac{6EI}{L^2} \\ -cs\frac{12EI}{L^3} & c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & cs\frac{12EI}{L^3} & -c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -s^2\frac{12EI}{L^3} & cs\frac{12EI}{L^3} & s\frac{6EI}{L^2} & s^2\frac{12EI}{L^3} & -cs\frac{12EI}{L^3} & s\frac{6EI}{L^2} \\ cs\frac{12EI}{L^3} & -c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} & -cs\frac{12EI}{L^3} & c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right] \end{array} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \\ \textcircled{5} \\ \textcircled{6} \end{matrix}$$



بردار بارگذاری P_f

با تفکیک باربهای وارده به سازه به باربهای گرهی (P_J) و باربهای میان دهانه می‌توان نوشت:

$$\begin{cases} M_{AB} = \frac{2EI}{L}(2\theta_A + \theta_B - \frac{3\Delta}{L}) + FEM_{AB} \\ M_{AC} = \frac{2EI}{L}(2\theta_A + \theta_C - \frac{3\Delta}{L}) + FEM_{AC} \end{cases}$$

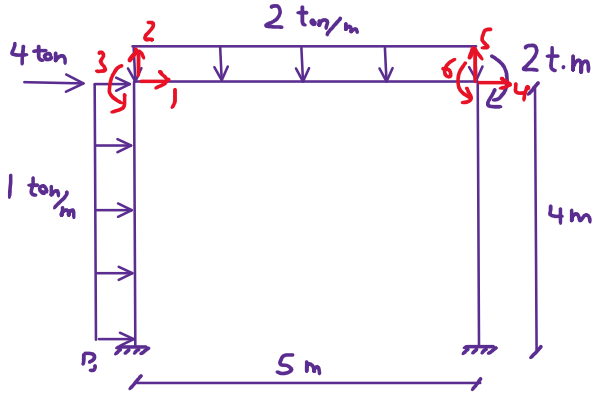
$$\Sigma M_B = 0 \Rightarrow (\quad)\theta_A + (\quad)\theta_B + (\quad)\Delta + FEM_{AB} + FEM_{AC} - M_J = 0$$

$$k\Delta = -FEM_{AB} - FEM_{AC} + M_J$$

$$k\delta + FER = P_J$$

$$\boxed{k\delta = P_J - FER}$$

مثال: بردار بارگذاری P_f را برای قاب شکل زیر به دست آورید.

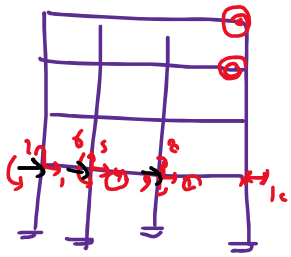


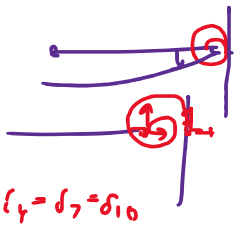
$$P_f = \begin{Bmatrix} 4 + \frac{1\times4}{2} \\ -\frac{2\times5}{2} \\ \frac{1\times4^2}{12} - \frac{2\times5^2}{12} \\ 0 \\ -\frac{2\times5}{2} \\ -2 + \frac{2\times5^2}{12} \end{Bmatrix} = \begin{Bmatrix} 6 \\ -5 \\ -2.83 \\ 0 \\ -5 \\ 2.17 \end{Bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \\ \textcircled{5} \\ \textcircled{6} \end{matrix}$$

حذف درجات آزادی یکسان (equal DOF) در ماتریس سختی

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}_{4\times4} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} \delta_1$$

فرض $\delta_1 = \delta_4$





$\delta_1 = \delta_4 = \delta_7 = \delta_{10}$

$$\begin{cases} P_1 = k_{11}\delta_1 + k_{12}\delta_2 + k_{13}\delta_3 + k_{14}\delta_1 = (k_{11}+k_{14})\delta_1 + k_{12}\delta_2 + k_{13}\delta_3 \\ P_2 = k_{21}\delta_1 + k_{22}\delta_2 + k_{23}\delta_3 + k_{24}\delta_1 = (k_{21}+k_{24})\delta_1 + k_{22}\delta_2 + k_{23}\delta_3 \\ P_3 = k_{31}\delta_1 + k_{32}\delta_2 + k_{33}\delta_3 + k_{34}\delta_1 = (k_{31}+k_{34})\delta_1 + k_{32}\delta_2 + k_{33}\delta_3 \\ P_4 = k_{41}\delta_1 + k_{42}\delta_2 + k_{43}\delta_3 + k_{44}\delta_1 = (k_{41}+k_{44})\delta_1 + k_{42}\delta_2 + k_{43}\delta_3 \end{cases}$$

$$\begin{cases} P_1 + P_4 = (k_{11}+k_{14}+k_{41}+k_{44})\delta_1 + (k_{12}+k_{42})\delta_2 + (k_{13}+k_{43})\delta_3 \\ P_2 = (k_{21}+k_{24})\delta_1 + k_{22}\delta_2 + k_{23}\delta_3 \\ P_3 = (k_{31}+k_{34})\delta_1 + k_{32}\delta_2 + k_{33}\delta_3 \end{cases}$$

$$\begin{Bmatrix} P_1+P_4 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} k_{11}+k_{14}+k_{41}+k_{44} & k_{12}+k_{42} & k_{13}+k_{43} \\ k_{21}+k_{24} & k_{22} & k_{23} \\ k_{31}+k_{34} & k_{32} & k_{33} \end{bmatrix}_{3\times3} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix}$$

* بنابراین اگر $\delta_1 = \delta_4$ باشد، برای تبدیل ماتریس سختی 4×4 به ماتریس سختی 3×3، ستون 4 را با ستون 1 و سطر 4 را با سطر 1 جمع می‌کنیم.

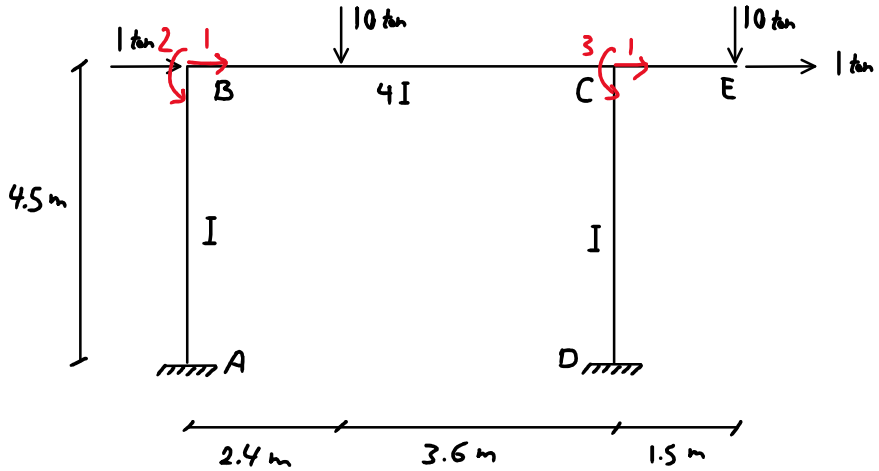
Video 52

Matrix Analysis 13

Tuesday, August 6, 2024 10:29

تحلیل ماتریسی سازه ها

مثال: قاب شکل زیر را به روش ماتریسی تحلیل کنید. (از تغییر طول محوری اعضا صرف نظر کنید.)



① $k_{ff}\,\delta_f = P_f$

② $P_L = k_L \delta_L + FER$

① تشکیل ماتریس کلی k_{ff}

$$[k_{ff}]_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} = EI \begin{bmatrix} \frac{12}{4.5^3} & \frac{6}{4.5^2} \\ \frac{6}{4.5^2} & \frac{4}{4.5} \end{bmatrix} = EI \begin{matrix} \\ ① \\ ② \end{matrix} \begin{bmatrix} ① & ② \\ 0.1317 & 0.2963 \\ 0.2963 & 0.8889 \end{bmatrix}$$

$$[k_{ff}]_{CD} = EI \begin{matrix} \\ ① \\ ③ \end{matrix} \begin{bmatrix} ① & ③ \\ 0.1317 & 0.2963 \\ 0.2963 & 0.8889 \end{bmatrix}$$

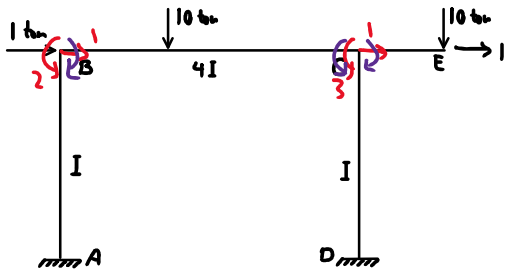
$$[k_{ff}]_{BC} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \begin{bmatrix} \frac{4(4EI)}{6} & \frac{2(4EI)}{6} \\ \frac{2(4EI)}{6} & \frac{4(4EI)}{6} \end{bmatrix} = EI \begin{matrix} \\ ② \\ ③ \end{matrix} \begin{bmatrix} ② & ③ \\ 2.6667 & 1.3333 \\ 1.3333 & 2.6667 \end{bmatrix}$$

سرهم بندی

$$k_{ff} = EI \begin{matrix} \\ ① \\ ② \\ ③ \end{matrix} \begin{bmatrix} ① & ② & ③ \\ 0.2634 & 0.2963 & 0.2963 \\ 0.2963 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix}$$

بردار بارگذاری P_f

$$P_f = \begin{Bmatrix} 1+1 \\ -\frac{10\times 2.4\times 3.6^2}{6^2} \\ \frac{10\times 3.6\times 2.4^2}{6^2} - 10\times 1.5 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -8.64 \\ -9.24 \end{Bmatrix}$$

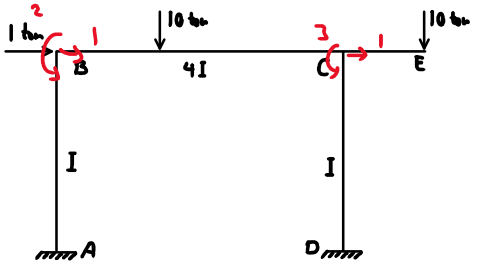


محاسبه مجهولات تغییر مکانی (δ)

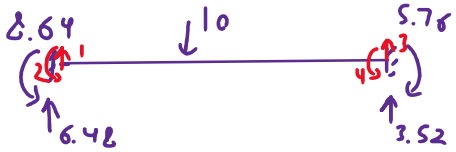
$$EI\begin{bmatrix} 0.2634 & 0.2963 & 0.2963 \\ 0.2967 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix}\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -8.64 \\ -9.24 \end{Bmatrix} \rightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} 13.555 \\ -2.515 \\ -2.785 \end{Bmatrix}$$

محاسبه نیروی اعضا

$$P_L = k_L \delta_L + FER$$



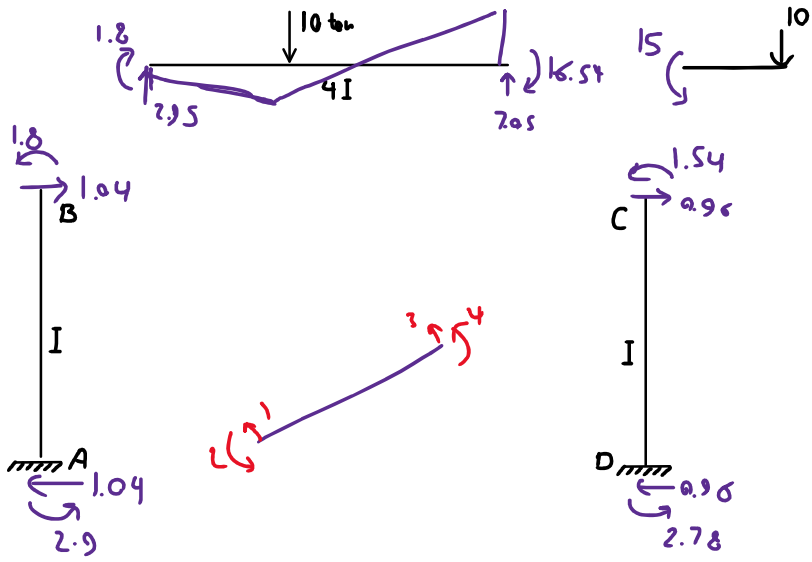
$$(P_L)_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -\delta_1 \\ \delta_2 \end{Bmatrix} + 0 = \begin{Bmatrix} 1.0398 \\ 2.8986 \\ -1.0398 \\ 1.7807 \end{Bmatrix}$$



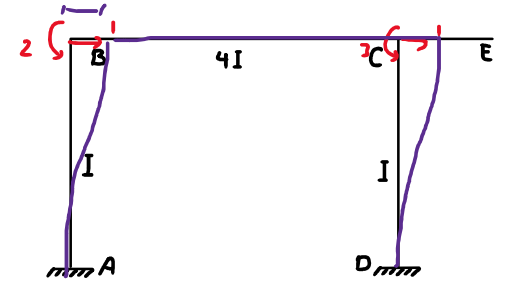
$$(P_L)_{CD} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -\delta_1 \\ \delta_3 \end{Bmatrix} + 0 = \begin{Bmatrix} 0.9598 \\ 2.7786 \\ -0.9598 \\ 1.5407 \end{Bmatrix}$$

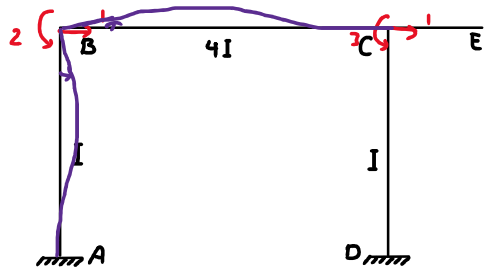
$$(P_L)_{BC} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{Bmatrix} 0 \\ \delta_2 \\ 0 \\ \delta_3 \end{Bmatrix} + \begin{Bmatrix} 6.48 \\ 8.64 \\ 3.52 \\ -5.76 \end{Bmatrix} = \begin{Bmatrix} 2.9484 \\ -1.7807 \\ 7.0536 \\ -16.5407 \end{Bmatrix}$$

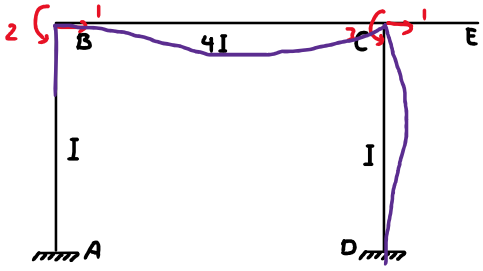
$$P_{AB} = \begin{Bmatrix} 1.0398 \\ 2.8986 \\ -1.0398 \\ 1.7807 \end{Bmatrix}, P_{CD} = \begin{Bmatrix} 0.9598 \\ 2.7786 \\ -0.9598 \\ 1.5407 \end{Bmatrix}, P_{BC} = \begin{Bmatrix} 2.9464 \\ -1.7807 \\ 7.0536 \\ -16.5407 \end{Bmatrix}$$



نوشتن ماتریس سختی k_{ff} به صورت مستقیم







$$k_{ff} = EI \begin{bmatrix} 2\times\frac{12EI}{4.5^3} & \frac{6EI}{4.5^2} & \frac{6EI}{4.5^2} \\ \frac{6EI}{4.5^2} & \frac{4EI}{4.5}+\frac{4(4EI)}{6} & \frac{2(4EI)}{6} \\ \frac{6EI}{4.5^2} & \frac{2(4EI)}{6} & \frac{4EI}{4.5}+\frac{4(4EI)}{6} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$$k_{ff} = EI \begin{bmatrix} 0.2634 & 0.2963 & 0.2963 \\ 0.2963 & 3.5556 & 1.3333 \\ 0.2963 & 1.3333 & 3.5556 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

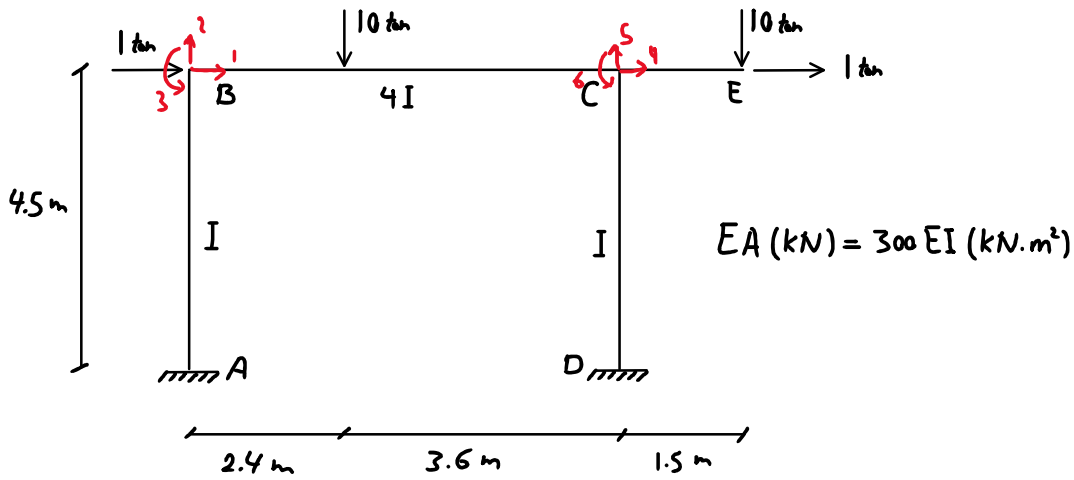
Video 53

Matrix Analysis 14

Tuesday, August 6, 2024 10:29

تحلیل ماتریسی سازه ها

مثال: قاب شکل زیر را به روش ماتریسی تحلیل کنید. (تغییر طول محوری اعضا را در نظر بگیرید)



① ماتریس k_{ff} برای اعضا

$$[k_{ff}]_{AB} = \begin{matrix} & (1) & (2) & (3) \\ (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} \frac{12EI}{4.5^3} & 0 & \frac{6EI}{4.5^2} \\ 0 & \frac{EA}{4.5} & 0 \\ \frac{6EI}{4.5^2} & 0 & \frac{4EI}{4.5} \end{bmatrix}$$

$$[k_{ff}]_{CD} = \begin{matrix} & (4) & (5) & (6) \\ (4) \\ (5) \\ (6) \end{matrix} \begin{bmatrix} \frac{12EI}{4.5^3} & 0 & \frac{6EI}{4.5^2} \\ 0 & \frac{EA}{4.5} & 0 \\ \frac{6EI}{4.5^2} & 0 & \frac{4EI}{4.5} \end{bmatrix}$$

$$[k_{ff}]_{BC} = \begin{matrix} & (1) & (2) & (3) & (4) & (5) & (6) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

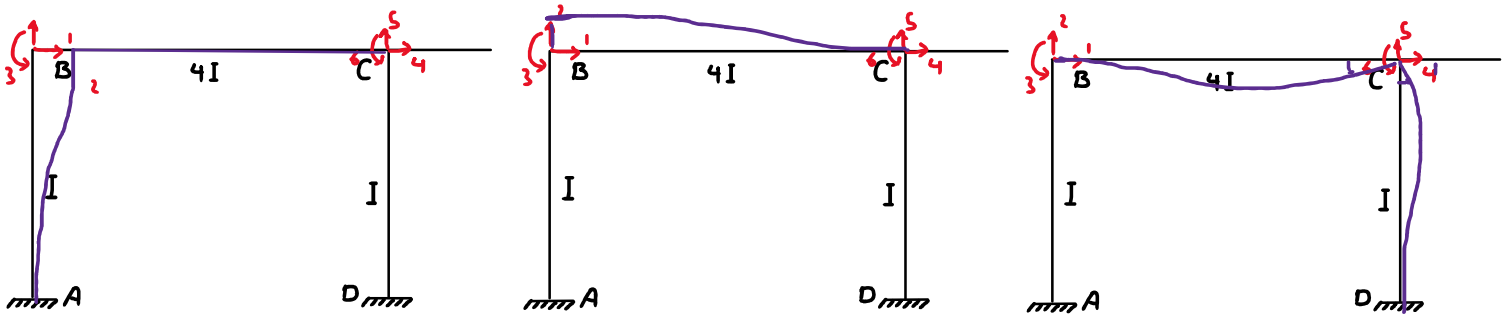
$L = 6$

$EI \to 4EI$

② ماتریس k_{ff} سازه

$$k_{ff} = \begin{matrix} & (1) & (2) & (3) & (4) & (5) & (6) \\ (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} \frac{4EA}{6} + \frac{12EI}{4.5^3} & 0 & \frac{6EI}{4.5^2} & -\frac{4EA}{6} & 0 & 0 \\ 0 & \frac{EA}{4.5} + \frac{12(4EI)}{6^3} & \frac{6(4EI)}{6^2} & 0 & -\frac{12(4EI)}{6^3} & \frac{6(4EI)}{6^2} \\ \frac{6EI}{4.5^2} & \frac{6(4EI)}{6^2} & \frac{4EI}{4.5} + \frac{4(4EI)}{6} & 0 & -\frac{6(4EI)}{6^2} & \frac{2(4EI)}{6} \end{bmatrix}$$

$$K_{ff} = \begin{bmatrix} \frac{6EI}{4.5^2} & \frac{-12(4EI)}{6^2} & \frac{6}{4.5}+\frac{12(4EI)}{6} & 0 & \frac{-6(4EI)}{6^2} & \frac{2(4EI)}{6} & (3) \\ \frac{-4EA}{6} & 0 & 0 & \frac{4EA}{6}+\frac{12EI}{4.5^3} & 0 & \frac{6EI}{4.5^2} & (4) \\ 0 & \frac{-12(4EI)}{6^3} & \frac{-6(4EI)}{6^2} & 0 & \frac{EA}{4.5}+\frac{12(4EI)}{6^3} & \frac{-6(4EI)}{6^2} & (5) \\ 0 & \frac{6(4EI)}{6^2} & \frac{2(4EI)}{6} & \frac{6EI}{4.5^2} & \frac{-6(4EI)}{6^2} & \frac{4EI}{4.5}+\frac{4(4EI)}{6} & (6) \end{bmatrix}$$



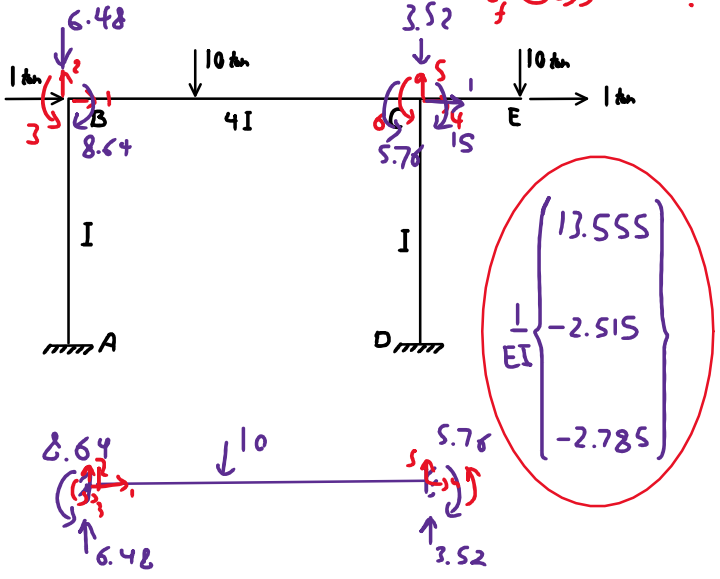
برنامه نویسی (حل نرم افزار)

$$K_G = \left[\begin{array}{ccc|ccc} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} & -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} & c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} \\ -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$

$K_{ff}\,\delta_f = P_f$

$$P_f = \begin{Bmatrix} 1 \\ -6.48 \\ -8.64 \\ 1 \\ -13.52 \\ -9.24 \end{Bmatrix} \rightarrow \delta_f = \frac{1}{EI}\begin{Bmatrix} 13.63 \\ -0.044 \\ -2.55 \\ 13.63 \\ -0.26 \\ -2.82 \end{Bmatrix}$$

(۳) بدست آوردن δ_f

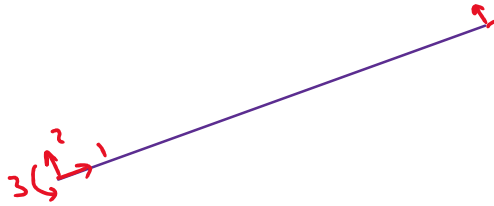


$$\frac{1}{EI}\begin{Bmatrix} 13.555 \\ -2.515 \\ -2.785 \end{Bmatrix}$$

④ محاسبه نیروی اعضا

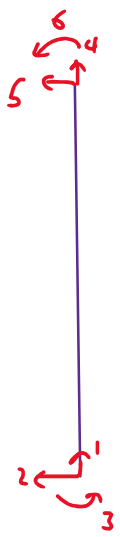
$$P_L = k_L \delta_L + FER$$

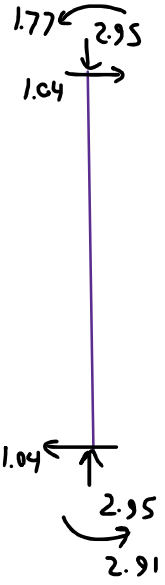
AB : L = 4.5



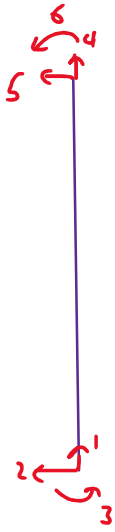
$$k_L = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}, \quad \delta_L = \frac{1}{EI} \begin{Bmatrix} c \\ 0 \\ 0 \\ -0.044 \\ -13.63 \\ -2.55 \end{Bmatrix}, \quad FER = 0$$

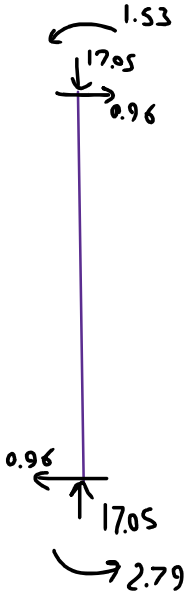
$$P_{L,AB} = \begin{Bmatrix} 2.95 \\ 1.04 \\ 2.91 \\ -2.95 \\ -1.04 \\ 1.77 \end{Bmatrix}$$



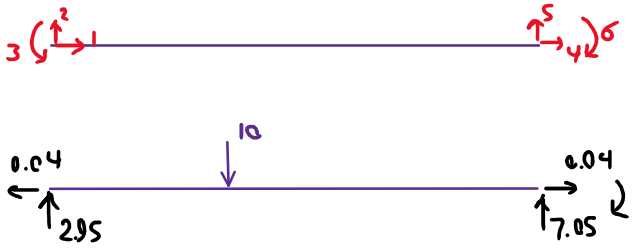


$$P_{L,CD} = \begin{Bmatrix} 17.05 \\ 0.96 \\ 2.79 \\ -17.05 \\ -0.96 \\ 1.53 \end{Bmatrix}$$





$$P_{L,BC} = \begin{Bmatrix} -0.04 \\ 2.95 \\ -1.77 \\ 0.04 \\ 7.05 \\ -16.53 \end{Bmatrix}$$



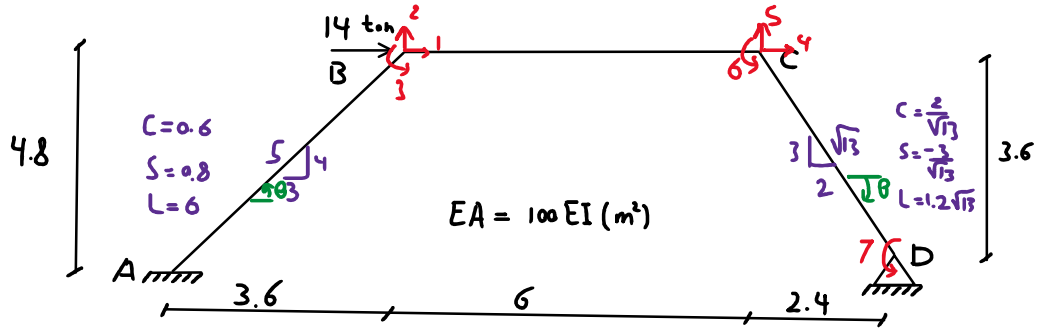
Video 54

Matrix Analysis 15

Tuesday, August 6, 2024 10:29

تحلیل ماتریسی سازه ها

مثال: قاب شکل زیر را با روش ماتریسی تحلیل کنید.



① ماتریس k_{ff} برای اعضا

AB: $c=0.6$, $s=0.8$, $L=6$

$$[k_{ff}]_{AB}=\begin{bmatrix} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{matrix}①\\②\\③\end{matrix}$$

$$k_e=\left[\begin{array}{ccc|ccc} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} & -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -c^2\frac{EA}{L}-s^2\frac{12EI}{L^3} & -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} & c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s\frac{6EI}{L^2} \\ -cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s^2\frac{EA}{L}-c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & -c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & s\frac{6EI}{L^2} & -c\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$

CD: $c=\frac{2}{\sqrt{13}}$, $s=\frac{-3}{\sqrt{13}}$, $L=1.2\sqrt{13}$

$$[k_{ff}]_{CD}=\begin{bmatrix} c^2\frac{EA}{L}+s^2\frac{12EI}{L^3} & cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -s\frac{6EI}{L^2} & -s\frac{6EI}{L^2} \\ cs\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & s^2\frac{EA}{L}+c^2\frac{12EI}{L^3} & c\frac{6EI}{L^2} & c\frac{6EI}{L^2} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{2EI}{L} \\ -s\frac{6EI}{L^2} & c\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}\begin{matrix}④\\⑤\\⑥\\⑦\end{matrix}$$

BC: $L=6$

$$[k_{ff}]_{BC}=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{matrix}①\\②\\③\\④\\⑤\\⑥\end{matrix}$$

② ماتریس k_{ff} سازه با سر هم بندی ماتریس های $k_{\#}$ اعضا

از نرم افزار MATLAB کمک می گیریم:

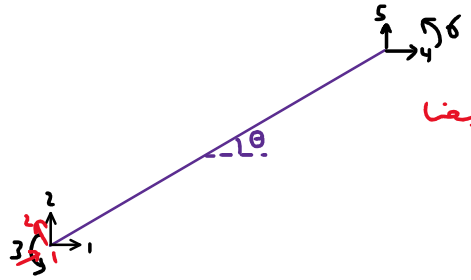
$k_{ff} = EI$

①	②	③	④	⑤	⑥	⑦	
22.7022	7.9733	0.1333	-16.6667	0	0	0	①
7.9733	10.7422	0.0667	0	-0.0556	0.1667	0	②
0.1333	0.0667	1.3333	0	-0.1667	0.3333	0	③
-16.6667	0	0	23.8808	-10.5989	0.2667	0.2667	④
0	-0.0556	-0.1667	-10.5989	16.1021	0.0111	0.1778	⑤
0	0.1667	0.3333	0.2667	0.0111	1.5912	0.4623	⑥
0	0	0	0.2667	0.1778	0.4623	0.9245	⑦

③ به دست آوردن δ_f

$$P_f = \begin{Bmatrix} 14 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \xrightarrow{k_{ff}\,\delta_f = P_f} \delta_f = \frac{1}{EI}\begin{Bmatrix} 56.5 \\ -41.7 \\ 0.68 \\ 56.2 \\ 37.1 \\ 1.56 \\ -24.1 \end{Bmatrix}$$

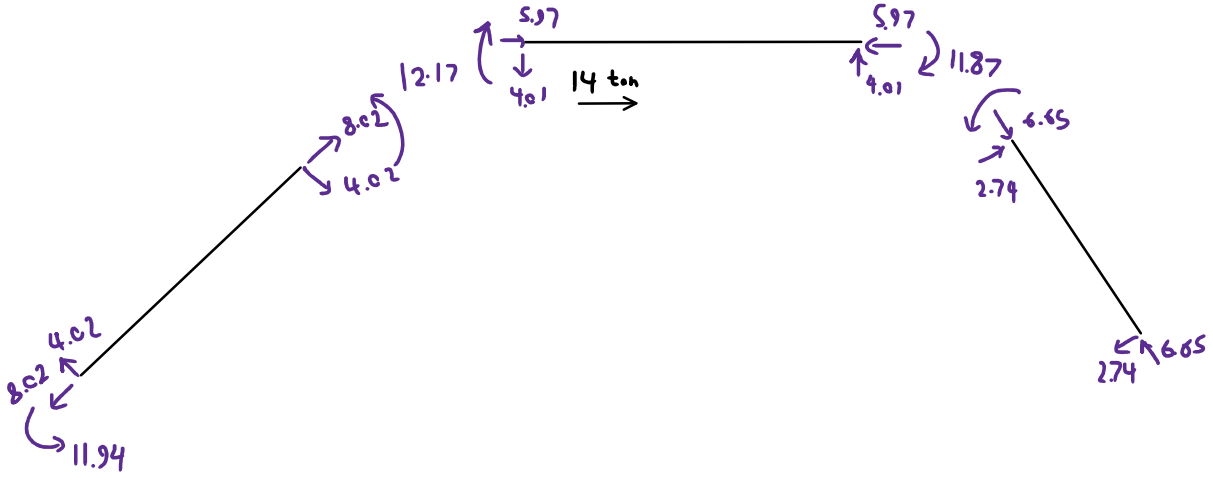
④ محاسبه نیروی اعضا



$$P_L = k_L R\,\delta_G + FER$$

AB: $C = 0.6$, $S = 0.8$, $L = 6$

$$R = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \delta_G = \frac{1}{EI}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 56.5 \\ -41.7 \\ 0.68 \end{Bmatrix}, \quad FER = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \xrightarrow{MATLAB} P_{L,AB} = \begin{Bmatrix} -8.02 \\ 4.02 \\ 11.94 \\ 8.02 \\ -4.02 \\ 12.17 \end{Bmatrix}$$



Video 55

Matrix Analysis 16

Tuesday, August 6, 2024 10:29

نشست تکیه گاهی در تحلیل ماتریسی

اگر تکیه گاه های سازه نشست و دوران داشته باشند، تعیین مجهولات تکیه گاهی δ_s به صورت زیر انجام می شود.

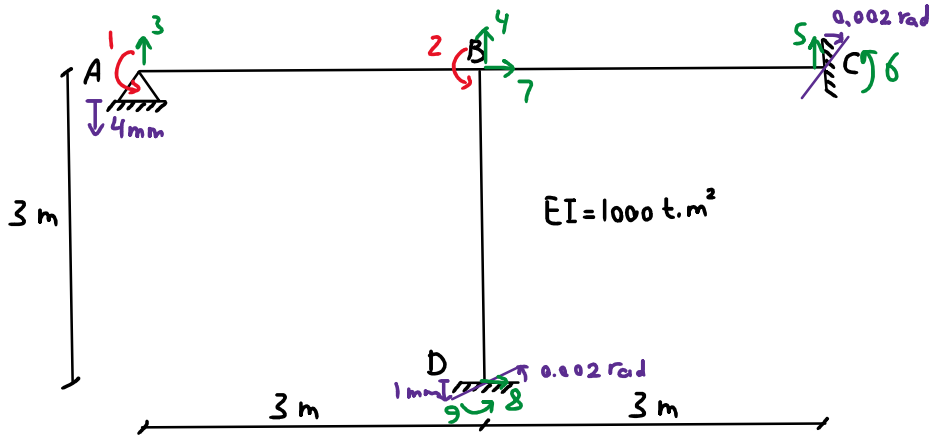
$$\begin{Bmatrix} P_f \\ P_s \end{Bmatrix} = \left[\begin{array}{c|c} k_{ff} & k_{fs} \\ \hline k_{sf} & k_{ss} \end{array}\right] \begin{Bmatrix} \delta_f \\ \delta_s \end{Bmatrix}$$

$$k_{ff}\delta_f + k_{fs}\delta_s = P_f$$

$$k_{ff}\delta_f = \underbrace{P_f - k_{fs}\delta_s}_{P_f^*}$$

* درحقیقت بردار بارگذاری به صورت فوق اصلاح می شود که در آن P_f ناشی از بار خارجی و $(-k_{sf}\delta_s)$ بار ناشی از نشست تکیه گاهی است.

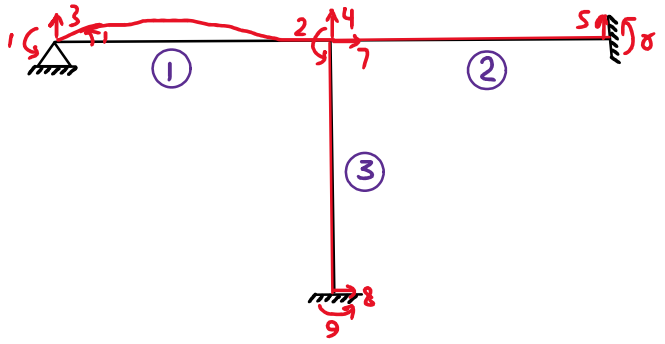
مثال: سازه شکل زیر را به روش ماتریسی تحلیل کنید. (از تغییر شکل های محوری صرف نظر کنید.)

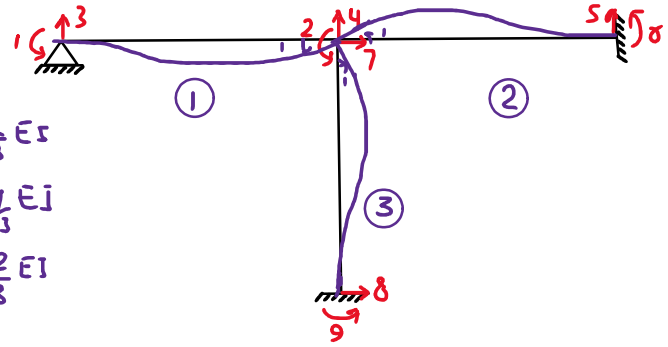


① نوشتن k_{ff} و k_{sf}

$$\begin{bmatrix} \frac{4EI_1}{L_1} & \frac{2EI_1}{L_1} \\ \frac{2EI_1}{L_1} & \frac{4EI_1}{L_1}+\frac{4EI_2}{L_2}+\frac{4EI_3}{L_3} \end{bmatrix} \quad (k_{ff})$$

$$\begin{bmatrix} \frac{6EI_1}{L_1^2} & \frac{6EI_1}{L_1^2} \\ -\frac{6EI_1}{L_1^2} & -\frac{6EI_1}{L_1^2}+\frac{6EI_2}{L_2^2} \\ 0 & -\frac{6EI_2}{L_2^2} \\ 0 & \frac{2EI_2}{L_2} \\ 0 & \frac{6EI_3}{L_3^2} \\ 0 & -\frac{6EI_3}{L_3^2} \\ 0 & \frac{2EI_3}{L_3} \end{bmatrix} \quad (k_{sf})$$





$\frac{2EI}{L} = \frac{2}{3}EI$

$\frac{4EI}{L} = \frac{4}{3}EI$

$\frac{6EI}{L^2} = \frac{2}{3}EI$

② تعیین بردار بارگذاری P_f^*

$$P_f^* = P_f^{\circ} - k_{fs}\delta_s = -EI \begin{bmatrix} ③ & ④ & ⑤ & ⑥ & ⑦ & ⑧ & ⑨ \\ \frac{2}{3} & -\frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & -\frac{2}{3} & \frac{2}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{Bmatrix} -0.004 \\ -0.001 \\ 0 \\ -0.002 \\ 0 \\ 0 \\ 0.002 \end{Bmatrix} \begin{matrix} ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \\ ⑧ \\ ⑨ \end{matrix} = \begin{Bmatrix} 2 \\ 8/3 \end{Bmatrix}$$

$$\boxed{P_f^* = \begin{Bmatrix} 2 \\ 8/3 \end{Bmatrix}}$$

③ محاسبه δ_f با حل معادله $k_{ff}\delta_f = P_f^*$

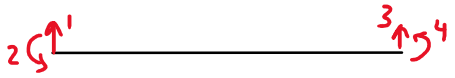
$$k_{ff}\delta_f = P_f^*$$

$$EI \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 4 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 8/3 \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 56/44 \\ 5/11 \end{Bmatrix}}$$

④ محاسبه نیروی اعضا

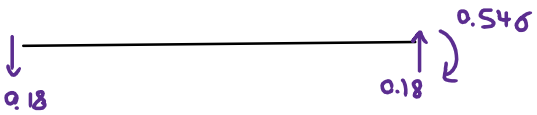
$$P_L = k_L \delta_L + FER$$

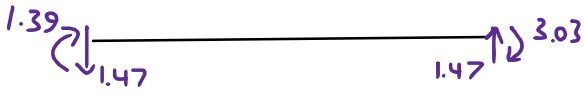
$$k_L = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} = EI \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix}$$

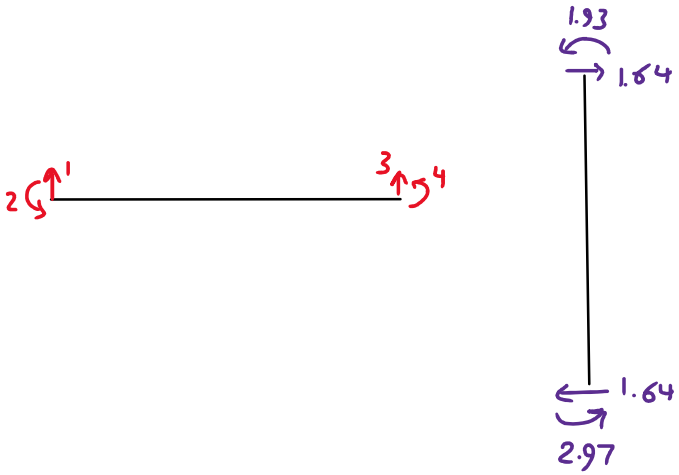


$$P_{L,AB} = EI \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix} \begin{Bmatrix} \delta_3 = -4\times10^{-3} \\ \delta_1 = \frac{56}{44}\times10^{-3} \\ \delta_4 = -1\times10^{-3} \\ \delta_2 = \frac{5}{11}\times10^{-3} \end{Bmatrix} = \begin{Bmatrix} -0.18 \\ 0 \\ 0.18 \\ -0.546 \end{Bmatrix}$$

$$\delta_{L,BC} = \begin{Bmatrix} \delta_4 \\ \delta_2 \\ \delta_5 \\ \delta_6 \end{Bmatrix} \begin{matrix} -0.001 \\ \frac{5}{11}\times10^{-3} \\ 0 \\ -0.002 \end{matrix} \qquad \delta_{L,CD} = \begin{Bmatrix} -\delta_8 \\ \delta_9 \\ -\delta_7 \\ \delta_2 \end{Bmatrix} \begin{matrix} 0 \\ 0.002 \\ 0 \\ \frac{5}{11}\times10^{-3} \end{matrix}$$







Video 56

Matrix Analysis 17

Tuesday, August 6, 2024 10:29

تغییر دما و نقص عضو در تحلیل ماتریسی

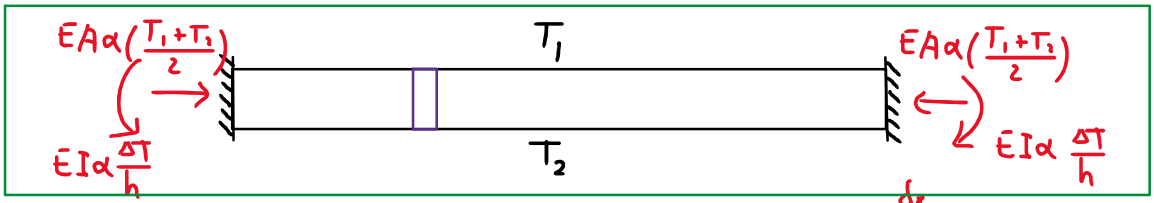
در مسائل تغییر دما و نقص عضو، تنها تفاوت در تعیین بردار بارگذاری است. چنانکه قبلاً در مورد باروی اعضا اشاره شد، عکس العمل یزودهای گیرداری (-FER) به صورت بارگذاری به گره‌های سازه وارد می‌شود.

$$P_f = P_J - FER$$ ← ناشی از باروی عضو و تغییر دما و نقص عضو

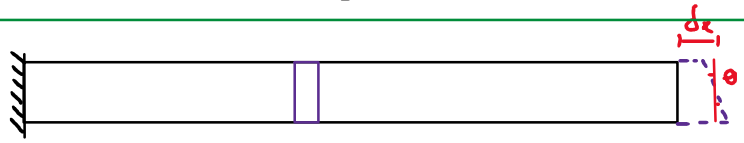
$$k_{ff}\,\delta_f = P_f$$

در مورد تغییر دما و نقص عضو نیز نیروهای انتهایی عضو دوسرگیردار به صورت زیر محاسبه می‌شود:

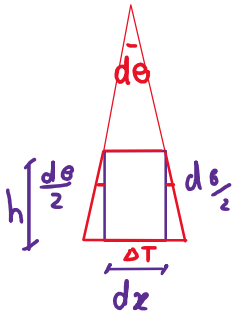
$\Delta T > 0$



$\delta_x = 0$
$\theta = 0$



$$\delta_x = 0 \rightarrow \alpha L\left(\frac{T_1+T_2}{2}\right) - \frac{PL}{EA} = 0 \rightarrow \boxed{P = EA\alpha\left(\frac{T_1+T_2}{2}\right)}$$



$$\frac{d\theta}{2} = \frac{\alpha\, dx\, \Delta T/2}{h}$$

$$\theta = 0 \rightarrow \theta_1 - \theta_2 = 0 \rightarrow$$

=

+

$$\begin{cases} \overbrace{1\times\theta_1 + 1\times\theta_1}^{2\theta_1} = \int m\,d\theta = \int_0^L 1\times\frac{\alpha\Delta T}{h}\,dx = \frac{\alpha L \Delta T}{h} \\ 2\theta_2 = \frac{ML}{EI} \end{cases} \rightarrow$$

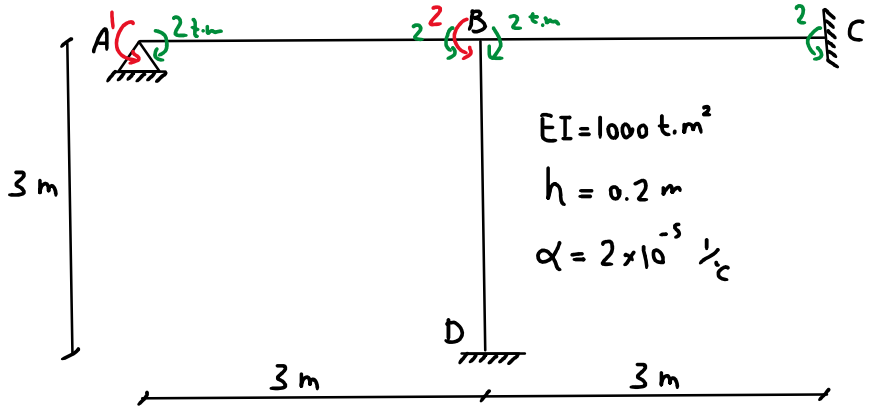
$$\frac{\alpha L \Delta T}{h} = \frac{ML}{EI} \rightarrow \boxed{M = EI\alpha\frac{\Delta T}{h}}$$

$M = \frac{2EI}{L}\theta_2$ ← کنش عضو متقارن

مثال: تیرهای AB و BC، 10°c افزایش دما در تار بالا و 30°c افزایش دما در تار پایین را تجربه می‌کنند. نیروهای داخلی اعضا را به دست آورید. (از تغییر شکل‌های محوری صرف نظر کنید.)

$\Delta T = +20$

$$EI\alpha\frac{\Delta T}{h} = 1000\times 2\times 10^{-5}\times\frac{20}{0.2} = 2\ t.m$$

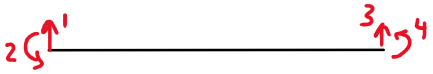


$$P_f = \begin{Bmatrix} -2 \\ 0 \end{Bmatrix}$$

$$k_{ff}\,\delta_f = P_f \longrightarrow EI \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 4 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -2 \\ 0 \end{Bmatrix} \longrightarrow \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -1.64\times10^{-3} \\ 2.73\times10^{-4} \end{Bmatrix}$$

$$P_L = k_L \delta_L + FER$$

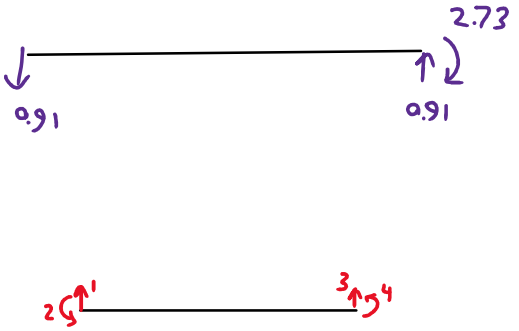
$$k_L = \begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} = EI \begin{array}{c} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix} \\ \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix} \end{array} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix}$$

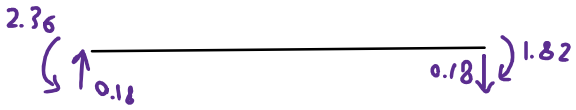


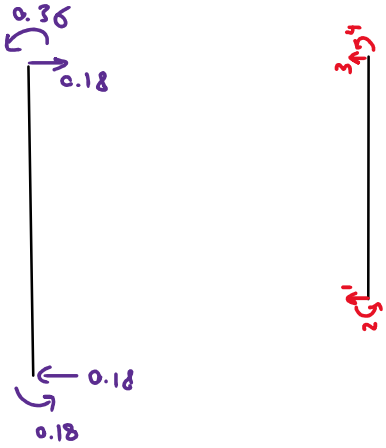
$$P_{L,AB} = EI \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix} \begin{Bmatrix} 0 \\ -1.64\times10^{-3} \\ 0 \\ 2.73\times10^{-4} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{Bmatrix} = \begin{Bmatrix} -0.91 \\ 0 \\ 0.91 \\ -2.73 \end{Bmatrix}$$

$$P_{L,BC} = EI \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix} \begin{Bmatrix} 0 \\ 2.73\times10^{-4} \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{Bmatrix} = \begin{Bmatrix} 0.18 \\ 2.36 \\ -0.18 \\ -1.82 \end{Bmatrix}$$

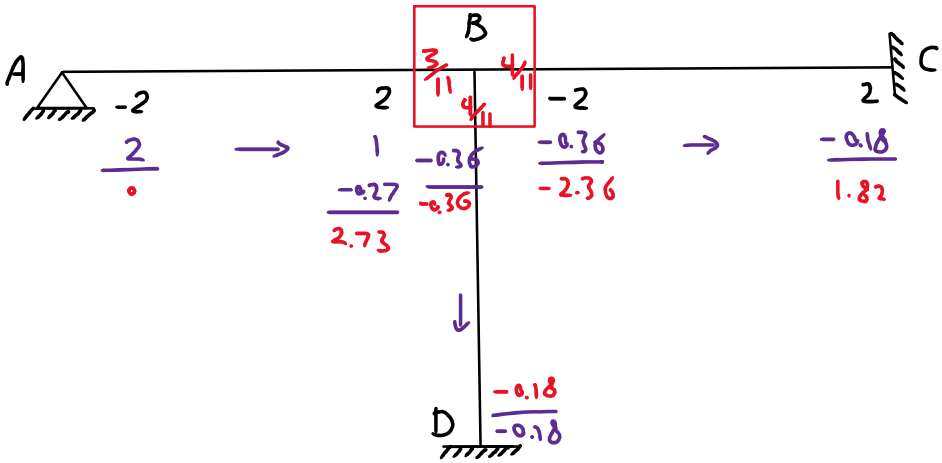
$$P_{L,CD} = EI \begin{bmatrix} 4/9 & 2/3 & -4/9 & 2/3 \\ 2/3 & 4/3 & -2/3 & 2/3 \\ -4/9 & -2/3 & 4/9 & -2/3 \\ 2/3 & 2/3 & -2/3 & 4/3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2.73\times10^{-4} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.18 \\ 0.18 \\ -0.18 \\ 0.36 \end{Bmatrix}$$







مقایسه با روش کرانی



Video 57

Matrix Analysis 18

Tuesday, August 6, 2024 10:30

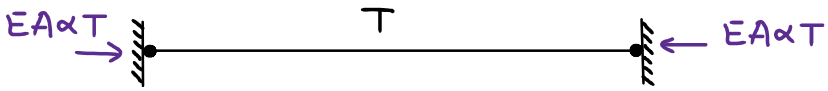
تغییر دما و نقص عضو در تحلیل ماتریسی

$$K_{ff}\,\delta_f = P_f$$

$$P_f = P_J - FER$$ ← ناشی از بارروی عضو و تغییر دما و نقص عضو

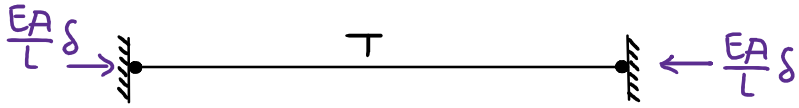
تغییر دما در عضو خرپایی

$\alpha_k T = \frac{PL}{EA}$

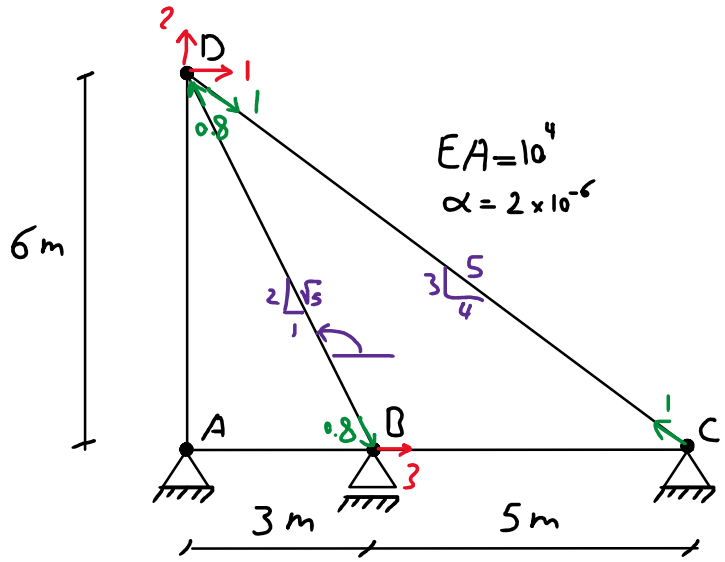


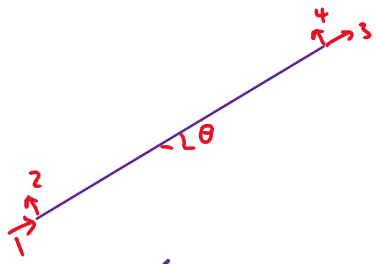
نقص عضو در عضو خرپایی

خطای مونتاژ $\delta = \frac{PL}{EA}$



مثال: اگر عضو BD، دچار 40°c افزایش دما شود و عضو CD در اثر خطای مونتاژ 1mm کوتاه باشد، نیروی اعضای BD و CD را به دست آورید.





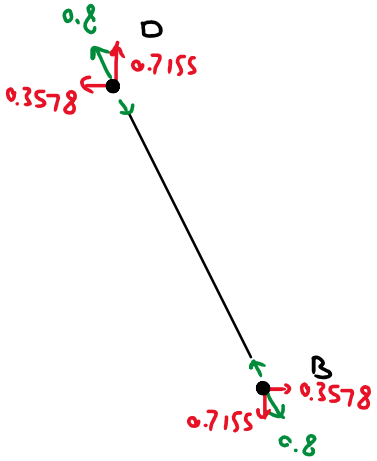
$$P_f = \begin{Bmatrix} 0.4422 \\ 0.1155 \\ 0.3578 \end{Bmatrix}$$

① تعیین بردار بارگذاری P_f

$BD:\ EA\alpha T = 10^4 \times 2\times10^{-6} \times 40 = 0.8\ ton$

$c = \frac{-1}{\sqrt{5}}\ ,\ s = \frac{2}{\sqrt{5}}$

$$P_{G,BD} = R^T P_l = \begin{bmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{bmatrix} \begin{Bmatrix} -0.8 \\ 0 \\ 0.8 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.3578 \\ -0.7155 \\ -0.3578 \\ 0.7155 \end{Bmatrix}$$



$CD:\ \frac{EA}{L}\delta = \frac{10^4}{10} \times 0.001 = 1\ ton$

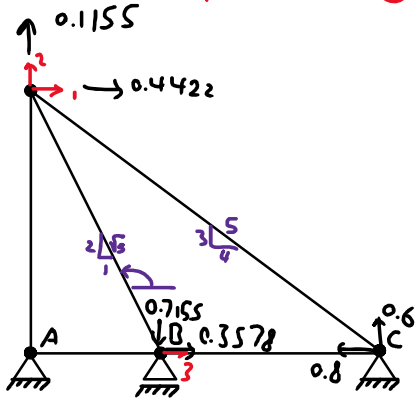
$C=-0.8\ ,\ S=0.6$

$$P_{G,CD} = R^T P_L = \begin{bmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.8 \\ 0.6 \\ 0.8 \\ -0.6 \end{Bmatrix}$$

② محاسبه مجهولات تغییرمکانی δ_f

$$k_{ff}\,\delta_f = P_f$$

$$\begin{bmatrix} 938 & -1076 & -298 \\ -1076 & 3220 & 596 \\ -298 & 596 & 5631 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} 0.4422 \\ 0.1155 \\ 0.3578 \end{Bmatrix} \longrightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} 8.44\times10^{-4} \\ 3.04\times10^{-4} \\ 7.61\times10^{-5} \end{Bmatrix}}$$



③ محاسبه نیروی اعضا

$$P_L = k_L R \delta_G + FER$$

$C=\frac{-1}{\sqrt{5}}\ ,\ S=\frac{2}{\sqrt{5}}$

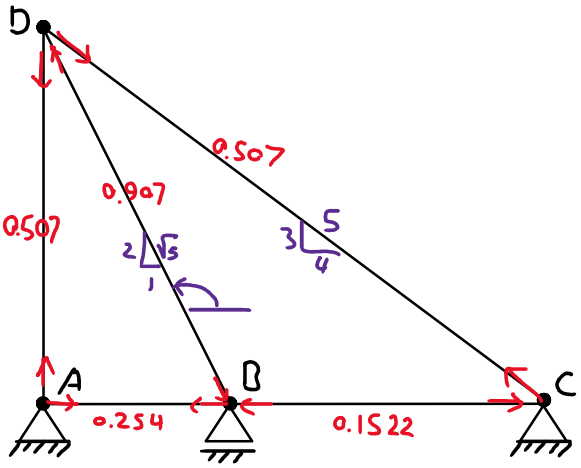
$$P_{L,BD} = k_L R \delta_G + FER = \frac{10^4}{3\sqrt{5}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} \begin{Bmatrix} \delta_3 \\ 0 \\ \delta_1 \\ \delta_2 \end{Bmatrix} + \begin{Bmatrix} 0.8 \\ 0 \\ -0.8 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.907 \\ 0 \\ -0.907 \\ 0 \end{Bmatrix}$$

$C=-0.8\ ,\ S=0.6$

$$P_{L,CD} = k_L R \delta_G + FER = 1000 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -0.8 & 0.6 & 0 & 0 \\ -0.6 & -0.8 & 0 & 0 \\ 0 & 0 & -0.8 & 0.6 \\ 0 & 0 & -0.6 & -0.8 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \delta_1 \\ \delta_2 \end{Bmatrix} + \begin{Bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.507 \\ 0 \\ 0.507 \\ 0 \end{Bmatrix}$$

$C=0\quad S=1$

$$P_{L,AD} = k_L \frac{\delta_L}{R} \delta_G + FER = \frac{10^4}{6} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \delta_1 \\ \delta_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.507 \\ 0 \\ 0.507 \\ 0 \end{Bmatrix}$$


D
0.507
0.907
0.507
2
√5
1
3
5
4
A
B
C
0.254
0.1522

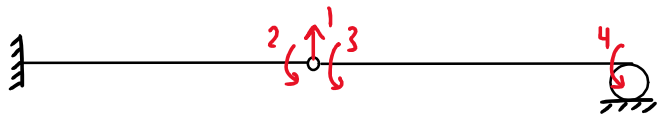
Video 58

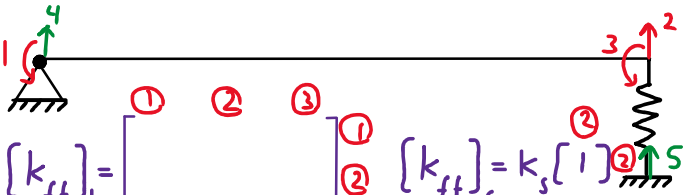
Matrix Analysis 19

Tuesday, August 6, 2024 10:30

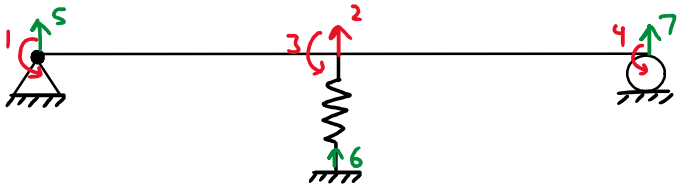
تحلیل ماتریسی سازه‌ها

حالت‌های دیگر:
- تغییر مقطع
- مفصل داخلی
- تکیه‌گاه ارتجاعی (فنری)
- فنر داخلی

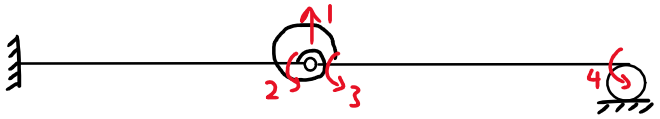




$$[k_{ff}]_b = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{matrix}①\\②\\③\end{matrix} \qquad [k_{ff}]_s = k_s [1]$$



$$k = k_s \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix}① \\ ③\end{matrix} \quad \begin{matrix}① & ② \end{matrix}$$



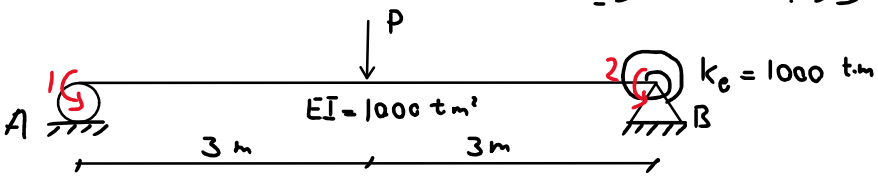
$$k = k_\theta \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix}① \\ ③\end{matrix} \quad \begin{matrix}① & ② \end{matrix}$$

مثال: نیروی فنر را به دست آورید.

روش توزیع لنگر

FEM DF

$$M_s = \frac{3}{2}\frac{PL}{8} \cdot \frac{2}{3} = \frac{3}{4}P$$



① نوشتن ماتریس k_{ff} اعضا

$$[k_{ff}]_{AB} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \begin{matrix}① & ② \\ \end{matrix} \begin{bmatrix} 666.7 & 333.3 \\ 333.3 & 666.7 \end{bmatrix} \begin{matrix}① \\ ②\end{matrix}$$

$$[k_{ff}]_s = \begin{matrix}② \\ \end{matrix} \begin{bmatrix} 1000 \end{bmatrix} ②$$

② نوشتن k_{ff} سازه با سرهم‌بندی

$$k_{ff} = \begin{matrix}① & ② \\ \end{matrix} \begin{bmatrix} 666.7 & 333.3 \\ 333.3 & 1666.7 \end{bmatrix} \begin{matrix}① \\ ②\end{matrix}$$

③ محاسبه مجهولات تغییرمکان δ_f

$$k_{ff}\,\delta_f = P_f$$

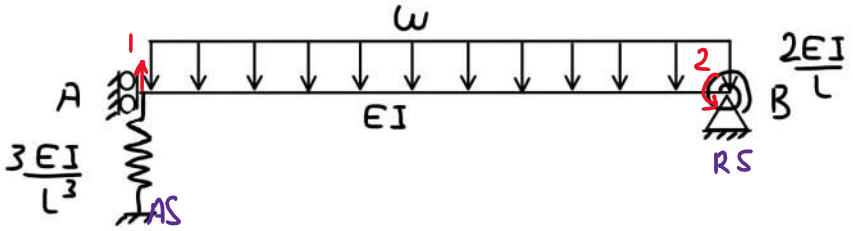
$$\begin{bmatrix} 666.7 & 333.3 \\ 333.3 & 1666.7 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{3}{4}P \\ \frac{3}{4}P \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -1.5\times10^{-3}P \\ 7.5\times10^{-4}P \end{Bmatrix}}$$

① محاسبه نیروی اعضا

$$P_L = k_L \delta_L \rightarrow P_L = \overset{1000}{k_\theta} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 7.5\times10^{-4}P \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.75P \\ -0.75P \end{Bmatrix}$$

0.75P ↺ ↙0.75P

مثال: نیروی میله‌ها را به دست آورید.



① نوشتن k_{ff} اعضا

$$[k_{ff}]_{AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix} \quad [k_{ff}]_{AS} = \left[\frac{3EI}{L^3}\right] ① \quad [k_{ff}]_{RS} = \left[\frac{2EI}{L}\right] ②$$

② نوشتن k_{ff} سازه با سرهم بندی

$$k_{ff} = \begin{bmatrix} \frac{15EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{6EI}{L} \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

③ محاسبه مجهولات تغییر مکان δ_f

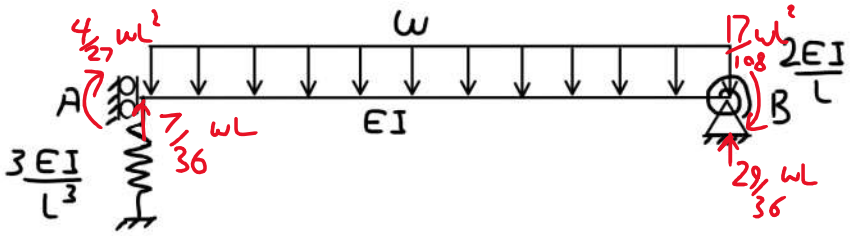
$$k_{ff}\delta_f = P_f$$

$$\begin{bmatrix} \frac{15EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{6EI}{L} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{wL}{2} \\ \frac{wL^2}{12} \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{Bmatrix} \frac{-7}{108}\frac{wL^4}{EI} \\ \frac{17}{216}\frac{wL^3}{EI} \end{Bmatrix}}$$

① محاسبه نیروی اعضا

① محاسبه نیروها اعضا

$$P_L = k_L \delta_L + FER = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} -\frac{7}{108}\frac{wL^4}{EI} \\ 0 \\ 0 \\ \frac{17}{216}\frac{wL^3}{EI} \end{Bmatrix} + \begin{Bmatrix} \frac{wL}{2} \\ \frac{wL^2}{12} \\ \frac{wL}{2} \\ -\frac{wL^2}{12} \end{Bmatrix} = \begin{Bmatrix} \frac{7}{36}wL \\ -\frac{4}{27}wL^2 \\ \frac{29}{36}wL \\ -\frac{17}{108}wL \end{Bmatrix}$$



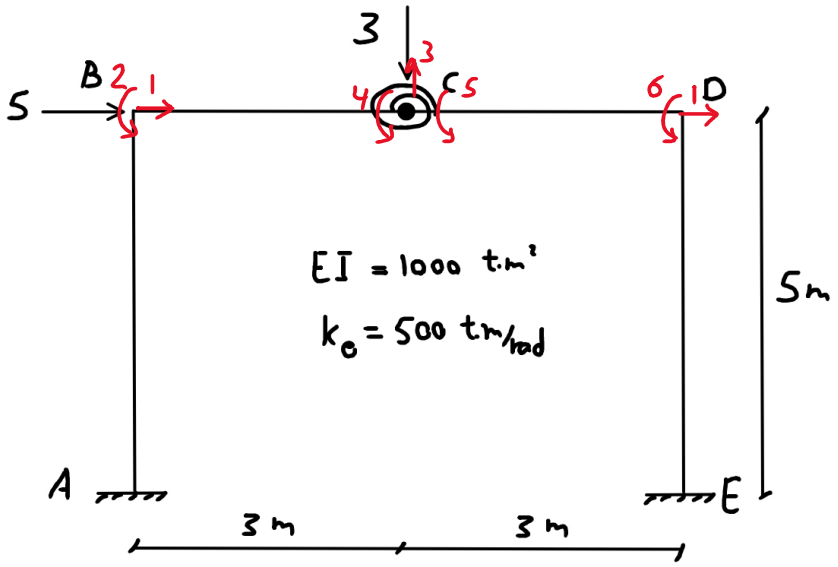
Video 59

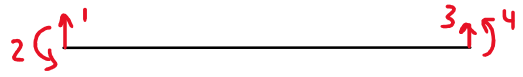
Matrix Analysis 20

Tuesday, August 6, 2024 10:30

تحلیل ماتریسی سازه‌ها

مثال : قاب شکل زیر را به روش ماتریسی تحلیل کنید. (از تغییر شکل محوری اعضا صرف نظر کنید.)





$$k_l = \begin{array}{c} \begin{matrix} ① & ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix} \end{array}$$

① ماتریس k_{ff} اعضا

ستون AB, DE : L = 5 m , EI = 1000

$$[k_{ff}]_{AB} = \begin{array}{c} \begin{matrix} ① & ② \end{matrix} \\ \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix} \end{array}$$

$$[k_{ff}]_{DE} = \begin{array}{c} \begin{matrix} ① & ⑥ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ① \\ ⑥ \end{matrix} \end{array}$$

فنر : k_θ = 500

$$[k_{ff}]_S = \begin{array}{c} \begin{matrix} ④ & ⑤ \end{matrix} \\ \begin{bmatrix} k_\theta & -k_\theta \\ -k_\theta & k_\theta \end{bmatrix} \begin{matrix} ④ \\ ⑤ \end{matrix} \end{array}$$

تیر BC, CD : L = 3 m , EI = 1000

$$[k_{ff}]_{BC} = \begin{array}{c} \begin{matrix} ② & ③ & ④ \end{matrix} \\ \begin{bmatrix} \frac{4EI}{l} & \frac{6EI}{l^2} & \frac{2EI}{l} \\ \frac{6EI}{l^2} & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{2EI}{l} & \frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ② \\ ③ \\ ④ \end{matrix} \end{array}$$

$$[k_{ff}]_{CD} = \begin{array}{c} \begin{matrix} ③ & ⑤ & ⑥ \end{matrix} \\ \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & \frac{2EI}{l} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & \frac{4EI}{l} \end{bmatrix} \begin{matrix} ③ \\ ⑤ \\ ⑥ \end{matrix} \end{array}$$

② ماتریس k_{ff} سازه با سرهم بندی به کمک MATLAB

$$k_{ff} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 192 & 240 & 0 & 0 & 0 & 240 \\ 240 & 2.1333e+03 & -666.6667 & 666.6667 & 0 & 0 \\ 0 & -666.6667 & 888.8889 & -666.6667 & 666.6667 & 666.6667 \\ 0 & 666.6667 & -666.6667 & 1.8333e+03 & -500 & 0 \\ 0 & 0 & 666.6667 & -500 & 1.8333e+03 & 666.6667 \\ 240 & 0 & 666.6667 & 0 & 666.6667 & 2.1333e+03 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \end{array}$$

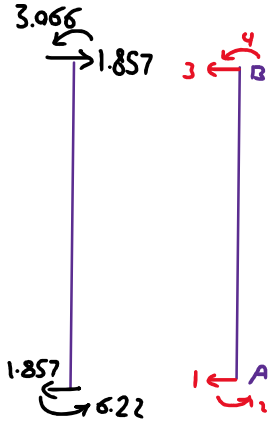
$$P_f = \begin{Bmatrix} 5 \\ 0 \\ -3 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$P_L = k_L \delta_L + \cancel{FER}$

③ حل $k_{ff}\delta_f = P_f$ و برزی اعضا به کمک MATLAB

$$\delta_f = \begin{Bmatrix} 0.0391 \\ -0.0079 \\ -0.0109 \\ 2.47\times10^{-4} \\ 0.005 \\ -0.0025 \end{Bmatrix}$$

$$P_{AB} = \begin{Bmatrix} 1.857 \\ 6.22 \\ -1.857 \\ 3.066 \end{Bmatrix}$$

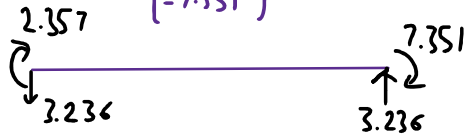


$$P_{DE} = \begin{Bmatrix} 3.1429 \\ 8.363 \\ -3.1429 \\ 7.351 \end{Bmatrix}$$

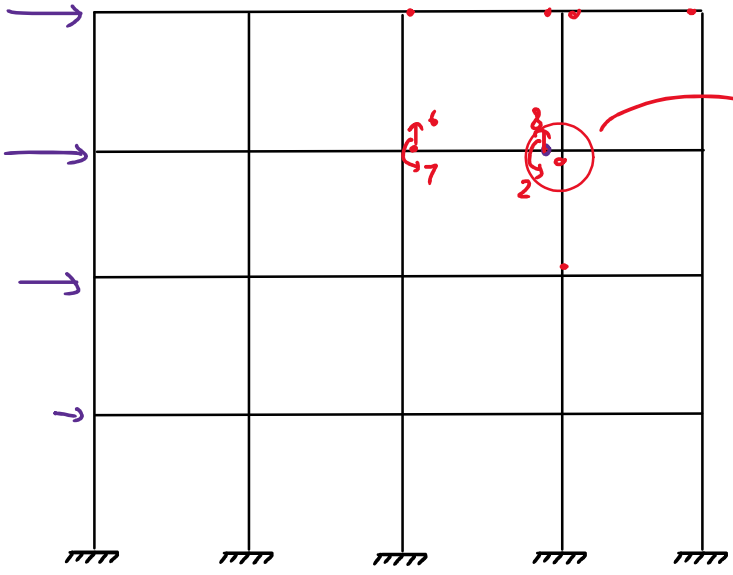
$$P_s = \begin{Bmatrix} -2.357 \\ 2.357 \end{Bmatrix}$$

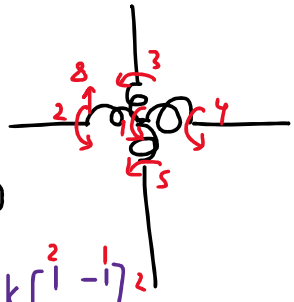
$$P_{BC} = \begin{Bmatrix} -0.236 \\ -3.066 \\ 0.236 \\ 2.357 \end{Bmatrix}$$

$$P_{CD} = \begin{Bmatrix} -3.236 \\ -2.357 \\ 3.236 \\ -7.351 \end{Bmatrix}$$

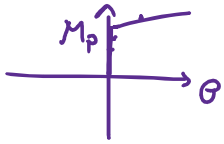


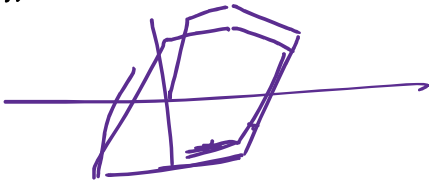
$k_{ff}\delta_f = \Delta P_f$





$$k_{ff} = k_0 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$





Video 60

Matrix Analysis 21

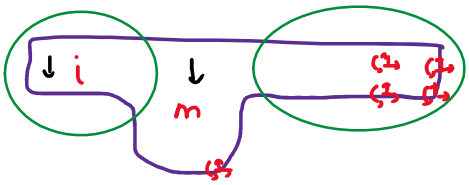
Sunday, September 15, 2024 13:12

تراکم استاتیکی ماتریس سختی (Static Condensation)

ماتریس سختی سازه را می‌توان با حذف یک سری از درجات آزادی و لحاظ کردن اثر آنها در سختی درجات آزادی دیگر متراکم نمود.

δ_m : درجات آزادی اصلی

δ_i : سایر درجات آزادی



$$\begin{bmatrix} k_{mm} & k_{mi} \\ k_{im} & k_{ii} \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_i \end{Bmatrix} = \begin{Bmatrix} P_m \\ P_i \end{Bmatrix} \rightarrow \begin{cases} (1)\ k_{mm}\delta_m + k_{mi}\delta_i = P_m \\ (2)\ k_{im}\delta_m + k_{ii}\delta_i = P_i \rightarrow \delta_i = k_{ii}^{-1}(P_i - k_{im}\delta_m) \end{cases}$$

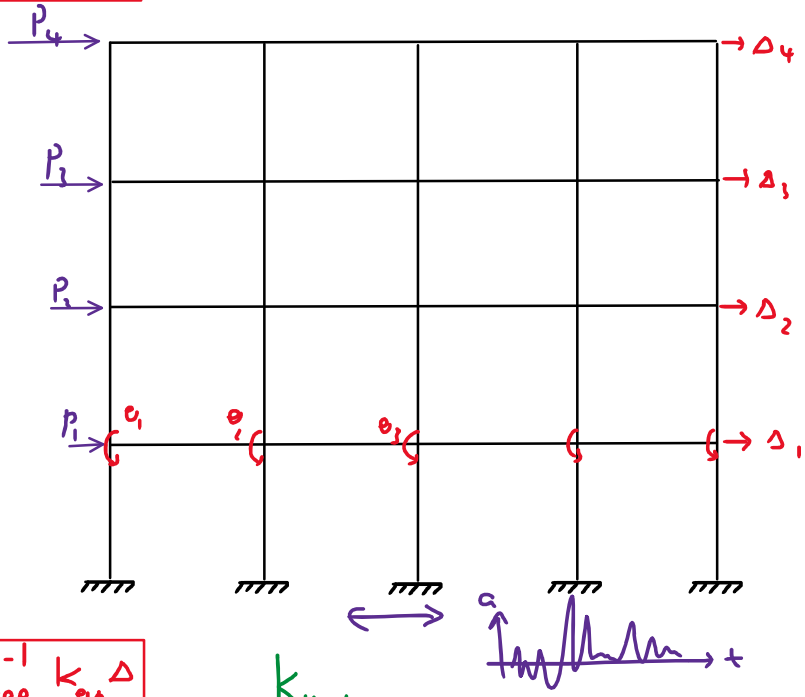
جایگذاری δ_i در معادله (1):

$$k_{mm}\delta_m + k_{mi}k_{ii}^{-1}(P_i - k_{im}\delta_m) = P_m$$

$$\underbrace{\left(k_{mm} - k_{mi}k_{ii}^{-1}k_{im}\right)}_{k}\delta_m = \underbrace{P_m - k_{mi}k_{ii}^{-1}P_i}_{P}$$

با صرف نظر از تغییر شکل محوری اعضا

δ_f : 24 ⟨ 4Δ, 20θ



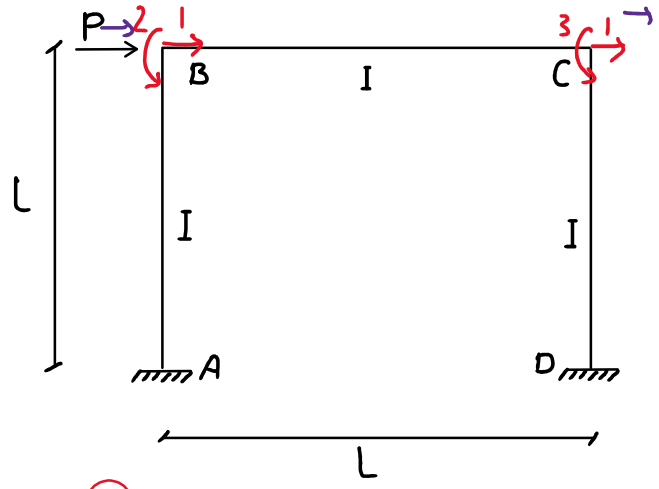
$$\begin{bmatrix} k_{\Delta\Delta}\ (4\times4) & k_{\Delta\theta}\ (4\times20) \\ k_{\theta\Delta}\ (20\times4) & k_{\theta\theta}\ (20\times20) \end{bmatrix} \begin{Bmatrix} \Delta\ (4\times1) \\ \theta\ (20\times1) \end{Bmatrix} = \begin{Bmatrix} P\ (4\times1) \\ 0\ (20\times1) \end{Bmatrix}$$

(2) $k_{\theta\Delta}\Delta + k_{\theta\theta}\theta = 0 \rightarrow \theta = -k_{\theta\theta}^{-1}k_{\theta\Delta}\Delta$

(1) $k_{\Delta\Delta}\Delta + k_{\Delta\theta}\left(-k_{\theta\theta}^{-1}k_{\theta\Delta}\right)\Delta = P \rightarrow \underbrace{\left(k_{\Delta\Delta} - k_{\Delta\theta}k_{\theta\theta}^{-1}k_{\theta\Delta}\right)}_{k_{4\times4}}\Delta = P$

(4×4 , 4×20 , 20×20 , 20×4)

مثال : ماتریس سختی قاب شکل زیر را روی درجه آزادی ۵ متراکم کنید.



$$k_{ff} = EI \begin{bmatrix} \frac{24}{L^3} & \frac{6}{L^2} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{8}{L} & \frac{2}{L} \\ \frac{6}{L^2} & \frac{2}{L} & \frac{8}{L} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$k_{\Delta\Delta}$, $k_{\Delta\theta}$, $k_{\theta\Delta}$, $k_{\theta\theta}$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$k_{ff,Con} = k_{\Delta\Delta} - k_{\Delta\theta} k_{\theta\theta}^{-1} k_{\theta\Delta} = \frac{24EI}{L^3} - \left\{ \frac{6EI}{L^2} \quad \frac{6EI}{L^2} \right\} \frac{1}{\left(\frac{EI}{L}\right)^2 (8^2 - 2^2)} \begin{bmatrix} \frac{8EI}{L} & -\frac{2EI}{L} \\ -\frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix} \left\{ \begin{matrix} \frac{6EI}{L^2} \\ \frac{6EI}{L^2} \end{matrix} \right\}$$

$$= \frac{24EI}{L^3} - \left(\frac{6EI}{L^2}\right)\left(\frac{1}{60\left(\frac{EI}{L}\right)^2}\right)\left(\frac{EI}{L}\right)\left(\frac{6EI}{L^2}\right) \{1 \quad 1\} \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \begin{Bmatrix} 6 \\ 6 \end{Bmatrix}$$

$0.6 \frac{EI}{L^3}$, 10

$$k_{ff,Con} = \frac{24EI}{L^3} - 0.6\frac{EI}{L^3} \{1 \quad 1\} \begin{Bmatrix} 6 \\ 6 \end{Bmatrix} = (24 - 0.6 \times 12)\frac{EI}{L^3} = \boxed{16.8\frac{EI}{L^3}} \qquad \frac{16}{10} = \frac{84}{5}$$

$$\boxed{P = k\Delta \;\; ! \;\; P = \frac{84}{5}\frac{EI}{L^3}\Delta}$$

$$\theta = -k_{\theta\theta}^{-1} k_{\theta\Delta} \Delta = -\frac{1}{60\left(\frac{EI}{L}\right)^2} \begin{bmatrix} \frac{8EI}{L} & -\frac{2EI}{L} \\ -\frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix} \begin{Bmatrix} \frac{6EI}{L^2} \\ \frac{6EI}{L^2} \end{Bmatrix} \Delta$$

$$\theta = \frac{-1}{60\left(\frac{EI}{L}\right)^2}\left(\frac{EI}{L}\right)\left(\frac{6EI}{L^2}\right)\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\Delta = -\frac{6}{10}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\frac{\Delta}{L}$$

$$\boxed{\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{6}{10}\frac{\Delta}{L} \\ -\frac{6}{10}\frac{\Delta}{L} \end{Bmatrix}}$$

$$P_{L,AB} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}\begin{Bmatrix} a \\ o \\ -\Delta \\ -\frac{6}{10}\frac{\Delta}{L} \end{Bmatrix} + FER^{o} = \begin{Bmatrix} 8.4\frac{EI}{L^3}\Delta \\ 4.8\frac{EI}{L^2}\Delta \\ -8.4\frac{EI}{L^3}\Delta \\ 3.6\frac{EI}{L^2}\Delta \end{Bmatrix} = \begin{Bmatrix} P/2 \\ \frac{2}{7}PL \\ -P/2 \\ \frac{3}{14}PL \end{Bmatrix}$$

$\frac{5}{84}\frac{PL^3}{EI}$